“Spatial Object Representation: Some Issues”

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What do we want?

Clean, closed and rigorous logic.

Automatic and reliable interchange of data.

Transfinite Set

(the theory)

"A possibly infinite set; … This is actually the usual definition of set in mathematics, but programming languages restrict the term to mean finite set" (ISO19107)

(space is "smooth")

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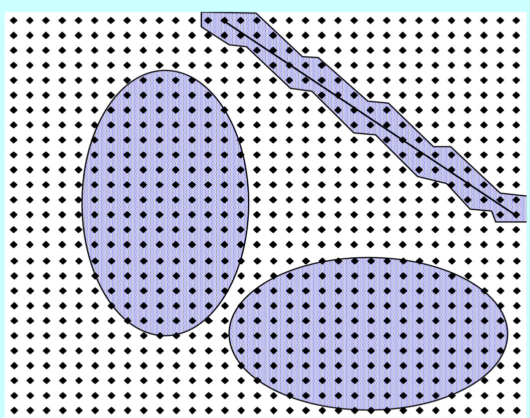
Point-set Definition of Regions

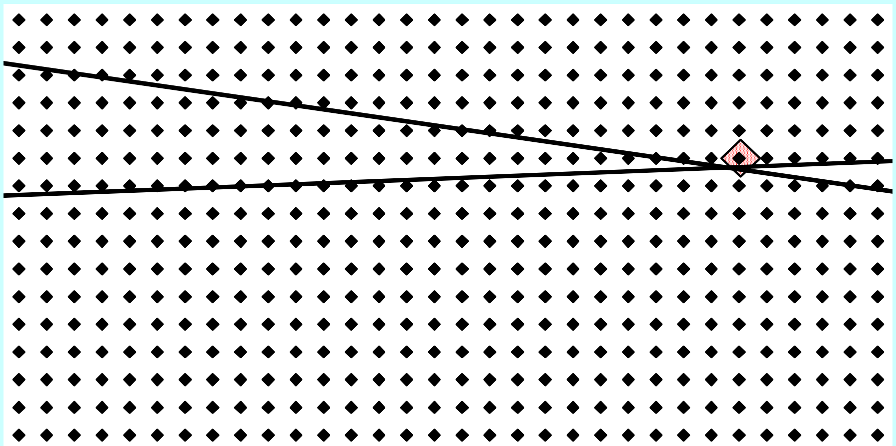
(the practice)



The volume, area, or line represents a continuous **infinite** set, but only a **finite** number of points within it can be expressed in the digital representation (integer or floating point).

4

Intersection of Two Lines or Planes



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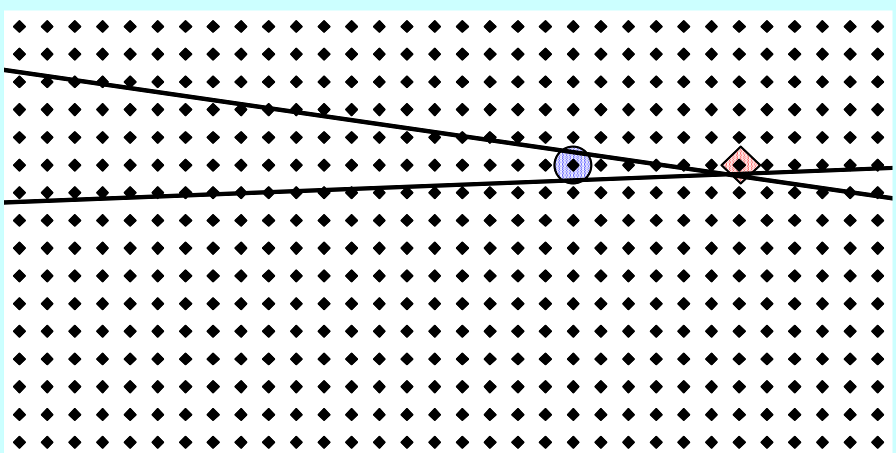
Intersection of Two Lines



We would prefer a unique point of intersection!

