



# 3D Topography

## A simplicial Complex-based Solution in a Spatial DBMS

Friso Penninga

GIS-t lunch meeting January 18, 2008

# Presentation outline

- Introduction
- Previous presentation (01-09-06)
  - Characteristics simplicial complex-based approach
  - Simplicial complexes applied to 3D Topography
  - Implementation details
- Update operations (feature insertion)
- Test data sets
  - Tetrahedronisation of models
  - Storage requirements, comparison to Oracle 11g polyhedrons
- Future research & conclusions



# Introduction (1/2)

PhD research within RGI-011

Objective RGI-011:

*Enforcing a major break-through in the application of 3D Topography in corporate ICT environments due to structural embedding of 3D methods and techniques.*

Research question my PhD:

*How can a 3D topographic representation be realised in a feature-based triangular data model?*



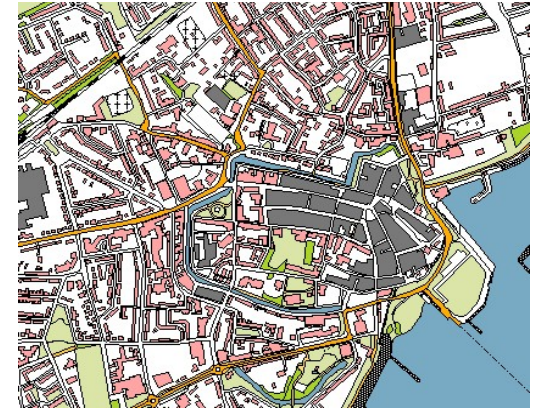
# Introduction (2/2)

## Two-step approach:

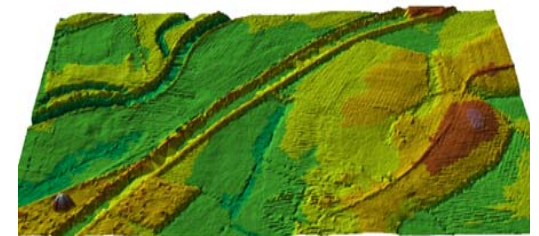
- How to develop a conceptual model that describes the real world phenomena (the topographic features)?
- How to implement this conceptual model, i.e. how to develop a suitable DBMS data structure?

## Focus:

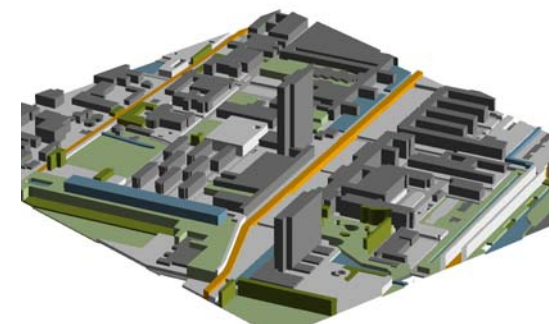
storage, analysis, validation



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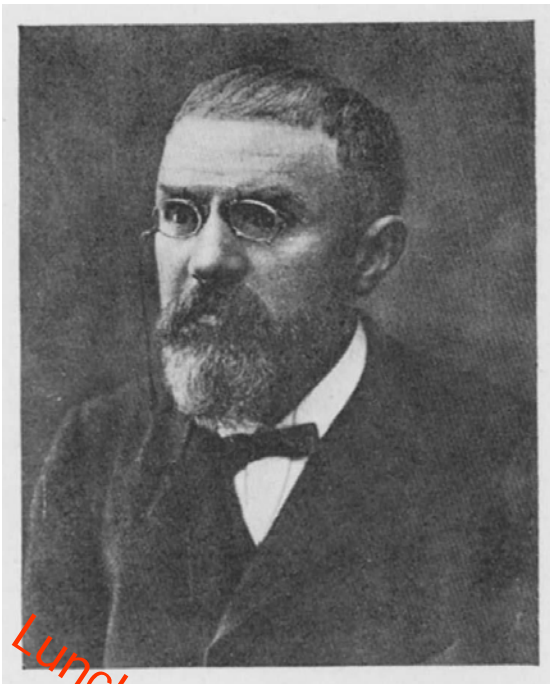


# Presentation outline

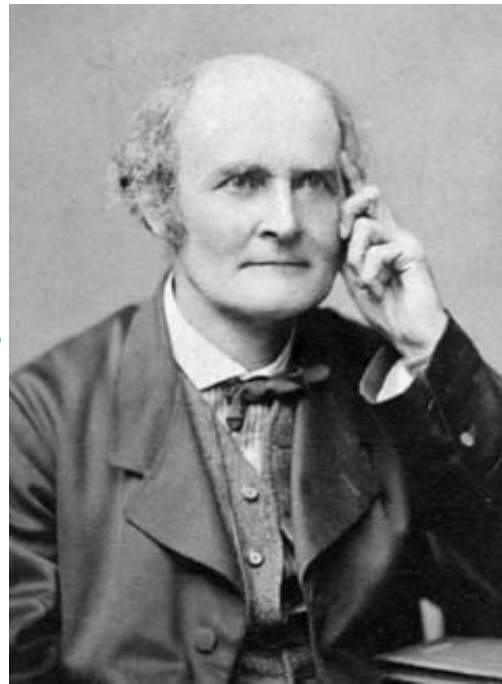
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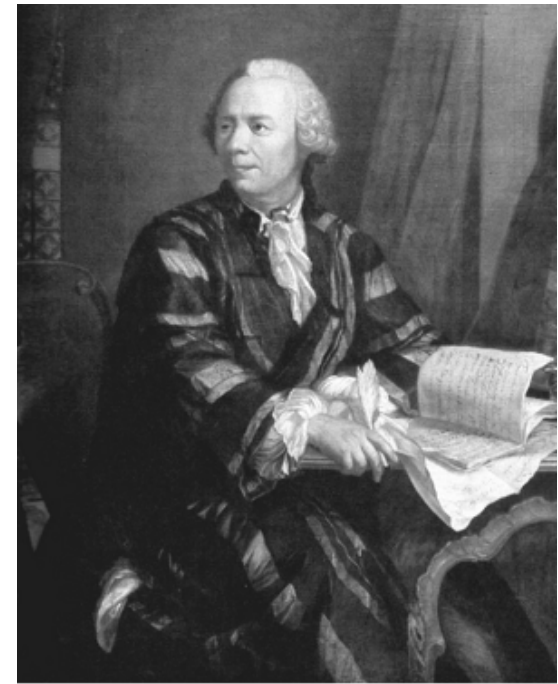
# Previous presentation



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= 3D GIS?

Lunchmeeting 01-09-06  
Available at [www.frisopenninga.nl](http://www.frisopenninga.nl)

# Previous presentation

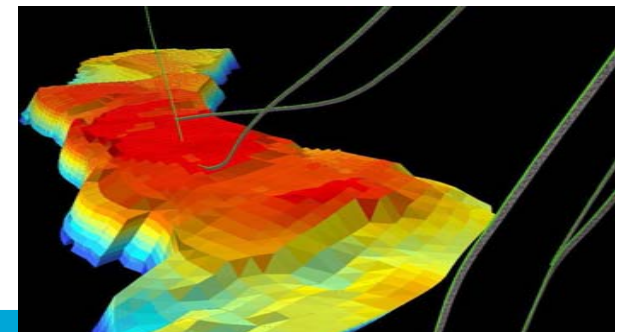
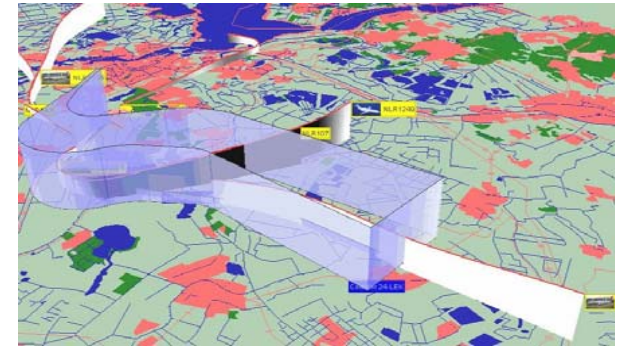
## Characteristics sc-based approach (1/3)

### Characteristic 1: Full decomposition of space

Two fundamental observations (Cosit'05 paper):

- ISO19101: a feature is an 'abstraction of real world phenomena'. These real world phenomena have by definition a **volume**
- Real world can be considered to be a **volume partition** (analogous to a planar partition: a set of non-overlapping volumes that form a closed modelled space)

