Constrained tetrahedral models and update algorithms for topographic data

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Introduction

Most current topographic products are limited to representing the real world in only two dimensions. As the real world exists of three dimensional objects, which are becoming more and more complex due to increasing multiple land use, accurate topographic models have to cope with the third dimension. The overall goal of this research is to extend current topographic modeling into the third dimension. Applications of 3D modeling are not limited to the terrain surface and objects built directly on top or beneath it, as geological features and air traffic or telecommunication corridors can be modeled too.

Most initiatives on developing 3D GIS focus on supporting visualization, often in Virtual Reality-like environments. One of the objectives of this 3D modeling research is to enable 3D analysis as well, as this traditional GIS-strength lacks until now in most 3D GIS approaches. Another important assumption within this research follows from the required wide variety of applications of topographic data. As topography is ranked high in the spatial data infrastructure hierarchy, one cannot optimize the data model for one specific purpose. One has to be able to serve the complete range of user applications, regardless whether these applications require for instance optimal visualization capabilities or optimal analytical capabilities.

3D Topography

The development of a 3D topographic data model is both demand and supply driven. 3D modeling is not only required for accurate modeling of increasingly complex real world objects, as is the case in multiple land use areas as illustrated in Figure 1.

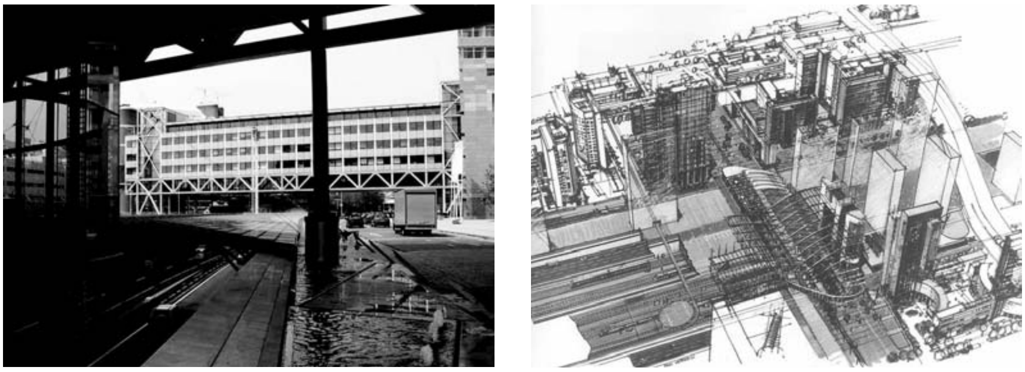

Figure 1. Multiple land use: offices at Utrechtse baan Den Haag (left) and plans for Amsterdam WTC (right).

Other important aspects in the increasing need for 3D modeling are the rising awareness of the importance of sustainable urban environments (requiring 3D planning and 3D analysis) and the need for better data for emergency services and disaster response. Disaster management (both natural and non-natural disasters) gained a lot of attention since 9/11, the Christmas 2004 tsunami in Asia and the flooding after hurricane Kathrina in New Orleans.

On the demand side the availability of the AHN (Actueel Hoogtebestand Nederland), a high density height point data set obtained with airborne laser altimetry, is an important factor in the development of 3D models. Figure 2 shows a part of the AHN in Zuid-Limburg. As the AHN contains on average one height point for every $16m^2$ the spatial resolution is high enough to enable extraction of for instance building heights. Building extraction can even be automated when laser scan data of higher resolution is available. Due to current developments in laser scanning technology laser scanners can measure up to 50 times more points per second compared to the mid-nineties, when the AHN resolution was chosen (Vosselman, 2005). Also terrestrial laser scanning offers the possibility to model objects in 3D with more detail, even indoor topography can be introduced.

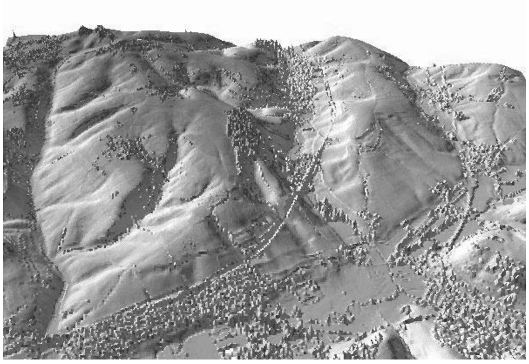

Figure 2. AHN offers high resolution height data.