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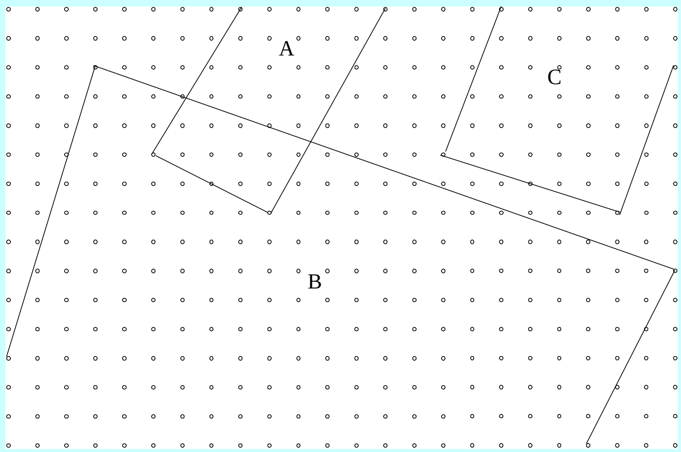
Breakdown of Logic
Non-associativity of Operations



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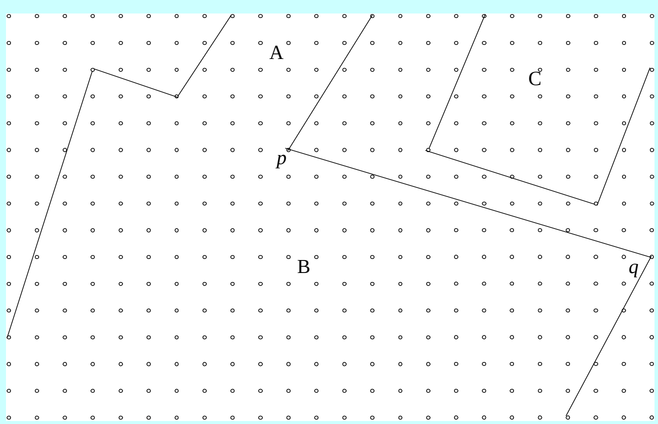
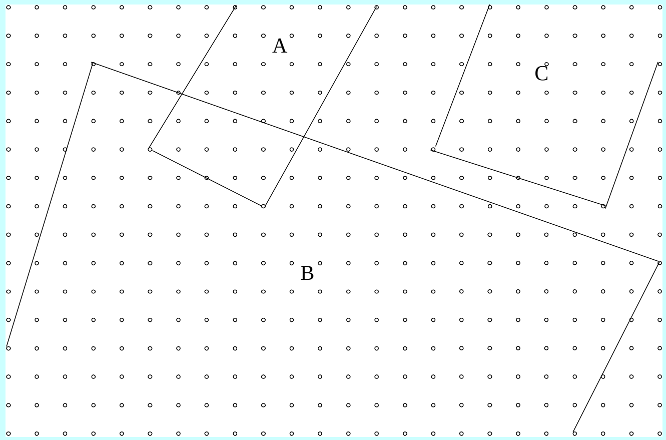
Non-associativity of Operations



Forming the union as (A∪B)∪C.

8

Non-associativity of Operations



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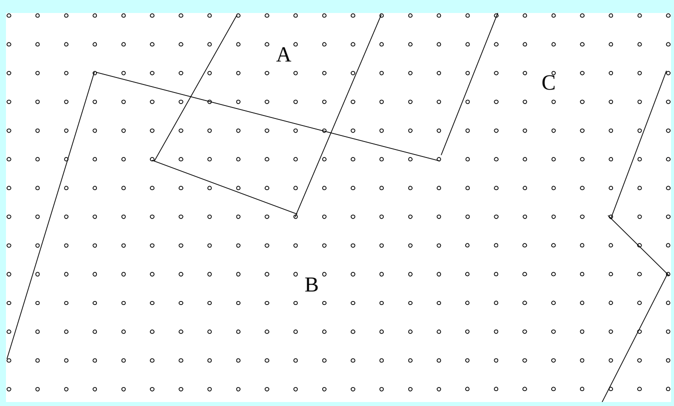
Non-associativity of Operations



Forming the union B∪C first

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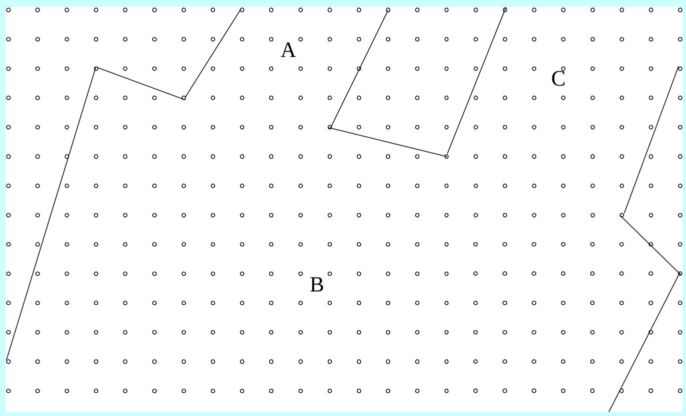
Non-associativity of Operations



Forming the union as A∪(B∪C).

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We would like:

The union and intersection operations to be associative and distributive.