**Result:** explicit inclusion of earth and air



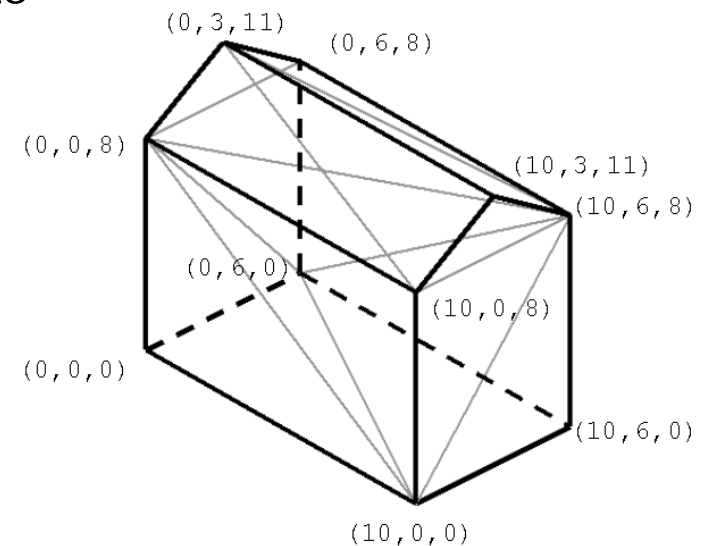
# Previous presentation

## Characteristics sc-based approach (2/3)

**Characteristic 2:** constrained TEN  
object boundaries represented by constraints

Advantages of TEN:

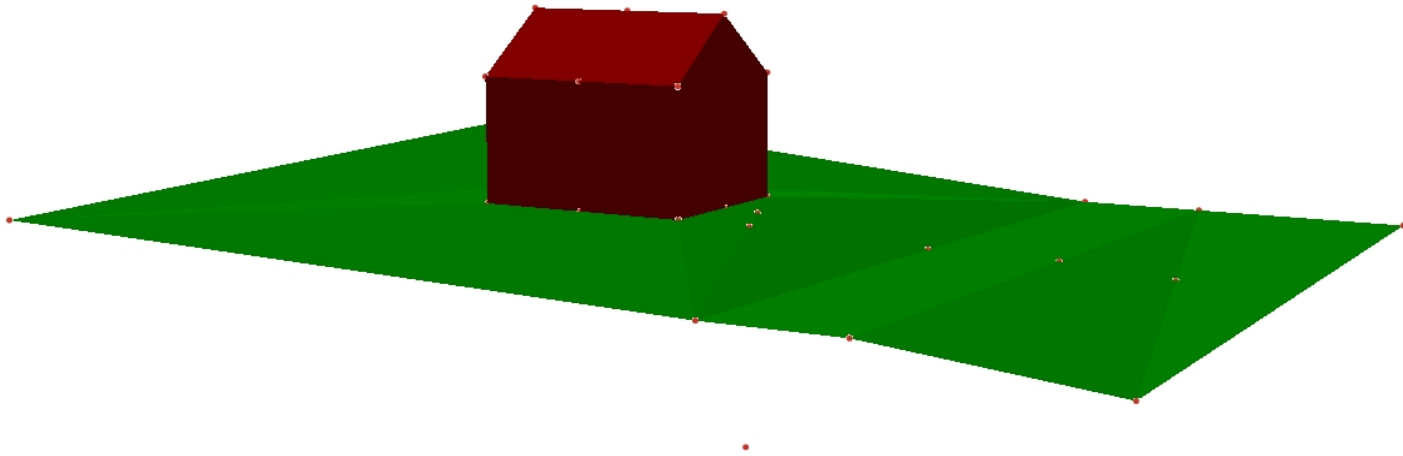
- Well defined: a  $n$ -simplex is bounded by  $n + 1$   $(n - 1)$ -simplexes.
- Flatness of faces: every face can be described by three points
- A  $n$ -simplex is convex (which simplifies amongst others point-in-polygon tests)





# Previous presentation

Example: small data set



# Previous presentation

## Characteristics sc-based approach (3/3)

**Characteristic 3:** based on Poincaré simplicial homology  
solid mathematical foundation (SDH'06 paper):

Simplex  $S_n$  defined by  $(n+1)$  vertices:  $S_n = \langle v_0, \dots, v_n \rangle$

The boundary  $\hat{\partial}$  of simplex  $S_n$  is defined as sum of  $(n-1)$  dimensional  
simplexes (note that 'hat' means skip the node):

$$\hat{\partial} S_n = \sum_{i=0}^n (-1)^i \langle v_0, \dots, \hat{v}_i, \dots, v_n \rangle$$

remark: sum has  $n+1$  terms

$$S_1 = \langle v_0, v_1 \rangle$$

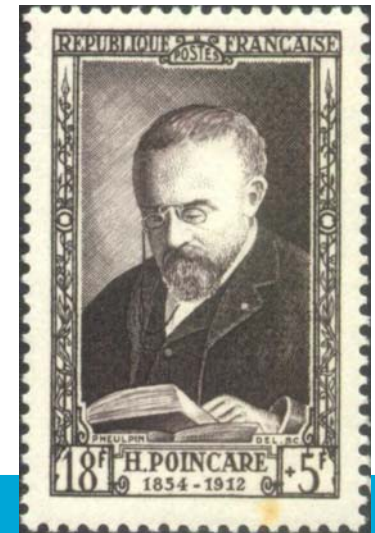
$$\hat{\partial} S_1 = \langle v_1 \rangle - \langle v_0 \rangle$$

$$S_2 = \langle v_0, v_1, v_2 \rangle$$

$$\hat{\partial} S_2 = \langle v_1, v_2 \rangle - \langle v_0, v_2 \rangle + \langle v_0, v_1 \rangle$$

$$S_3 = \langle v_0, v_1, v_2, v_3 \rangle$$

$$\hat{\partial} S_3 = \langle v_1, v_2, v_3 \rangle - \langle v_0, v_2, v_3 \rangle + \langle v_0, v_1, v_3 \rangle - \langle v_0, v_1, v_2 \rangle$$