3D modeling

Selection of 3D primitive

In 3D modeling one needs a 3D primitive (a volume) beside points, lines and faces to represent 3D objects accurately. Earlier research proposed amongst others using simplexes (point, line, triangle, tetrahedron) (Carlson 1987), points, lines, surfaces and bodies (3D Formal Data Structure (FDS)) (Molenaar 1990, Molenaar 1992), combining Constructive Solid Geometry (CSG) and a B-rep. (de Cambray 1993) and integrating a 2.5D Triangulated Irregular Network (TIN) with 3D FDS (Pilouk 1996). In applications polyhedrons are often used as 3D primitive (Zlatanova 2000, Stoter 2004).

In this research simplexes are the primitives of choice. A great advantage of using these simplexes is the well-defined character of the mutual relationships: a kD simplex is bounded by k+1 (k-1)D-simplexes (Pilouk 1996). This means that for instance a 2D simplex (a triangle) is bounded by three 1D simplexes (edges) and a 3D simplex (tetrahedron) is bounded by four 2D simplexes (triangles). The second important advantage of simplexes is the flatness of the faces, which enables one to describe a face using only three points. The third advantage is that every simplex, regardless its dimension, is convex, thus making convexity testing unnecessary. This quality simplifies point-in-polygon test significantly. The price for this comes with increased modeling complexity. Compared to for instance using polyhedrons as 3D primitive it will be clear that there exists a 1:1 relationship between a 3D feature (for instance a building) and its representation (the polyhedron), but that there will be a 1:n relationship between this 3D feature and its tetrahedrons. However, as long as one is able to hide this complexity from the average user, the advantages will overcome this drawback. To further illustrate the strength of using well-defined primitives, consider a real estate tax application that determines the tax assessment based on the volume of the building. In order to automate this process, a formula for determining volumes is required. Designing a formula capable of determining a polyhedron's volume is more complex due to the unlimited variation in shape. Contrarily, implementing a formula for the volume of a tetrahedron is straightforward, it only has to be applied several times as a building will be represented as a set of tetrahedrons. This repetition is however exactly what computers are good for. As a result the TEN (Tetrahedronized Irregular Network) is selected as data structure.

Modeling concept

The development of the current modeling concept is described in (Penninga, 2005). The modeling concept is based on two basic observations:

- The ISO 19101 Geographic information - Reference model defines a feature as an 'abstraction of real world phenomena'. These real world phenomena have by definition a volumetric shape. In modeling often a less-dimensional representation is used in order to simplify the real world. Fundamentally there are no such things as point, line or area features; there are only features with a point, line or area representation (at a certain level of abstraction/generalization).
- The real world can be considered to be a volume partition. A volume partition can be defined (analogously to a planar partition) as a set of non-overlapping volumes that form a closed modeled space. As a consequence objects like 'air' or 'earth' are explicitly part of the real world and thus have to be modeled.

As a result the real world features will be modeled as volumes (set of tetrahedrons) in a TEN structure. The option to represent certain feature types in less dimensional representations belongs in the digital cartographic model and not already in the digital landscape model. Based on this one might wonder whether less-dimensional representations are even allowed in the new modeling approach, for instance using a face instead of a volume. The answer is positive, but only in special cases. Looking at the real world one can see that the features that are represented by faces are actually marking a border between two volume objects. For instance an area labeled as 'wall' might still be represented as a face, despite its actual thickness in reality. However it's important to realize that this face marks the transition between two volumes: 'building' at one side and 'air' at the other side of the face. In the new modeling approach these volumes play a central role. The faces marking the borders between volumes might still be labeled, for instance as 'wall' or 'roof', but semantically they do not bound the building anymore, as the building in itself is represented by a volume, with neighboring volumes that represent air, earth or perhaps another adjacent building. One can say that area features, such as walls, are derivatives of volume features, as they cannot exist without the presence of these volume features. At this time it is not decided yet whether only first order derivatives (area features) are useful or that second (line features) and third order derivates (point features) should be supported too.

The UML class diagram of the proposed method is shown in Figure 3. The concept of derived features is modeled by treating these classes as association classes. For instance an area feature is an association class between two (adjacent) volume features. Also visible in the UML class diagram is that features are stored as constraints in the TEN structure. On algorithm level these constraints are all on the edge level, as current triangulation / tetrahedronization algorithms are only capable of handling constraint edges. A third aspect that is shown in the diagram is that every tetrahedron should represent a volume feature, whereas for instance not every triangle represents an area feature. This full volume partition idea results in modeling 'air' and 'earth' in between of topographic features as well.