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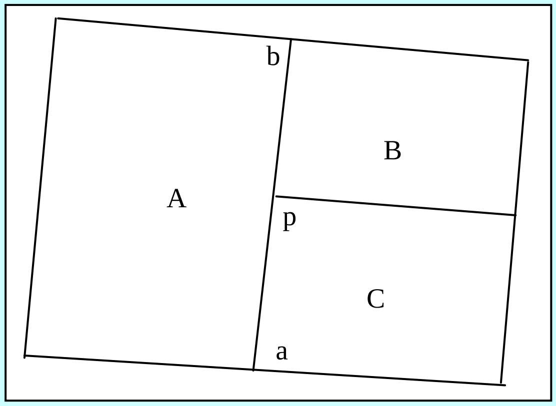
equals()
(meaning identically defined)



After subdivision of B/C, A is not equal to its before image.

In 3D there are many more variants on how to define an object.

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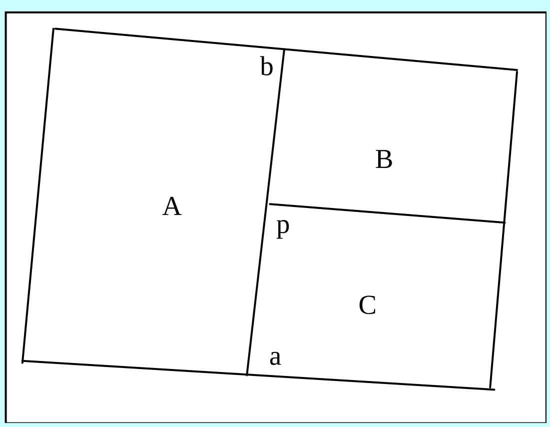
equals()
(defining the same transfinite set)



Subdivision of B/C does not change the point set definition of A

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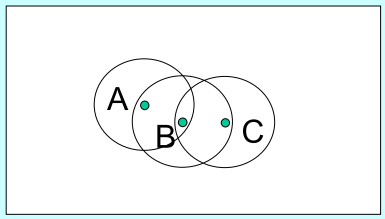
equals()
(by ISO 19107 definiton)

"Application schemas may define a **tolerance** that returns true if the two GM_Objects have the same dimension and each direct position in this GM_Object is within a tolerance distance of a direct position in the passed GM_Object and visa-versa"

(ISO 19107 section 6.2.2.18.3).



A.equals(B), B.equals(C), but not A.equals(C)

We would like equals() to be an equivalence relation

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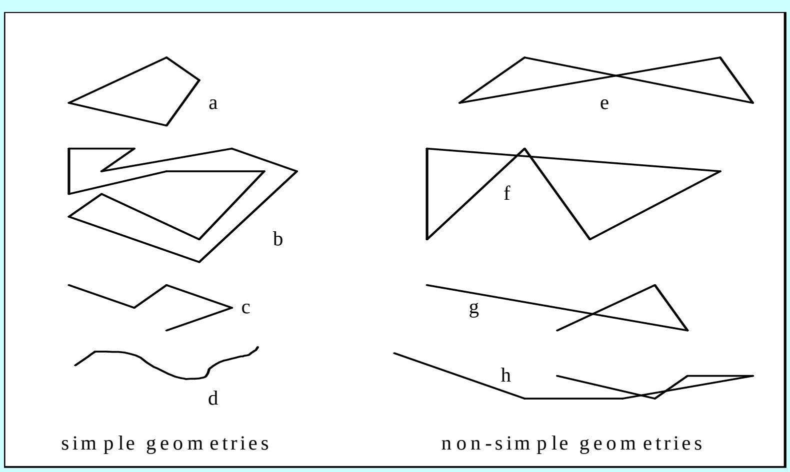
isSimple()



ISO 19107 definition.

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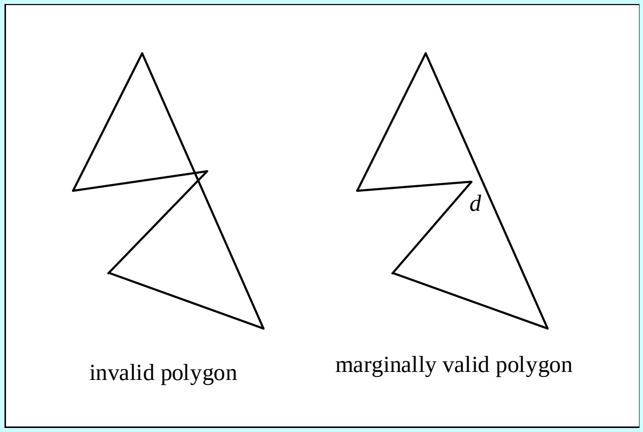
isSimple()



In practice, the right figure is almost certainly "wrong" if *d* is small, but not detected as invalid