# Previous presentation

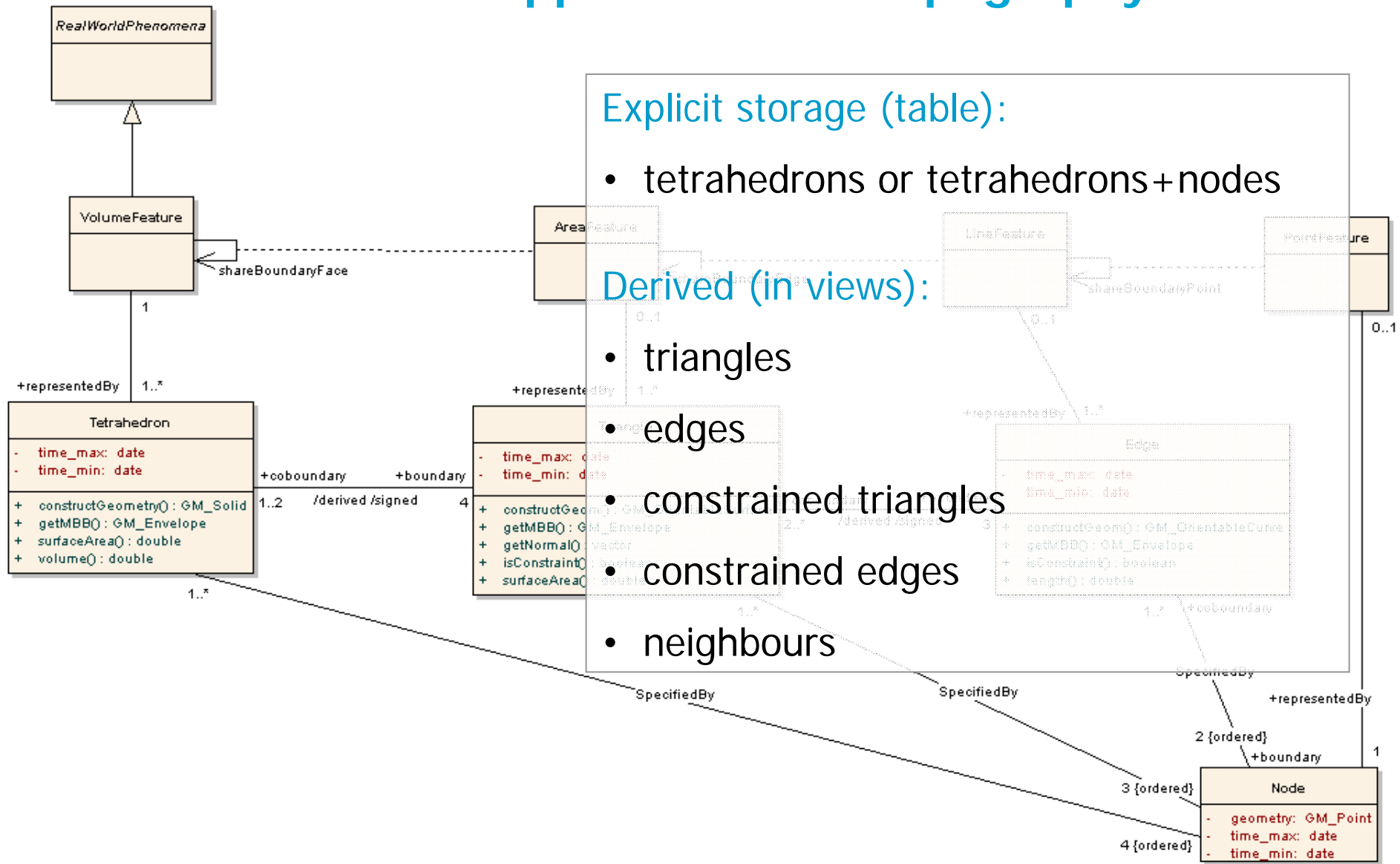
## SC-based approach to 3D topography

Explicit storage (table):

- tetrahedrons or tetrahedrons+nodes

Derived (in views):

- triangles
- edges
- constrained triangles
- constrained edges
- neighbours



# Previous presentation

## Implementation details

$$\partial S_n = \sum_{i=0}^n (-1)^i \langle v_0, \dots, \hat{v}_i, \dots, v_n \rangle$$

Boundary operator implemented in PL/SQL procedure

Procedure used to define views with triangles, edges, constrained triangles (object boundaries!), constrained edges, e.g.:

```
create or replace view triangle as
  select deriveboundarytriangle1(tetcode) tricode,
         tetcode fromtetcode from tetrahedron
  UNION ALL
  select deriveboundarytriangle2(tetcode) tricode,
         tetcode fromtetcode from tetrahedron
  UNION ALL
  ...
```

# Presentation outline

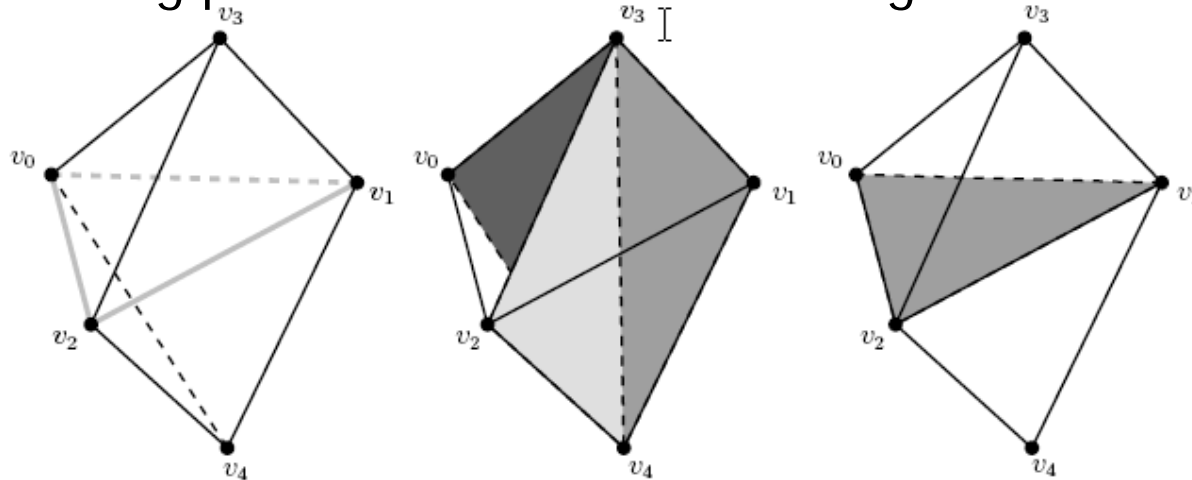
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# Update operations

## Four steps in feature insertion

1. Feature boundary triangulation: input for step 2
2. Inserting constrained edges with new approach (following slides)
3. Ensuring presence of constrained triangles



4. Interior modelling + reclassifying tetrahedrons

# Update operations

## Insertion of constrained edge in a TEN

Usual approach: insert nodes, using flipping for edge recovery

Might fail in topographic TEN:

many constraints in close proximity → flipping not always possible

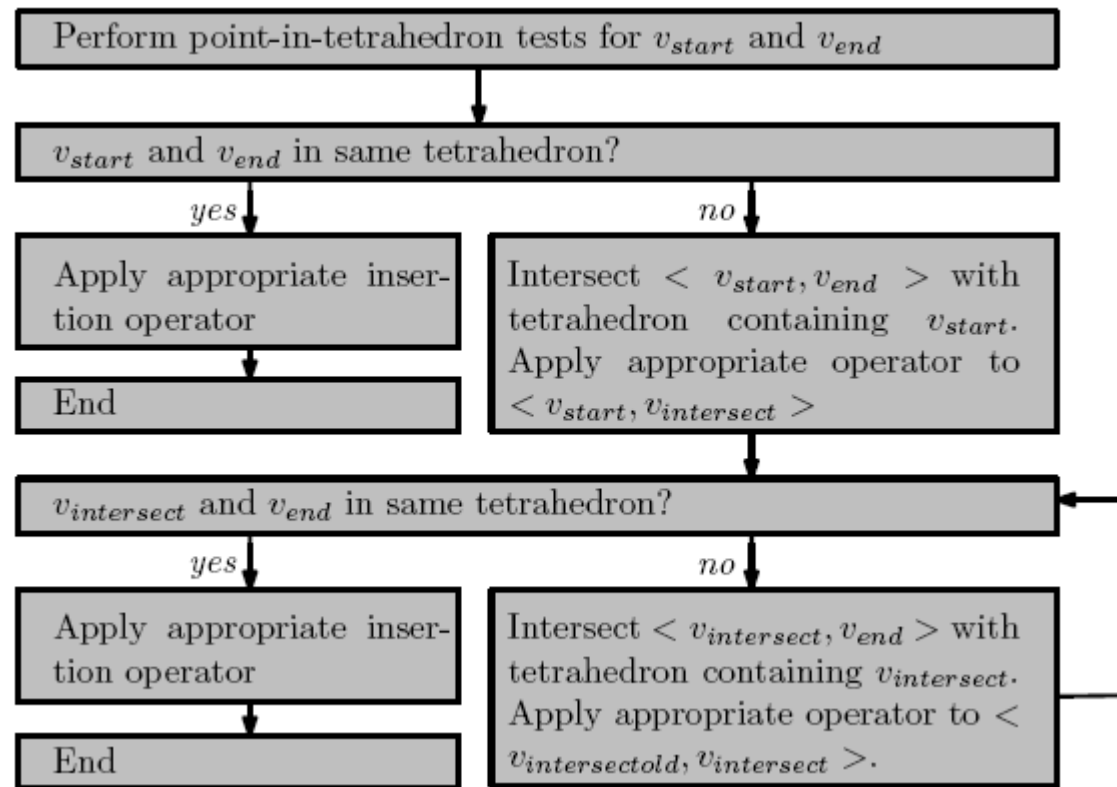
New approach: insert a complete constrained edge

Nine unique cases: *exhaustive + mutually exclusive*

<i>Node lies on</i>	<b>Node</b>	<b>Edge</b>	<b>Triangle</b>	<b>Tetrahedron</b>
<b>Node</b>	$I_{00}$	$I_{01}$	$I_{02}$	$I_{03}$
<b>Edge</b>	$(I_{01})$	$I_{11}$	$I_{12}$	$I_{13}$
<b>Triangle</b>	$(I_{02})$	$(I_{12})$	$I_{22}$	$I_{23}$
<b>Tetrahedron</b>	$(I_{03})$	$(I_{13})$	$(I_{23})$	$I_{33}$