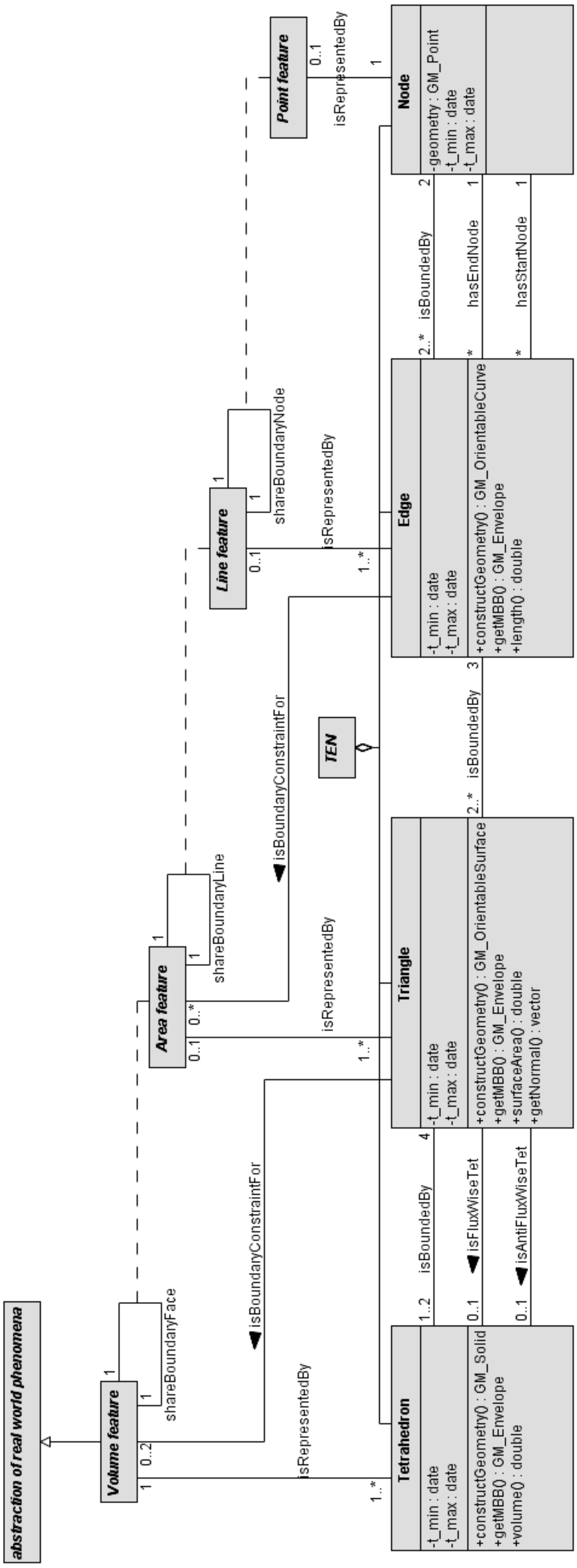

Figure 3. UML class diagram of the 3D Topography TEN model.

At this point it might seem that also modeling 'air' and 'earth' in addition to all common topographic features is a very rigid approach of modeling, more serving the abstract goal of 'clean' modeling than an actual useful goal. This is however not the case. These air and earth objects do not just fill up the space between features of the other types, but are often also subject of analyses, such as noise and odor modeling and flooding analysis. Another great advantage is the flexibility introduced by these features, as they enable future extensions of the data model. Figure 4 shows some examples of features that can be included in the model in the future, such as air traffic corridors and petroleum reservoirs.

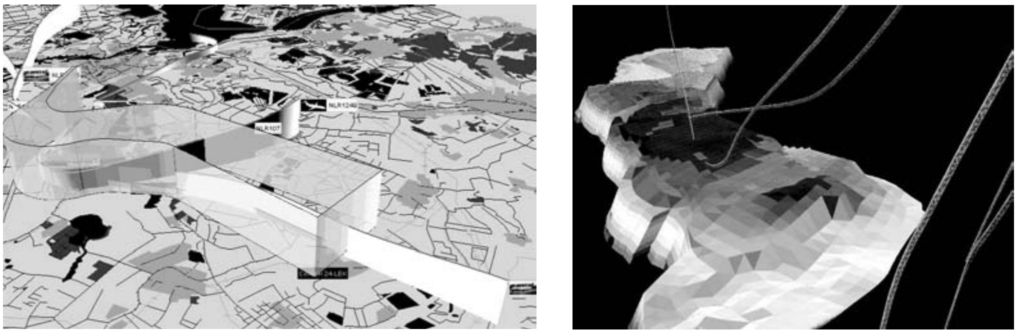

Figure 4. Air traffic corridors near Schiphol Amsterdam Airport (left) and a 3D petroleum reservoir (Ford and James, 2005) (right).