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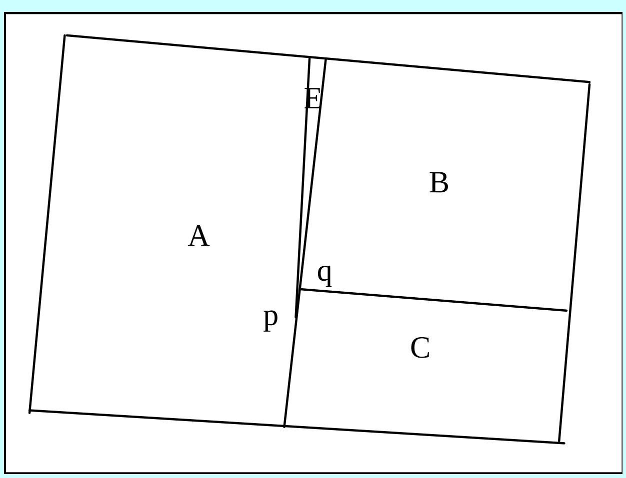
Cadastral Cases



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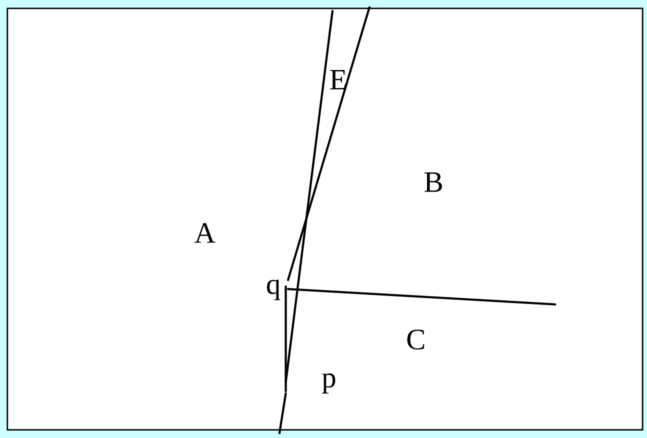
Region around q (enlarged)



Small perturbations may cause topological breakdown

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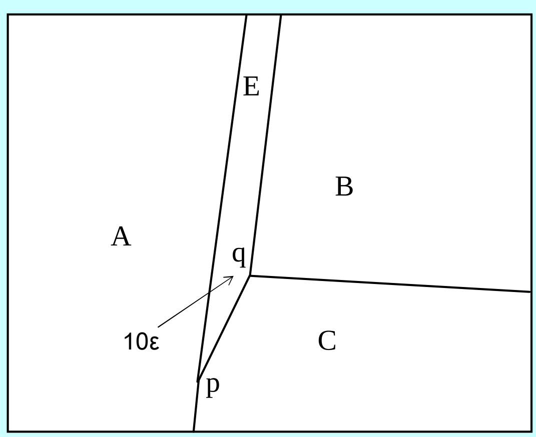
Milenkovic Normalization



Leads to the development of a complete coverage with no possibility of logic failure within that coverage.

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Milenkovic Normalization

BUT

Logic not necessarily closed and associative. The final coverage can vary depending on the order of addition of features to it.

No help with the difficulties with equals() already discussed.

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Realms Approach

Addresses the problem of logic closure.

In effect, pre-calculates the intersections of all objects (in all layers) in the database, to ensure no “surprises” later.

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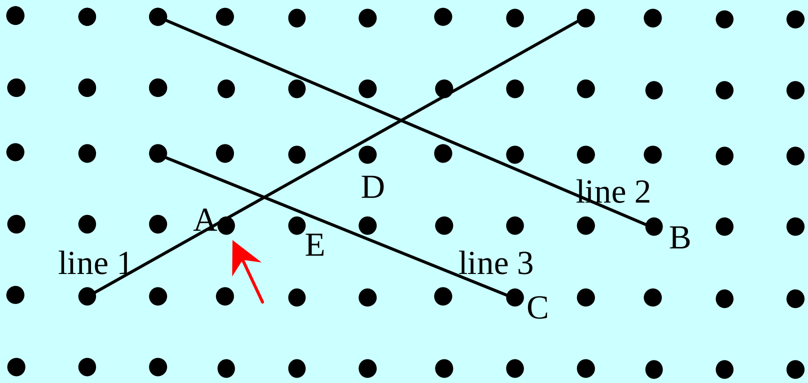
Realms Approach



From: Guting, R. H. & Schneider, M. 1993, 'Realms: A Foundation for Spatial Data Types in Database Systems' *3rd International Symposium on large Spatial Databases (SSD)*.