# Update operations

## Insertion of constrained edge in a TEN

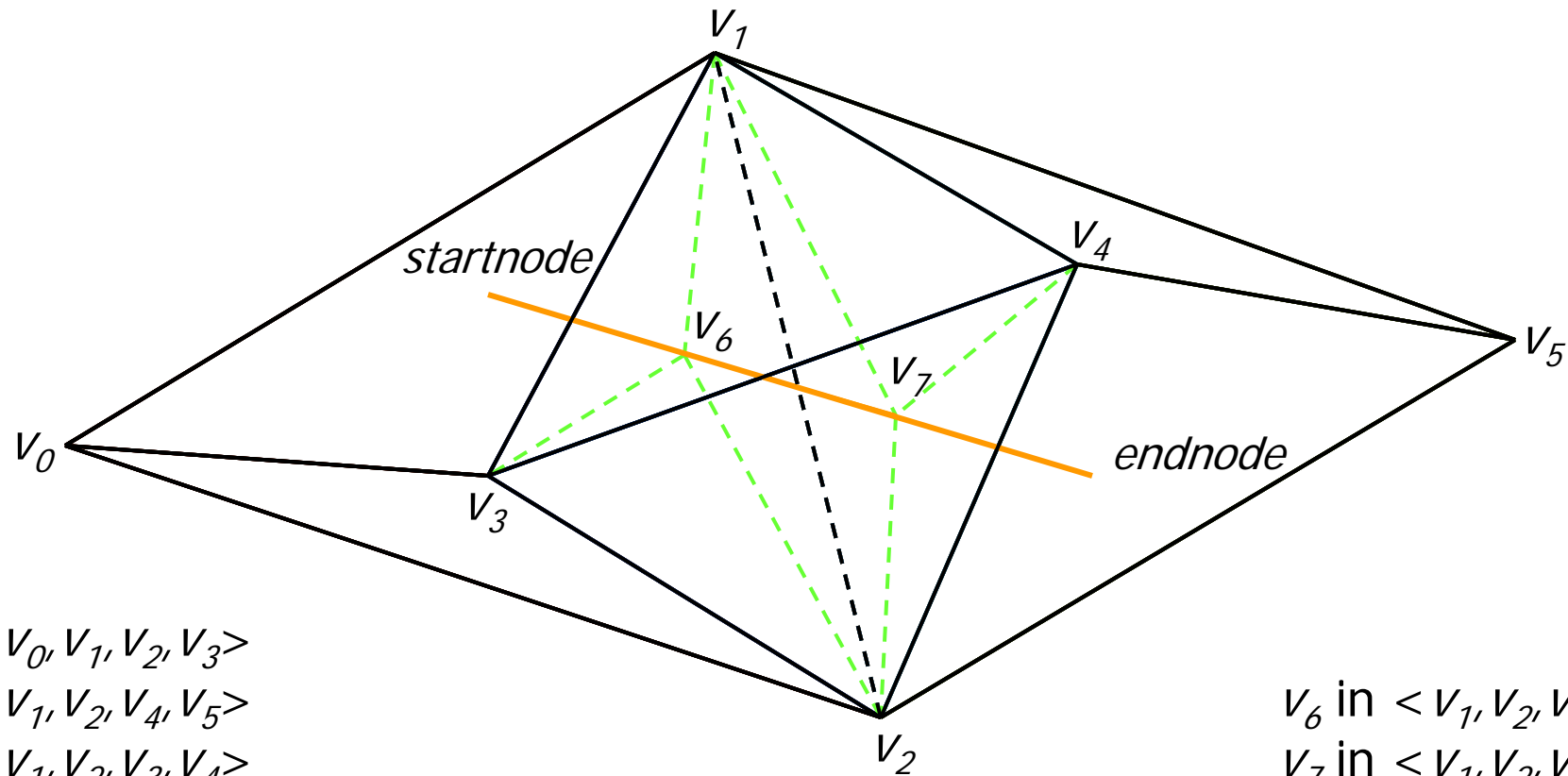


New approach: act as local as possible with minimal impact



# Update operations

## Example: constrained edge insertion



$\langle V_0, V_1, V_2, V_3 \rangle$

$\langle V_1, V_2, V_4, V_5 \rangle$

$\langle V_1, V_2, V_3, V_4 \rangle$

$V_6$  in  $\langle V_1, V_2, V_3 \rangle$

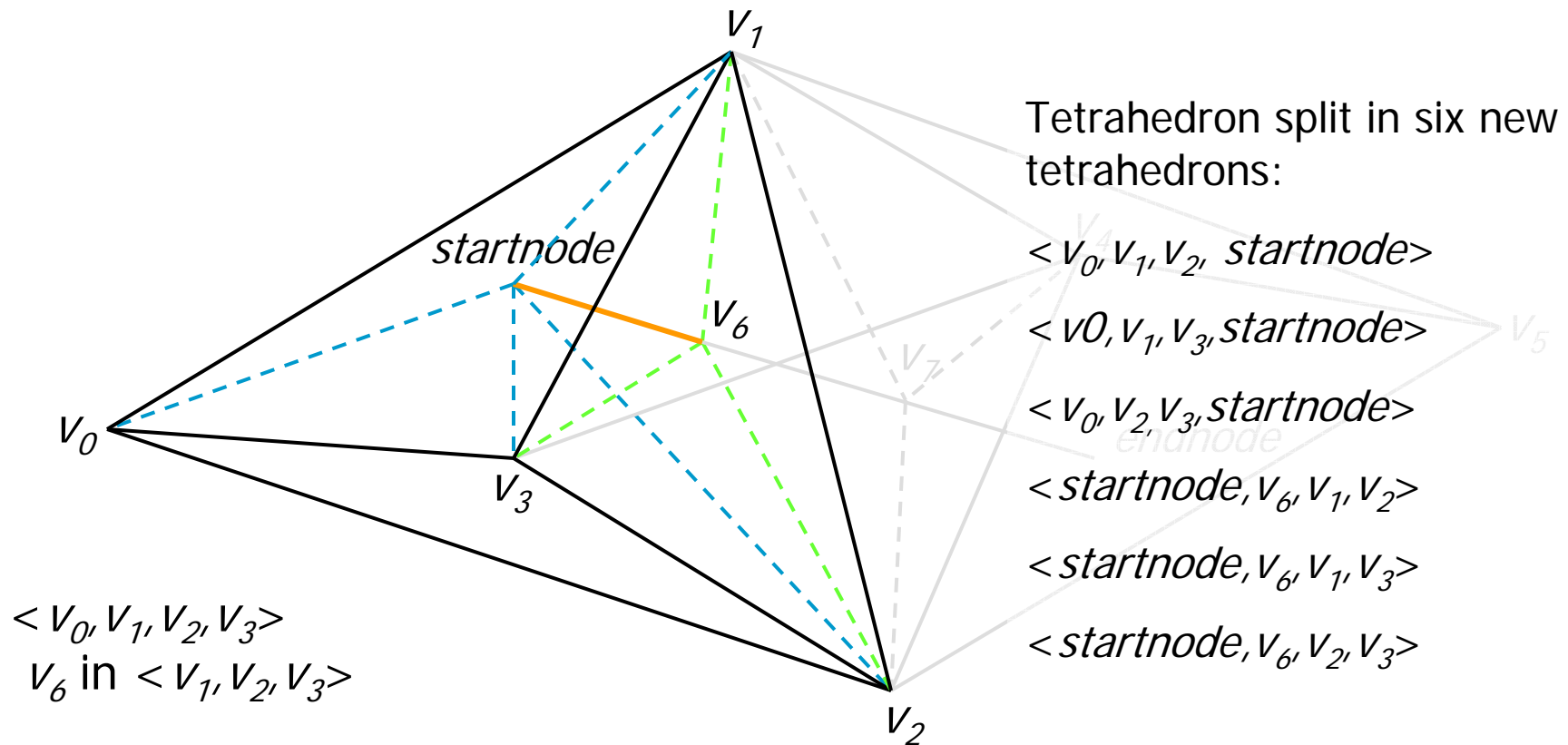
$V_7$  in  $\langle V_1, V_2, V_4 \rangle$

i.e. combination of intersection cases:

$$I_{23} - I_{22} - I_{23}$$

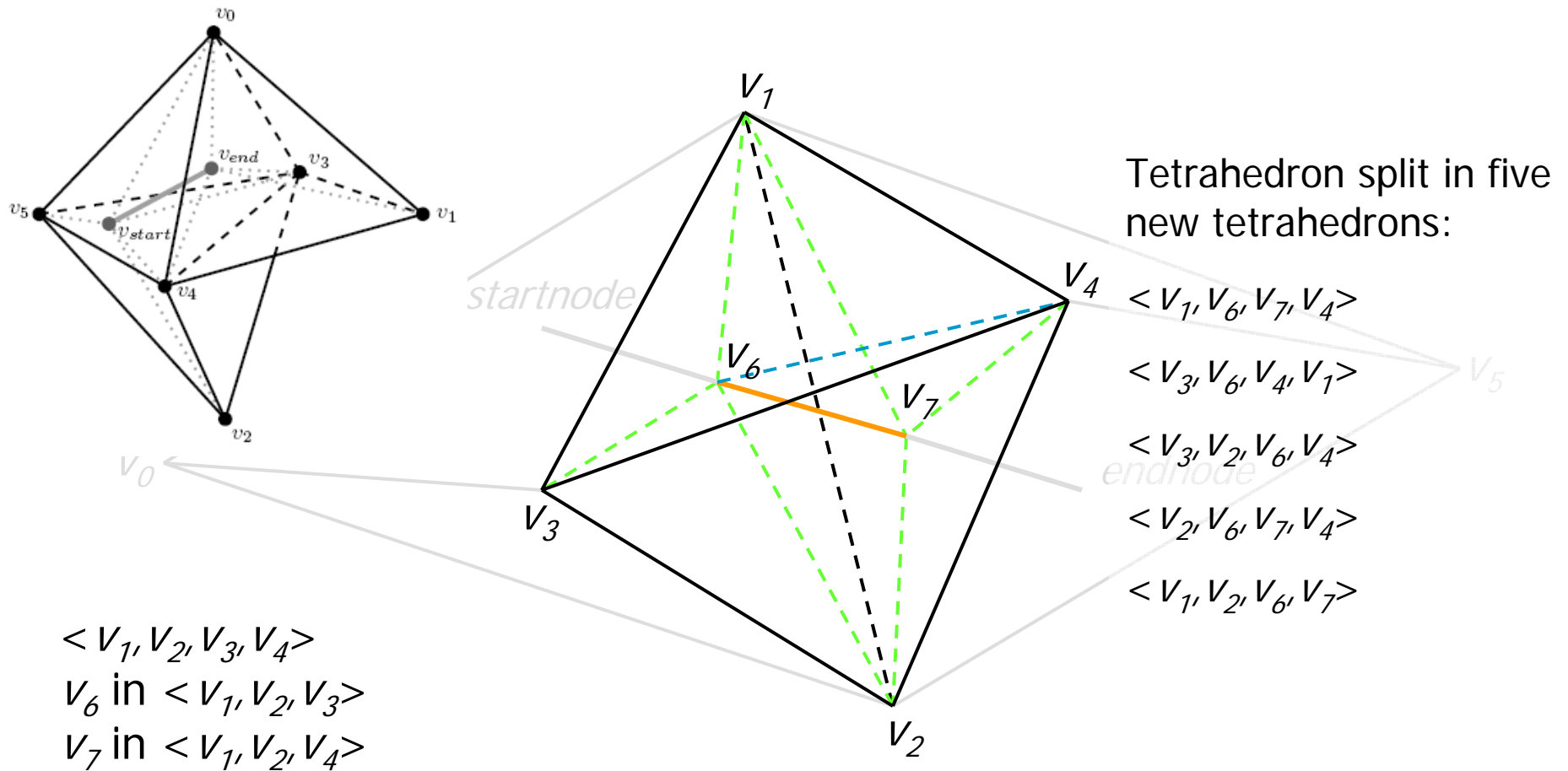
# Update operations

## Example: constrained edge insertion



# Update operations

## Example: constrained edge insertion



# Update operations

## Example: constrained edge insertion

Tetrahedron split in six new tetrahedrons:

$\langle V_2, V_4, \text{endnode}, V_5 \rangle$

$\langle V_4, V_5, \text{endnode}, V_1 \rangle$

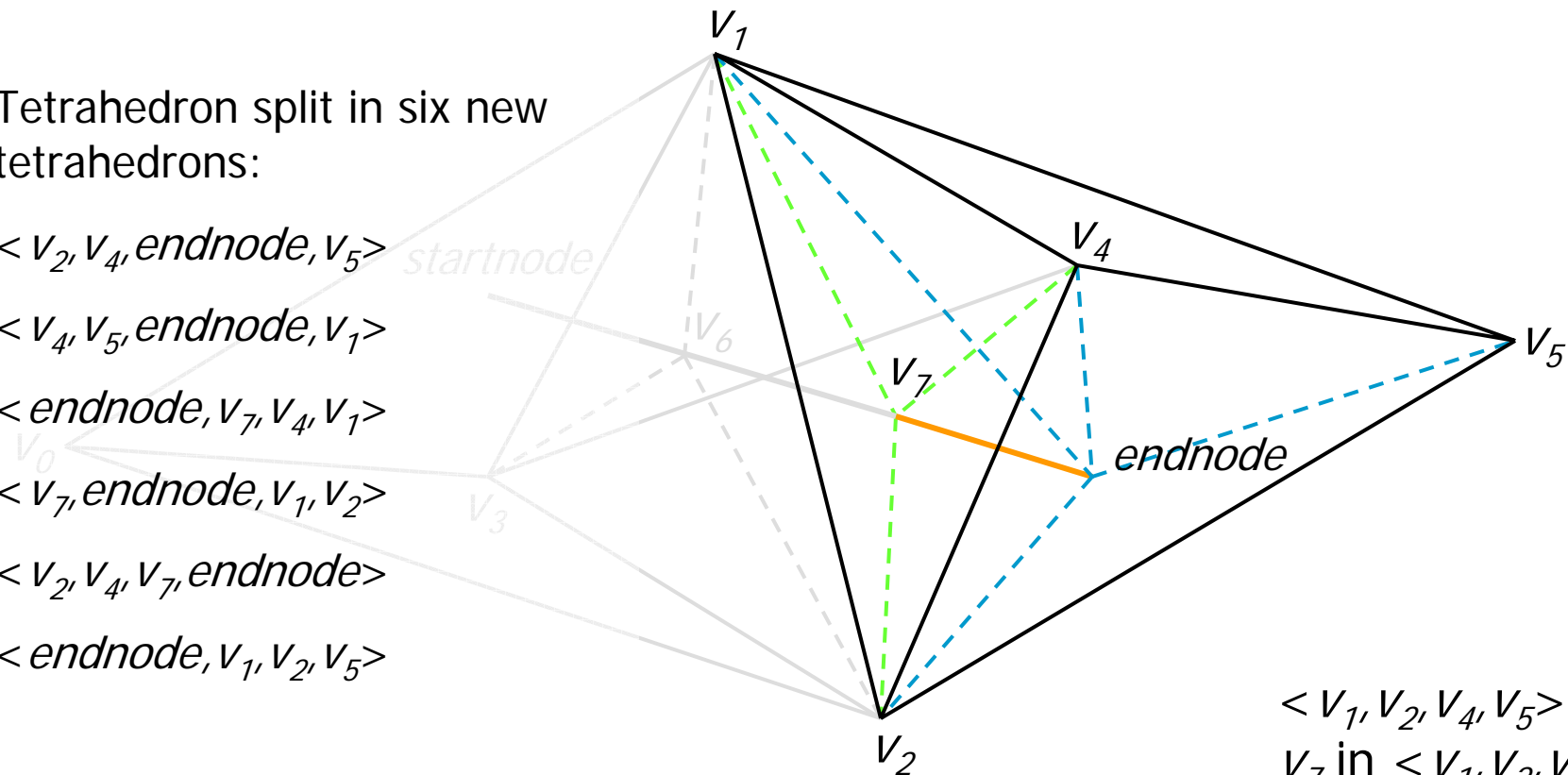
$\langle \text{endnode}, V_7, V_4, V_1 \rangle$

$\langle V_7, \text{endnode}, V_1, V_2 \rangle$

$\langle V_2, V_4, V_7, \text{endnode} \rangle$

$\langle \text{endnode}, V_1, V_2, V_5 \rangle$

$\langle V_1, V_2, V_4, V_5 \rangle$   
 $V_7$  in  $\langle V_1, V_2, V_4 \rangle$