Another possible future extension is the addition of indoor topography to the model. Information on the basic interior layout of a building can be very useful for emergency services, especially the fire brigade. Indoor geo information will become more and more available due to the introduction of terrestrial laser scanning. This technique also enables measurements in complex 3D situations such as highway interchanges, see Figure 5.

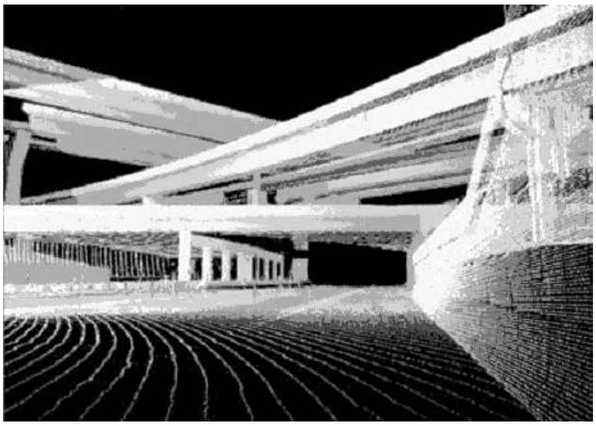

Figure 5. Scan of highway interchange obtained by terrestrial laser scanning.

3D Topography modeling: implementation

In the presented 3D Topography TEN model the topographic features are stored as constraints in the tetrahedronization. The creation of the model starts with four initial tetrahedrons, as can be seen in Figure 6: two air and two earth tetrahedrons with an initial earth surface. As a first step height data from digital elevation models will be used to refine the earth surface. If necessary ill-shaped tetrahedrons will be reshaped and Steiner points might be added. The last step is to insert separately tetrahedronized topographic features into the model. Therefore an incremental algorithm for updating the TEN model is required.

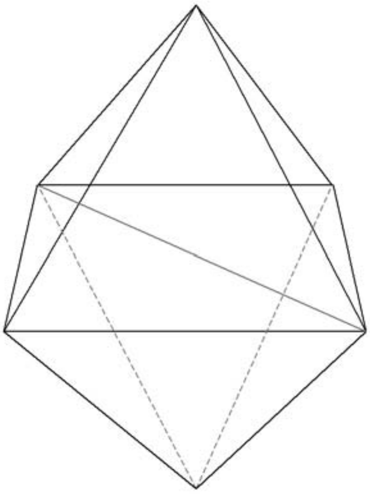

Figure 6. The four initial tetrahedrons (two 'air' and two 'earth').

3D Topography and Computational Geometry

Amount of data

If one considers the tetrahedronization of the building in Figure 7, it will be clear that storing the building in a TEN requires a lot of storage. In Table 1 the required number of tetrahedrons, triangles, edges and nodes is compared to the number of volumes, faces, edges and points in a polyhedron approach.

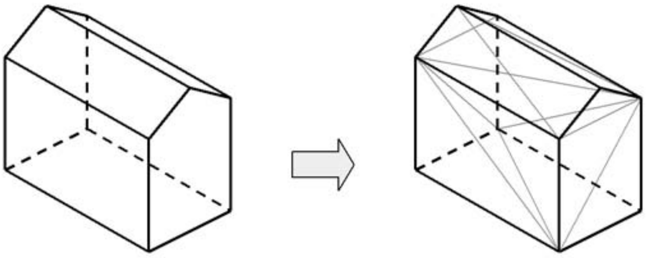

Figure 7. Tetrahedronized building.

Building as polyhedron	Building as TEN
(1 volume)	8 tetrahedrons
7 faces	24 triangles
(15 edges)	25 edges
(10 points)	10 nodes

Table 1. Comparison between polyhedron and TEN model of the building.

In order to reach acceptable performance it has to be decided which relationships (as modeled in the class diagram in Figure 3) will be stored explicitly, as performance requirements do not tolerate full storage of all possible relationships. Several approaches exist in 2D to reduce storage requirements of TINs by either working with an edge- or a triangle based approach, in which not both triangles, edges and nodes are stored explicitly. However, in the 3D situation and in the case of constraints in the TEN this is very difficult.

Updating the topography model: requirements from a computational geometry perspective

In the developed data model all data is stored in a spatial database. The most straight forward implementation consists of four tables with nodes, edges, triangles and tetrahedrons and a table with volume features. The set of representing simplexes is stored in this last table for every feature, so for the building from Figure 7 references are present to the eight tetrahedrons. To ensure the correct representation of the building in the TEN model one needs to enforce the boundary faces of this building to be present. As triangulation algorithms can only handle constrained edges, so a set of constrained edges is required. This set of constrained edges forms a complete surface triangulation of the building.

