Towards a Rigorous Logic for Spatial Data Representation

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Overview

- The problem
- The regular polytope approach
- Implementation issues
- Conclusion

Reasoning from Data

- Is it possible to determine the correctness of propositions from data stored in a computer?

    E.g. accounts with balance ≥ 0 are solvent. This account has a balance of 5 Euros – is this account solvent?

- Is this possible for spatial data?

    E.g. Land within 5km of the city centre is classed “urban”. This parcel of land is 2km from the city centre – is it “urban”?

Design by Contract

- Is computer software prepared to "trust" other software?

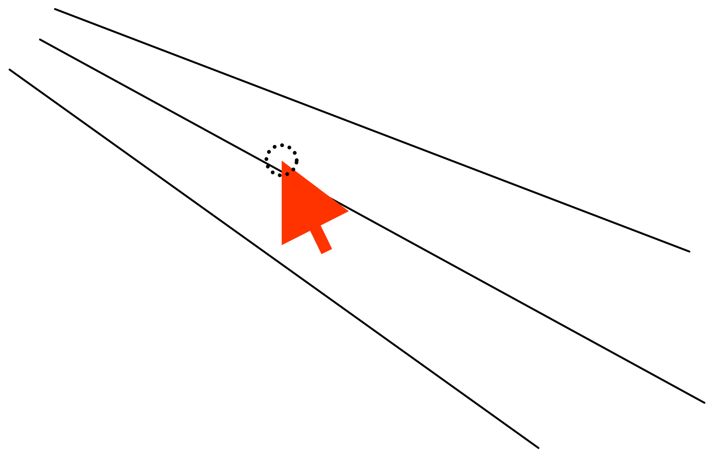
  (The alternative is "defensive programming" – for example, before using a polygon, it must be validated).

- Defensive programming is very expensive – especially for spatial data.

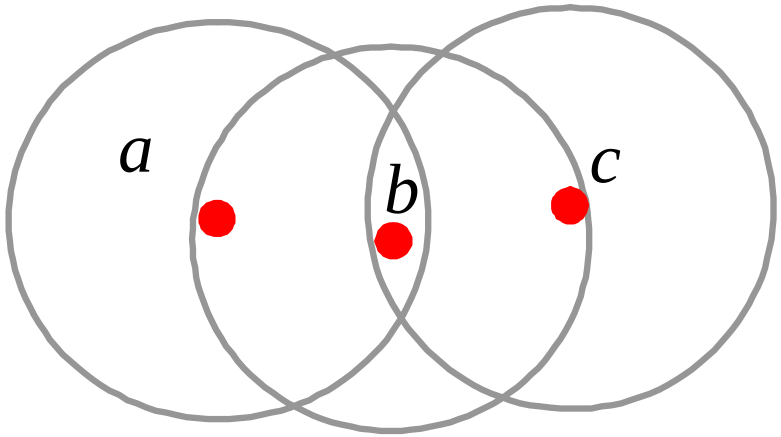
  (A particular example is spatial data interchange – is it necessary to re-validate data on receipt?)

Imprecision in Calculations

- Computer calculations do not use real numbers.
- Precision is finite. Rounding happens.
- It is common to use “tolerance” in calculations to provide reasonable answers.