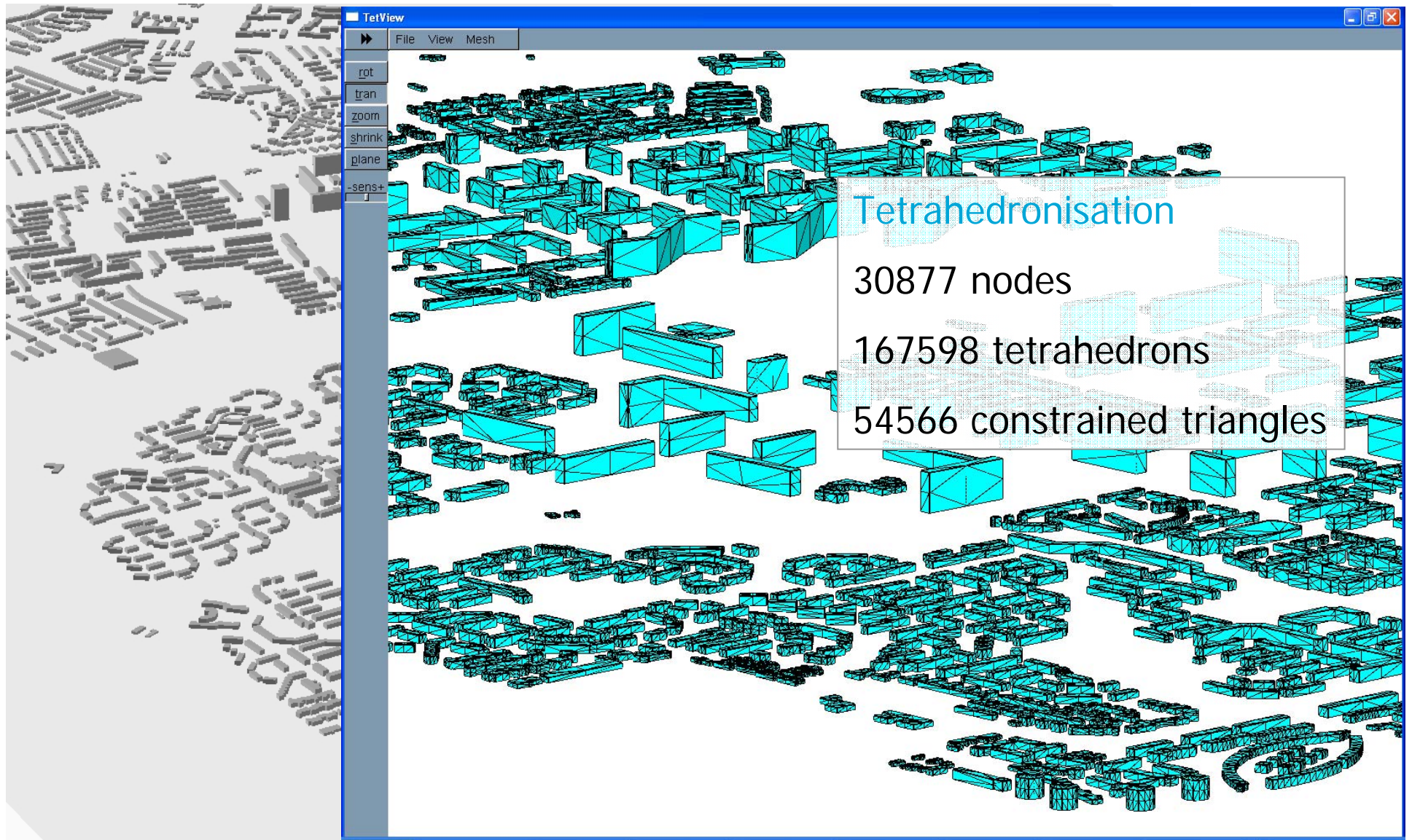
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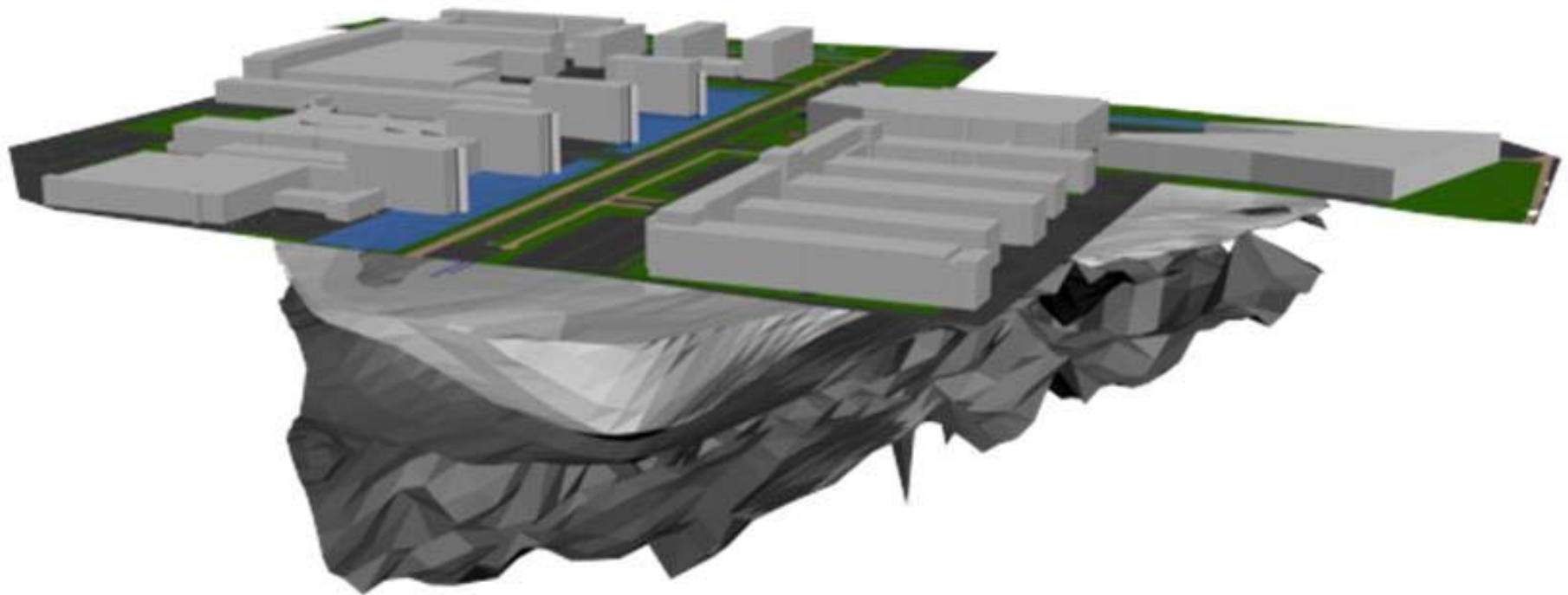
# Test data sets (1/2)