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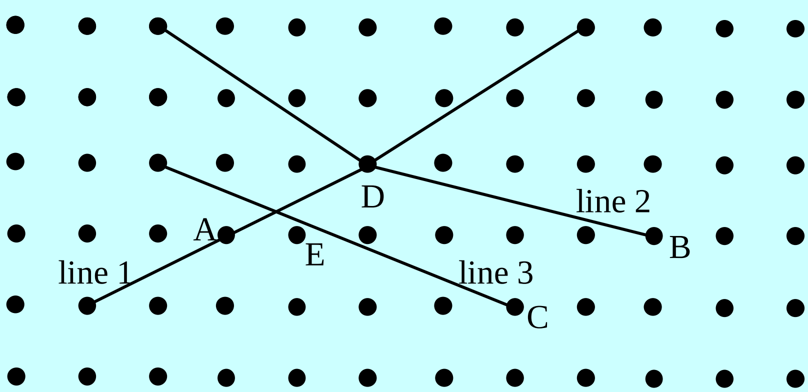
Realms Approach



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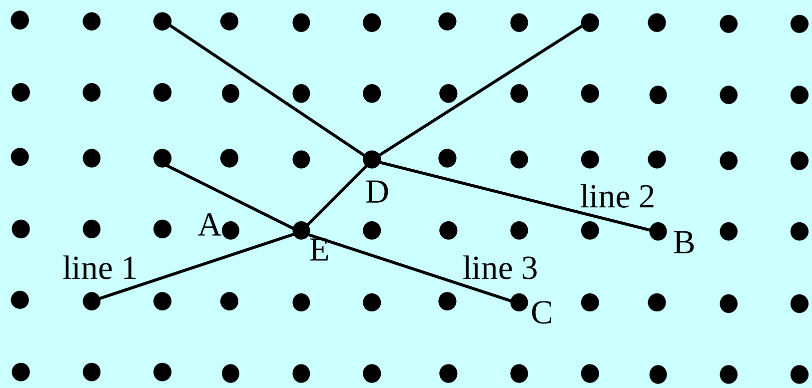
Realms Approach



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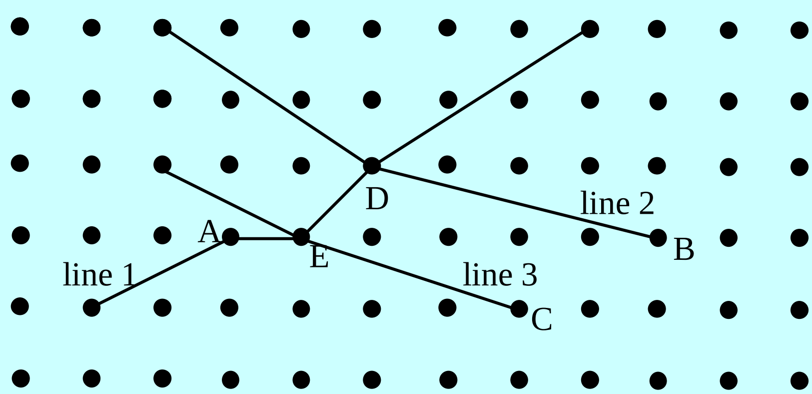
Realms Approach



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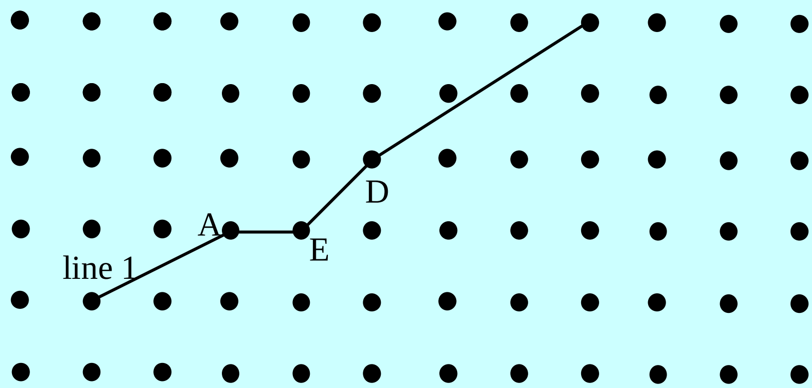
Realms Approach



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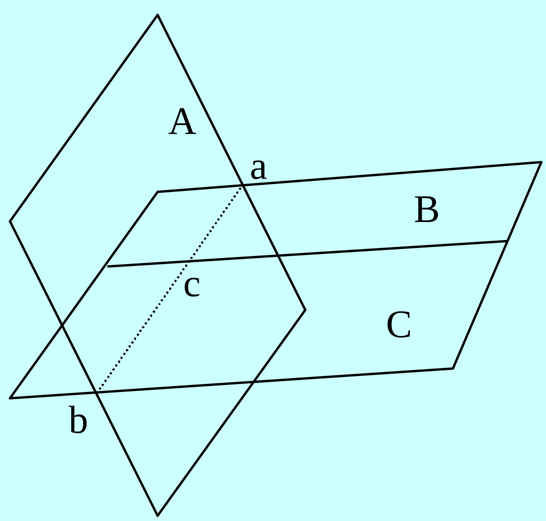
Realms in 3D