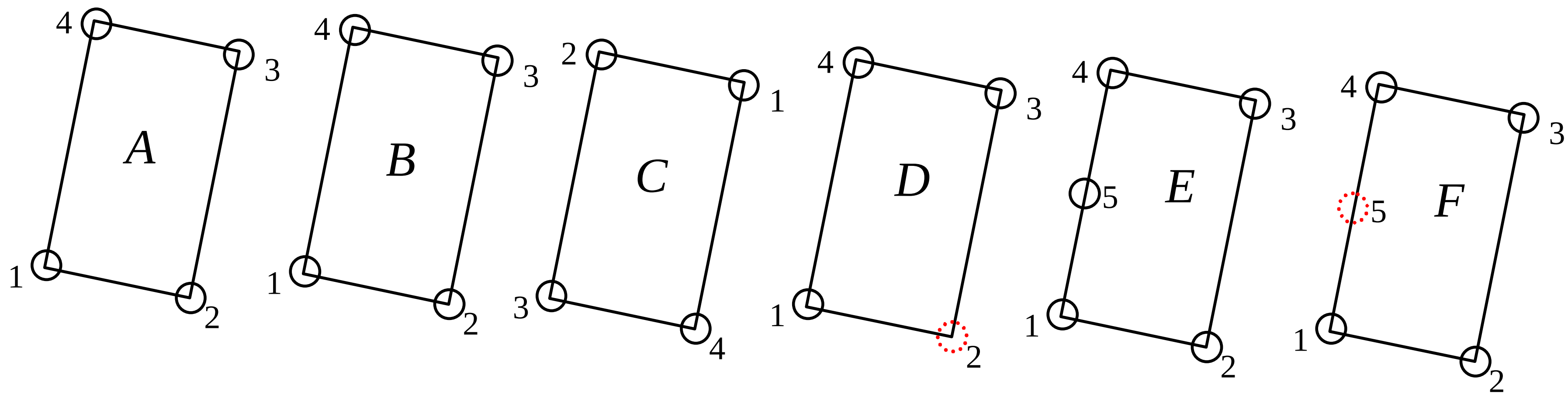


Test for Equality



$$a = b \quad b = c \quad \text{but} \quad a \neq c.$$

Equality



Points marked with a complete circle are exactly correct.

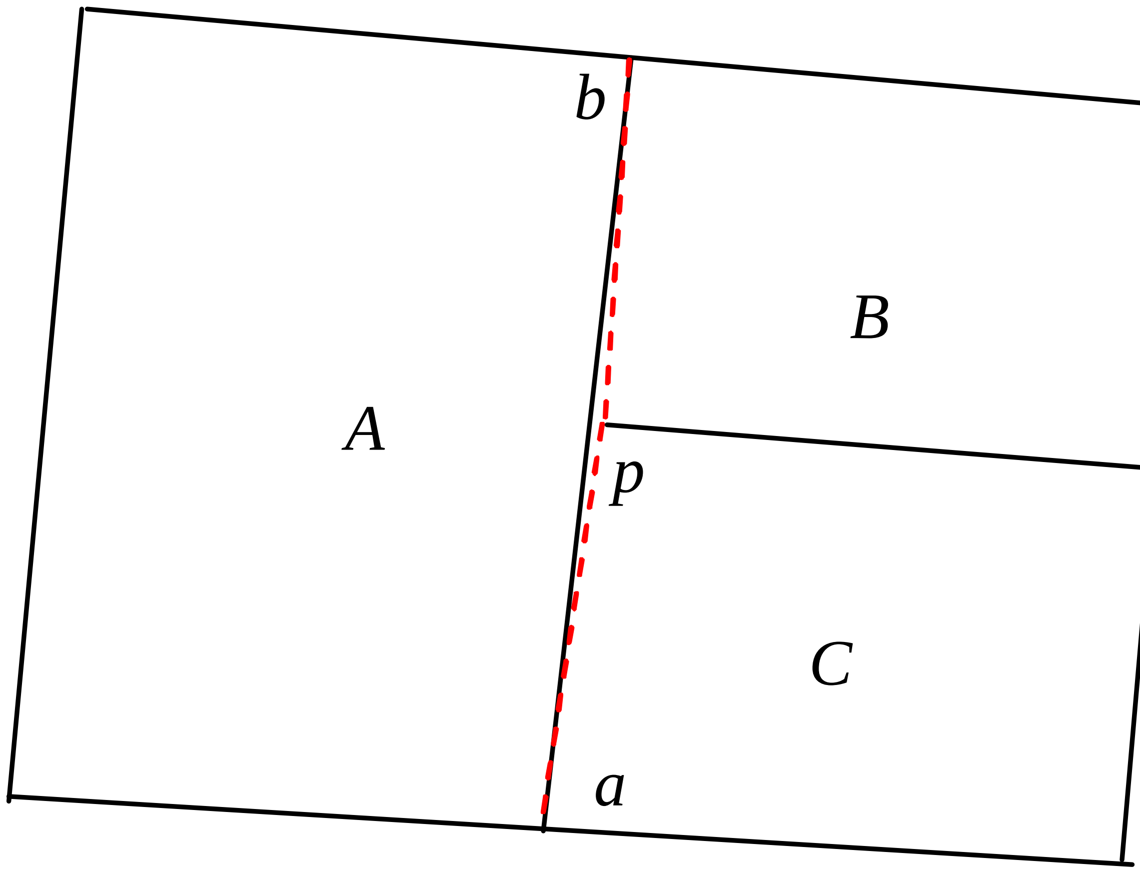
Points marked with a dashed circle are correct to within tolerance.