## Rotterdam data set



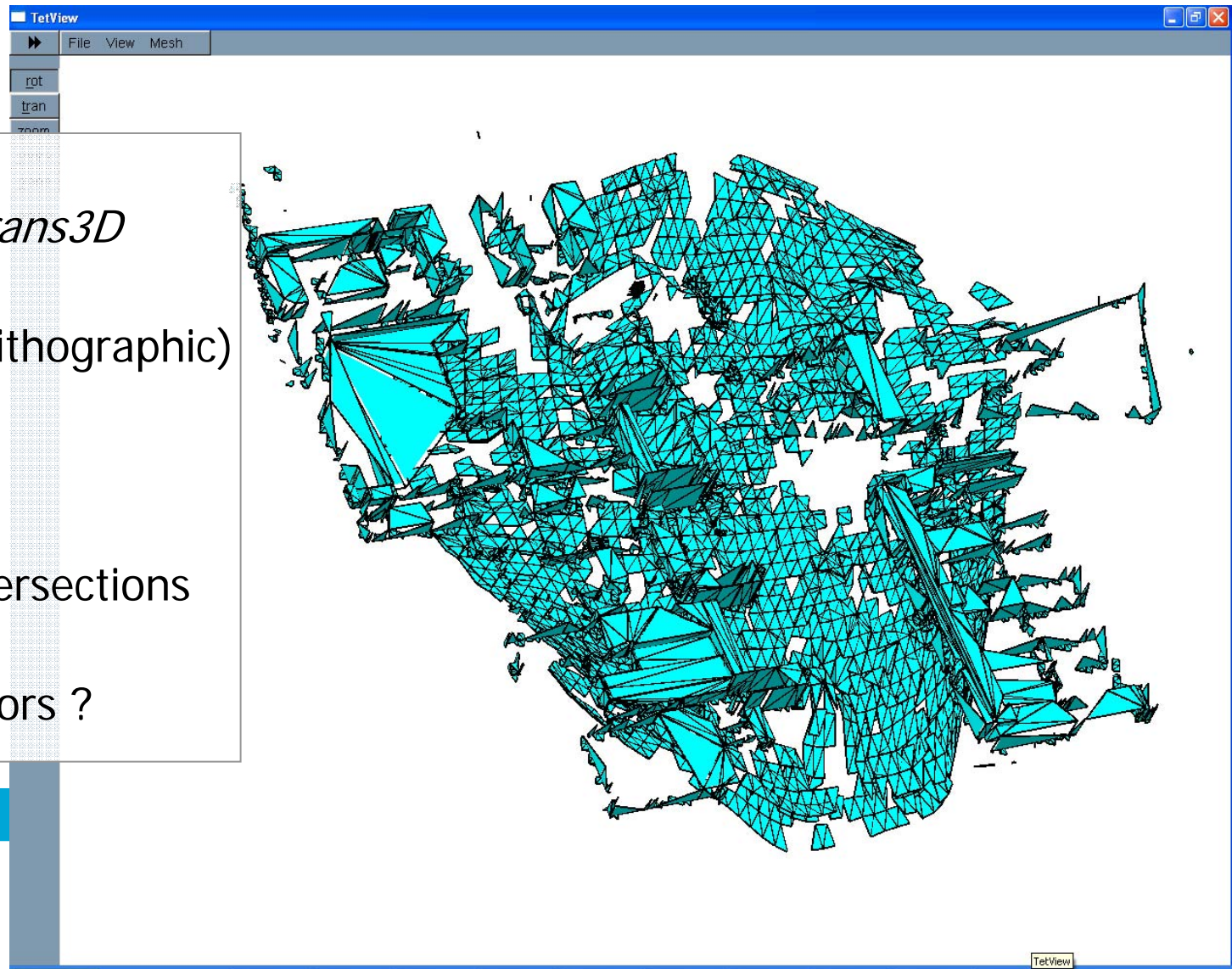
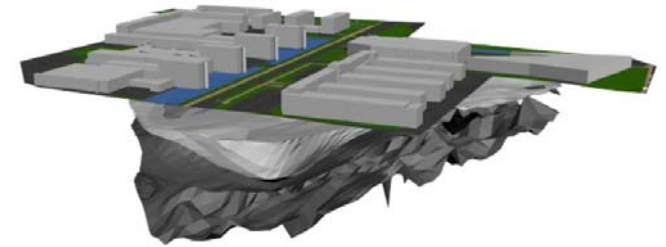
# Test data sets (2/2)

## Campus data set



# Test data sets (2/2)

## Campus data set

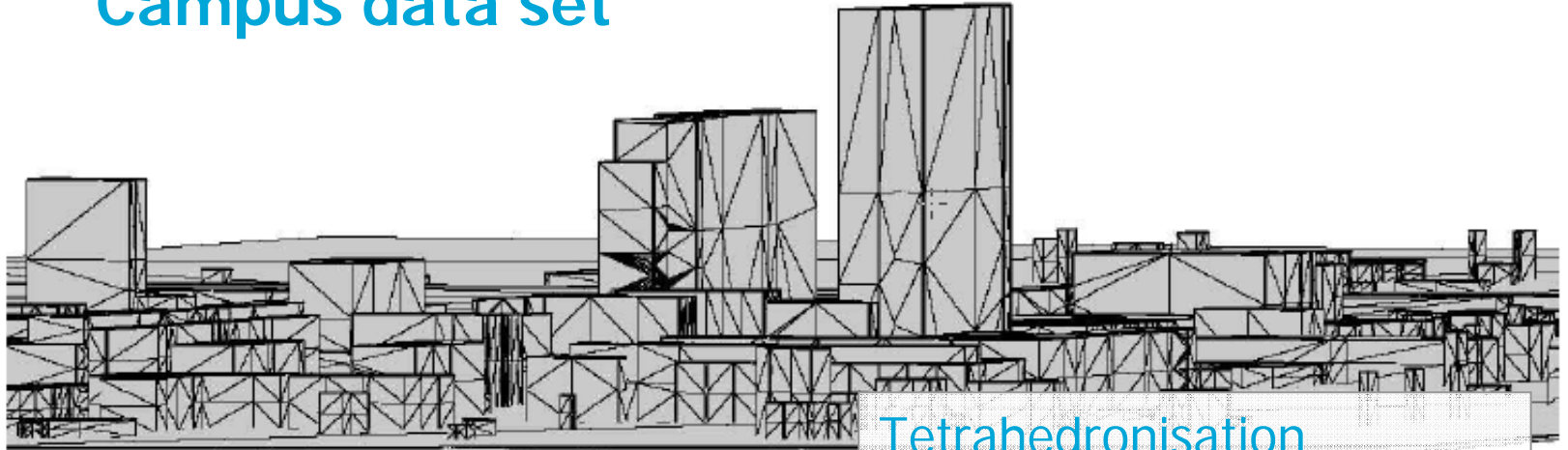


DXF  
↓  
STL  
↓ (StereoLithographic)  
TetGen  
↓  
errors...  
>50.000 intersections  
round off errors ?



# Test data sets (2/2)

## Campus data set

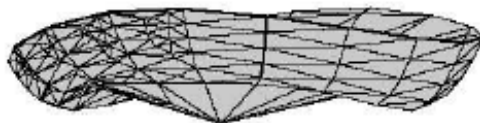
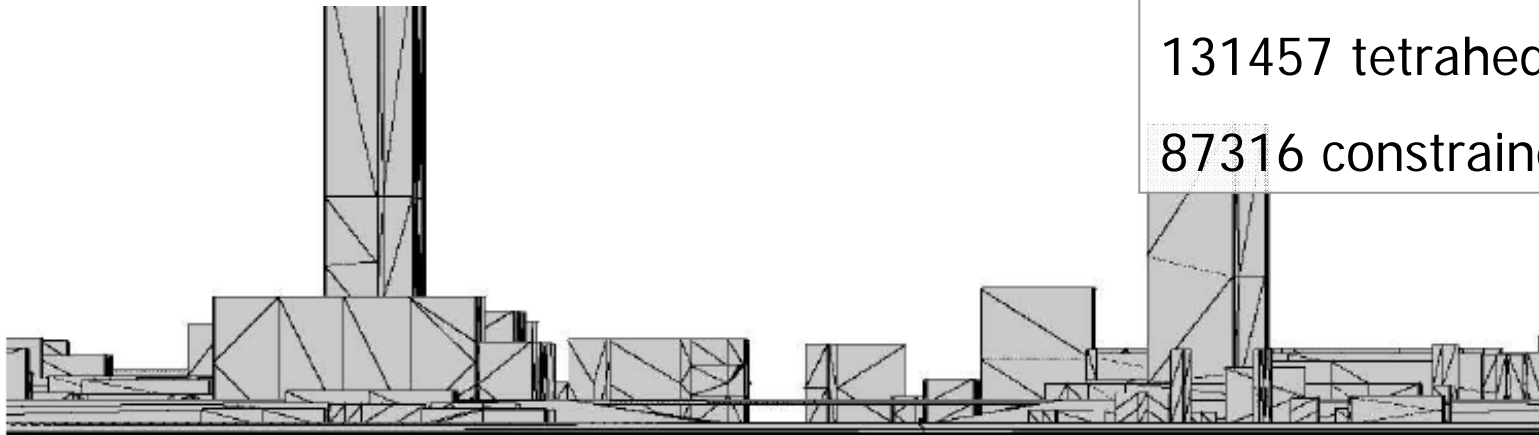
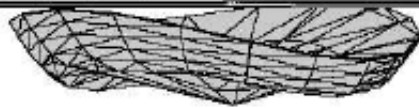