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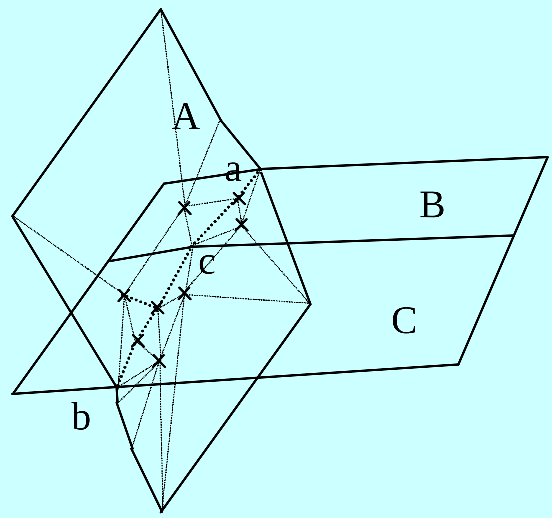
Realms in 3D



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Rational Polygons

(Lemon & Pratt 1998)

The logic is closed and correct.

All operations are associative.

Can lead to a definition of equals().

Define coordinates of spatial primitives using rational coordinate values. (Store each coordinate value as a pair of integers).

The intersection of two lines is defined exactly (no tolerance is required), and it falls on both lines.

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Rational Polygons

But:

This is only true if the integer values are potentially infinite in size – such as the “big integer” of java.

As the database matures, and gets more complex, the big integers get bigger.

It is not possible to deal with irrational numbers – for example where a circular arc intersects a straight line.

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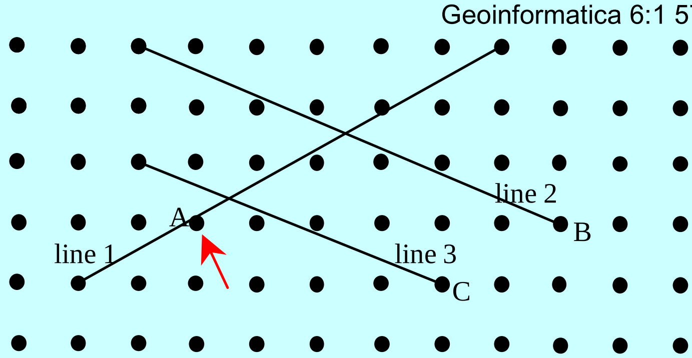
Dual Grid

Lema, J.A.C. and Gueting R.H.
Geoinformatica 6:1 57-67 2002



Lines are defined by points on a grid, but the intersections are points on a very much finer grid.
Cannot, for example, draw a line through the intersection of lines 1 and 2.

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Dual Grid

Provides a closed logic, based on the point-line-polygon paradigm.

Implements the ROSE algebra.

Requires only finite precision integer arithmetic. (But precision requirements are very large).

Does not easily extend to 3D.

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Regular Polytope

Regular – A topological term, referring to a set which is equal to the interior of its closure.

(loosely – a set with no spikes).

Polytope – the generalisation of polygon (2D) and polyhedron (3D) to nD. (not restricted to 2 or 3D).

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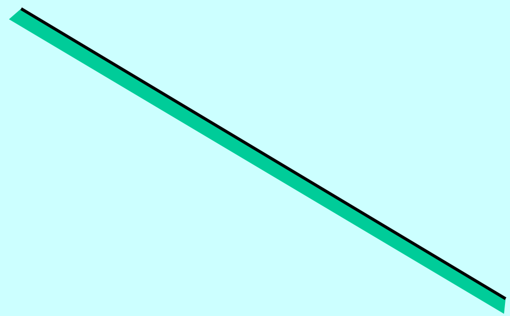
Definition – Half Plane (2D)



$H(A,B,C,S)=\{(X,Y)|((A\otimes X \oplus B\otimes Y \oplus C) > 0) = S,$

$-M<X\leq M, -M<Y\leq M\}$

The inverse of $H(A,B,C,S)$ is defined as $H(A,B,C,not(S))$.