If one wants to remove this building, for instance because it is demolished, the record from the volume feature table can be deleted. At the same time the TEN needs to be updated. The constraints on the edges of the surface triangulation can be removed, but only if this building is the only feature that is bounded by this constrained edge. In the case of the demolished building constrained edges on the building's floor also bound the earth surface and therefore need to remain present in the TEN model. The tetrahedrons that were previously representing the building now need to be re-classified, in this case most likely just as 'air'. This reclassification is necessary to maintain the volume partition. At this moment the building is entirely removed from the model, both on TEN and on feature level, but the deletion process is not finished. As a last step it is necessary to check whether the TEN can be simplified by creating larger tetrahedrons or can be optimized by creating better-shaped tetrahedrons. As an alternative one might delete directly all edges that were part of the building, except for (constrained) edges that also contribute to the shape of other features. The resulting hole in the TEN needs to be re-triangulated and the created tetrahedrons will be linked to the 'air' feature.

If one wants to add a feature (for instance the same building) to the model its surface first needs to be triangulated. The resulting edges are the input for the tetrahedronization. This tetrahedronization is performed separately from the TEN network. The complete set of edges is then inserted into the TEN model by an incremental tetrahedronization algorithm. As a last step the volume feature table needs to be updated. A new record is created which links the building to the representing tetrahedrons and the previous 'air' tetrahedrons on the specific location are removed.

Now that the update process is described the algorithm requirements can be extracted. For creating and maintaining the TEN an incremental algorithm is required. Due to the potential enormous amount of data this incremental algorithm has to work in the database and should preferably impact the TEN structure as locally as possible. In the TEN all simplexes should be explicitly available, as the tetrahedrons represent volume features, the triangles contain most topological relationships, the edges contain the constraints and the nodes contain the geometry. Attempts to work for instance with implicit edges as is done with TINs would seriously complicate some of the required analysis or editing operations. Being able to store a TEN as compact as possible is might be nice, but in this 3D topography research interest is not only in maintaining a TEN but also in altering and querying the structure, which requires more functionality. Another requirement is the need for numerical stability through detection and repair of ill-shaped triangles and tetrahedrons. Shewchuk has performed a lot of research (Shewchuk 1997, Shewchuk 2004) in the field of Delaunay mesh refinement in both two and three dimensions.

Conclusions

The volume approach in 3D topographic data modeling offers several advantages, amongst other good analytical and computational capabilities. However the TEN approach will lead to very large data sets. In order to overcome this drawback fast and reliable algorithms are required. The constrained tetrahedronized irregular network needs to be updated by an incremental algorithm that will also guarantee well-shaped triangles and tetrahedrons to avoid numerical instability. Despite the conceptual advantages of the 3D TEN approach the success of this approach completely depends on the degree in which the algorithms are capable of querying, analyzing and altering with acceptable performance.

References

Carlson, E., 1987. Three-dimensional conceptual modeling of subsurface structures. In: Auto-Carto 8, pp. 336-345.

de Cambray, B., 1993. Three-dimensional 3D) modeling in a geographical database. In: Auto-Carto 11, pp. 338-347.

Ford, A. and P. James, 2005. Integration of 3D petroleum datasets in commercial GIS. In: Agile 2005, 8th Conference on Geographic Information Science, pp. 411-418.

Molenaar, M., 1990. A formal data structure for three dimensional vector maps. In: 4th International Symposium on Spatial Data Handling, Zurich, pp. 830-843.

Molenaar, M., 1992. A topology for 3D vector maps. In: ITC Journal 2, pp. 25-33.

Penninga, F., 2005. 3D Topographic data modeling: Why rigidity is preferable to pragmatism. In: Spatial Information Theory, Cosit 2005, pp. 409-425.

Pilouk, M., 1996. Integrated Modeling for 3D GIS, PhD thesis, ITC Enschede.

Shewchuk, J.R., 1997. Delaunay refinement mesh generation, PhD thesis, Carnegie Mellon University.

Shewchuk, J., 2004. General-Dimensional Constrained Delaunay and Constrained Regular Triangulations I: Combinatorial Properties. To appear in: Discrete & computational Geometry. Available at: http://www-2.cs.cmu.edu/jrs.

Stoter, J., 2004. 3D Cadastre, PhD thesis, Delft University of Technology.

Vosselman, G., 2005. Sensing Geo-information, Inaugural address, ITC Enschede.

Zlatanova, S., 2000. 3D GIS for urban development, PhD thesis, Graz University of Technology.