### Tetrahedronisation

23260 nodes

131457 tetrahedrons

87316 constrained triangles



# Data storage requirements (1/2)

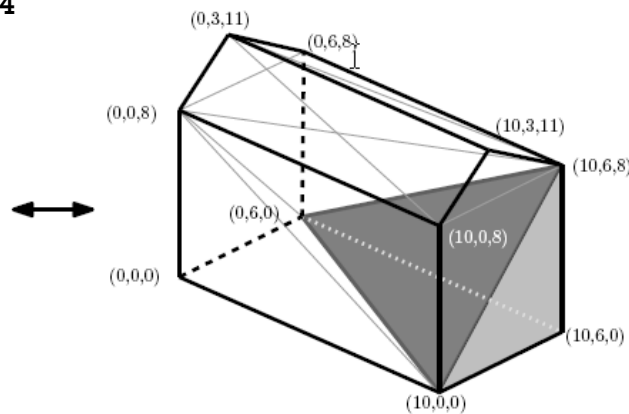
## Two alternative approaches

Coordinate concatenation:

describe tetrahedrons by node geometries:

$$\mathbf{x}_1 \mathbf{y}_1 \mathbf{z}_1 \mathbf{x}_2 \mathbf{y}_2 \mathbf{z}_2 \mathbf{x}_3 \mathbf{y}_3 \mathbf{z}_3 \mathbf{x}_4 \mathbf{y}_4 \mathbf{z}_4$$

```
000000000008000600100000
000008000311000608100608
000008000311000008100608
000600000008000608100608
000008000600100000100608
000008100000100008100608
000311100008100311100608
100000000600100600100608
```



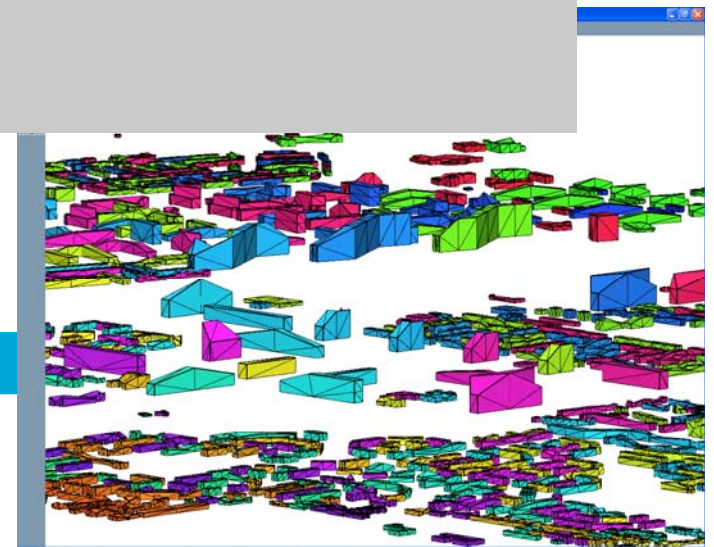
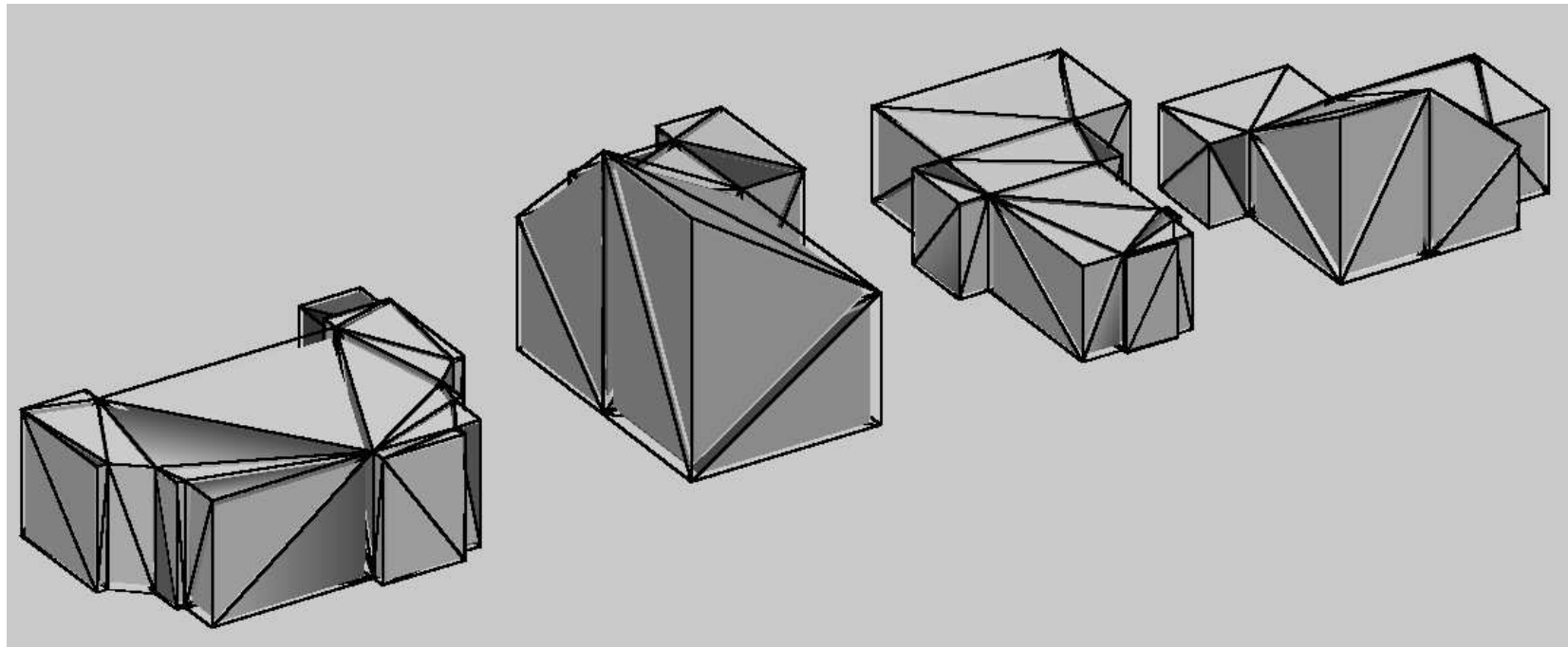
Identifier concatenation:

describe tetrahedrons by node id's:

$$\mathbf{id}_1 \mathbf{id}_2 \mathbf{id}_3 \mathbf{id}_4 \quad \text{with } \mathbf{id}_1 : \mathbf{x}_1 \mathbf{y}_1 \mathbf{z}_1, \mathbf{id}_2 : \mathbf{x}_2 \mathbf{y}_2 \mathbf{z}_2, \text{ etc.}$$

# Data storage requirements (2/2)

## Comparing alternatives and polyhedrons



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# Future research

## Spatial clustering and indexing

Basic idea:

Why add a meaningless unique id to a node, when its geometry is already unique?

1 Bitwise interleaving coordinates → Morton-like code → sorting these codes → spatial clustering

2 Use as spatial index → no additional indexes (R-tree/quad tree)

Objective: reducing storage requirements

# Future research

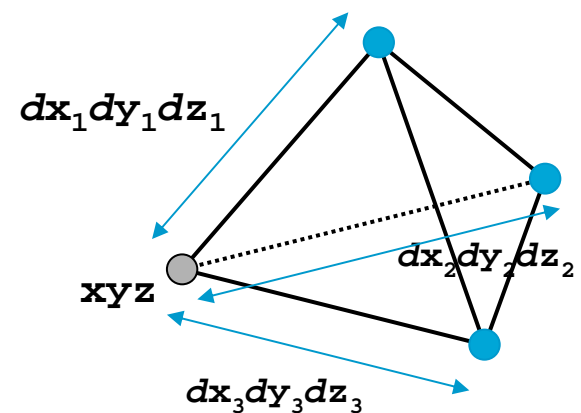
## Coordinates vs. coord. differences

Four nodes of a tetrahedron will be relatively close:  
only small differences in coordinates

Alternative tetrahedron description:

$$xyz dx_1 dy_1 dz_1 dx_2 dy_2 dz_2 dx_3 dy_3 dz_3$$

Description is based on geometry  
(so still unique) but smaller



# Conclusions

The PhD project results in a a new topological approach to data modelling, based on a tetrahedral network.

- **Operators and definitions** from the field of simplicial homology are used to define and handle this structure of tetrahedrons.
- Simplicial homology provides a **solid mathematical foundation**
- Simplicial homology enables one to **derive substantial parts** of the TEN structure efficiently, instead of explicitly storing all primitives.
- **DBMS characteristics** as the usage of views, functions and function-based indexes are extensively used to realise this potential data reduction.

# Conclusions

- A proof-of-concept implementation was created:  
prevailing view that tetrahedrons are **more expensive** in terms of storage, is **not correct** when using the proposed approach (incl. proposed improvements (binary, delta's, etc.)



Using tetrahedrons is easy!



# Thesis defence



Thursday June 19, 2008 – 15.00 h  
*Keep the evening free...*