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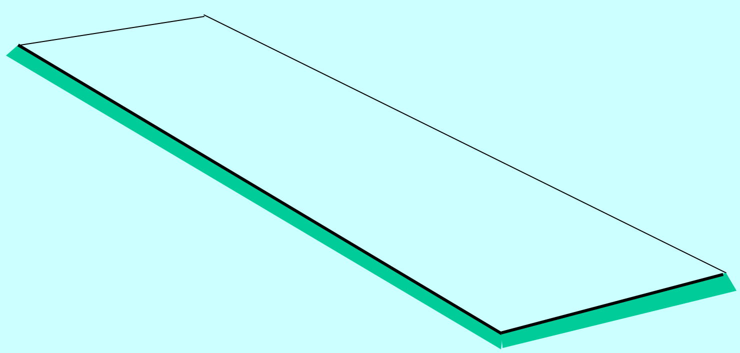
Definition – Half Space (3D)



$H(A,B,C,D,S)=\{(X,Y)|((A\otimes X \oplus B\otimes Y \oplus C \otimes Z \oplus D) > 0) = S,$

$-M<X\leq M, -M<Y\leq M, -M<Z\leq M\}$

The inverse of $H(A,B,C,D,S)$ is defined as $H(A,B,C,D,not(S))$.

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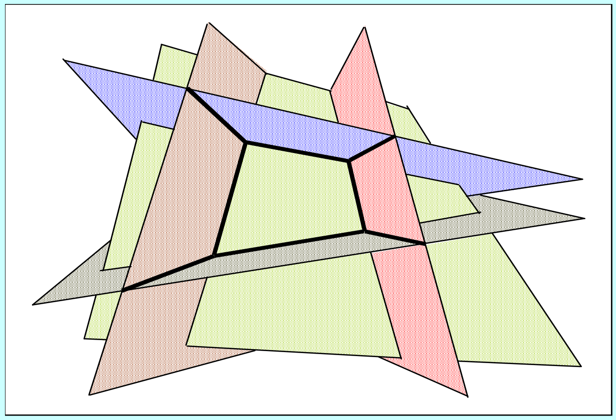
Definition – Convex Polytope



Defined as the intersection of a set of half spaces

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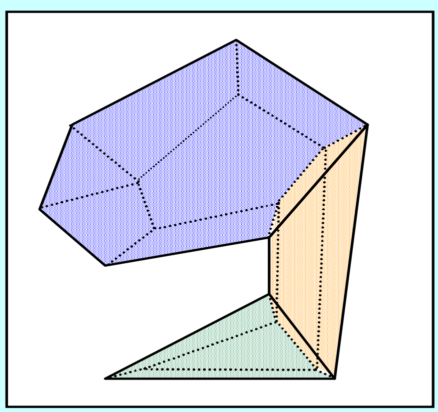
Definition – Regular Polytope



Defined as the union of a set of convex polytopes.

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(closely related to the polygon chain of constraint db approaches)

Regular Polytope

Regular Polytopes by this definition can be shown to form the basis of a **topological space**.

Note – the representations themselves span the topological space, not approximation to a topological space.

Axioms of a Topological Space

$O_0 \in \boldsymbol{O}$ and $O_\infty \in \boldsymbol{O}$

if $O_1 \in \boldsymbol{O}$ and $O_2 \in \boldsymbol{O}$ then $O_1 \cap O_2 \in \boldsymbol{O}$

if $O_i \in \boldsymbol{O}$ for all $i \in I$ then $\bigcup_{i \in I} O_i \in \boldsymbol{O}$

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Regular Polytope

Regular Polytopes support a closed, rigorously defined logic, and the operations are consistent.

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Regular Polytope

• All regular polytopes are finite point sets – there is only a finite number of values for x, y, and z that satisfy the criterion of the regular polytope.

• For this reason, the Regular Polytope is not continuous – it is a grid of points. (Closely related to the finer grid of the "Dual grid").

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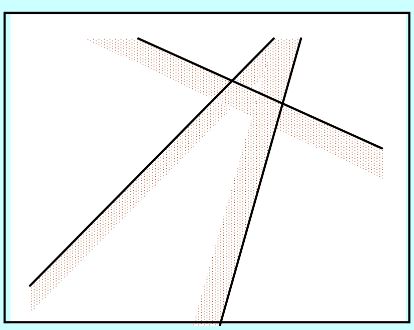
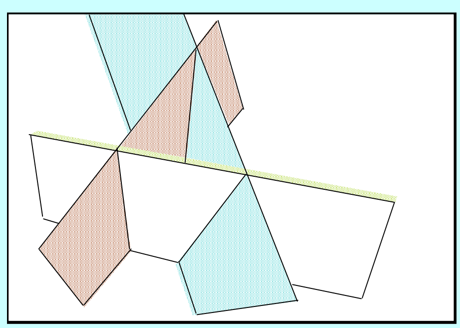
Regular Polytope



2D



3D

• A convex polytope need not be fully bounded, thus a Regular Polytope need not be fully bounded. (This is a consequence of every Regular Polytope having an inverse)

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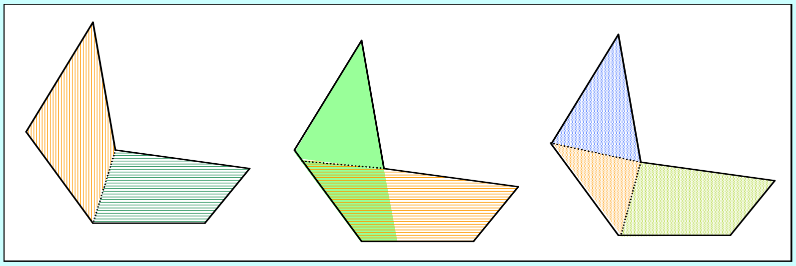
Regular Polytope



The way of defining a region as a regular polytope is not unique.

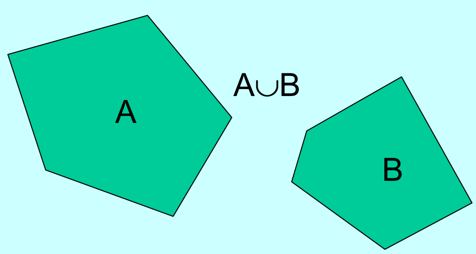
The convex polytopes can overlap, and can meet in different ways.

Many definitions can produce the same point set.

equals() can be defined as point set equality.

Regular Polytope - Contiguity

The definition of RP does not imply contiguity, and in fact cannot – since the closure under union prevents this.



Regular Polytope Usefulness

• No value in the Regular Polytope concept if it cannot be used to represent real world features.

• Can be converted to/from conventional polyhedra.

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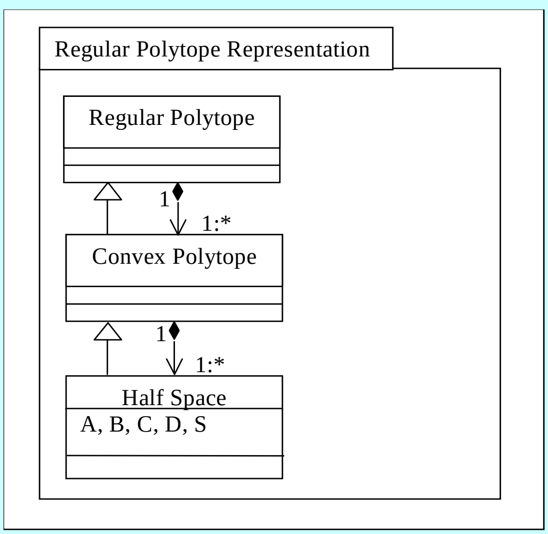
Storage Schema



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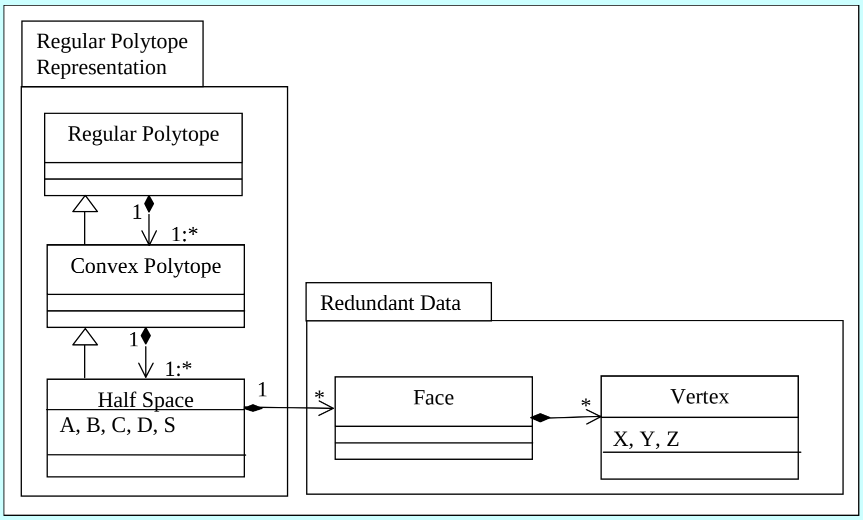
Storage – with redundant vertices



Storage Requirements

In summary – using redundant vertices, about double the requirements of conventional polygon or polyhedral storage.

But

The approach leads to a closed predictable logic, and a robust digital representation.

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Further Research

- Practical proof of concept
  (development of a prototype)
- Extension to floating point.
- Lower dimensionality objects
  (lines,points)
- “Topology” in GIS sense.
- “Approximated Polytope” – a variant
  storage structure.
- Other than straight lines

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