All these polygons are equal to A by the ISO 19107 definition.

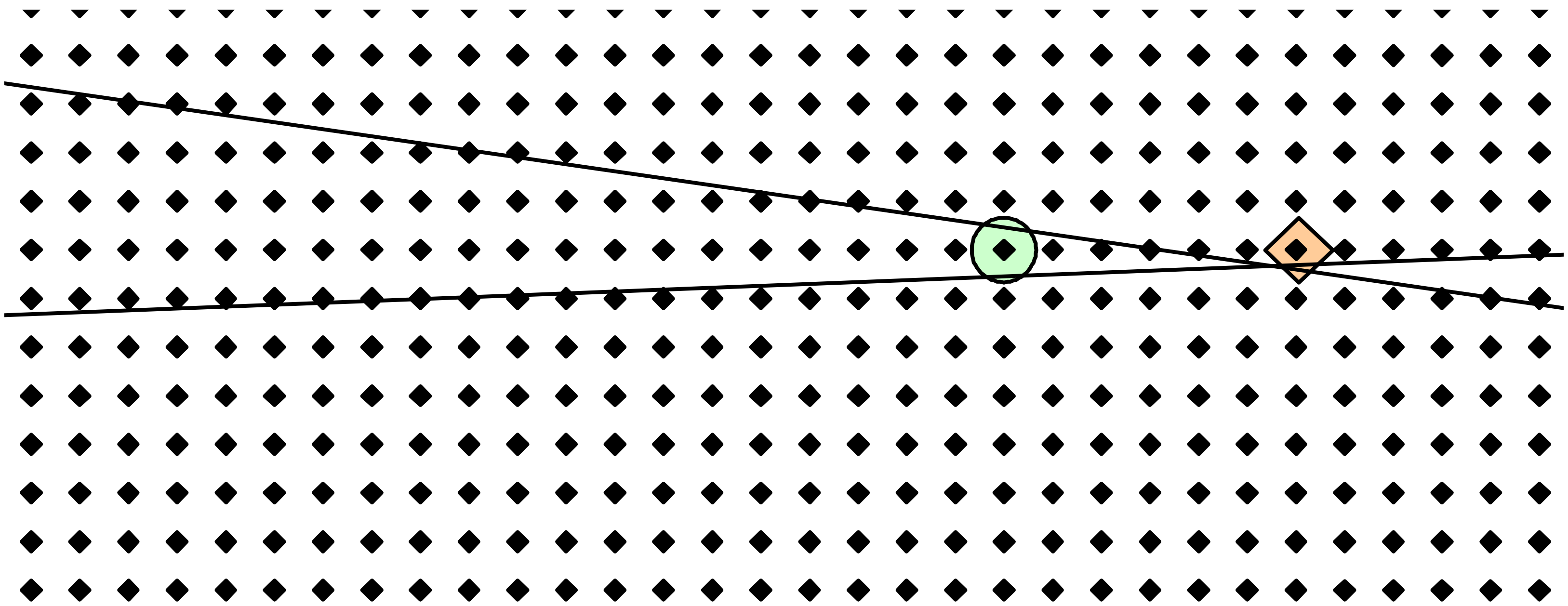
(Note – in all cases the sense is the same).

Adjoining Polygons

To allow for point *p* not being exactly on the line, the definition of *A* changes.

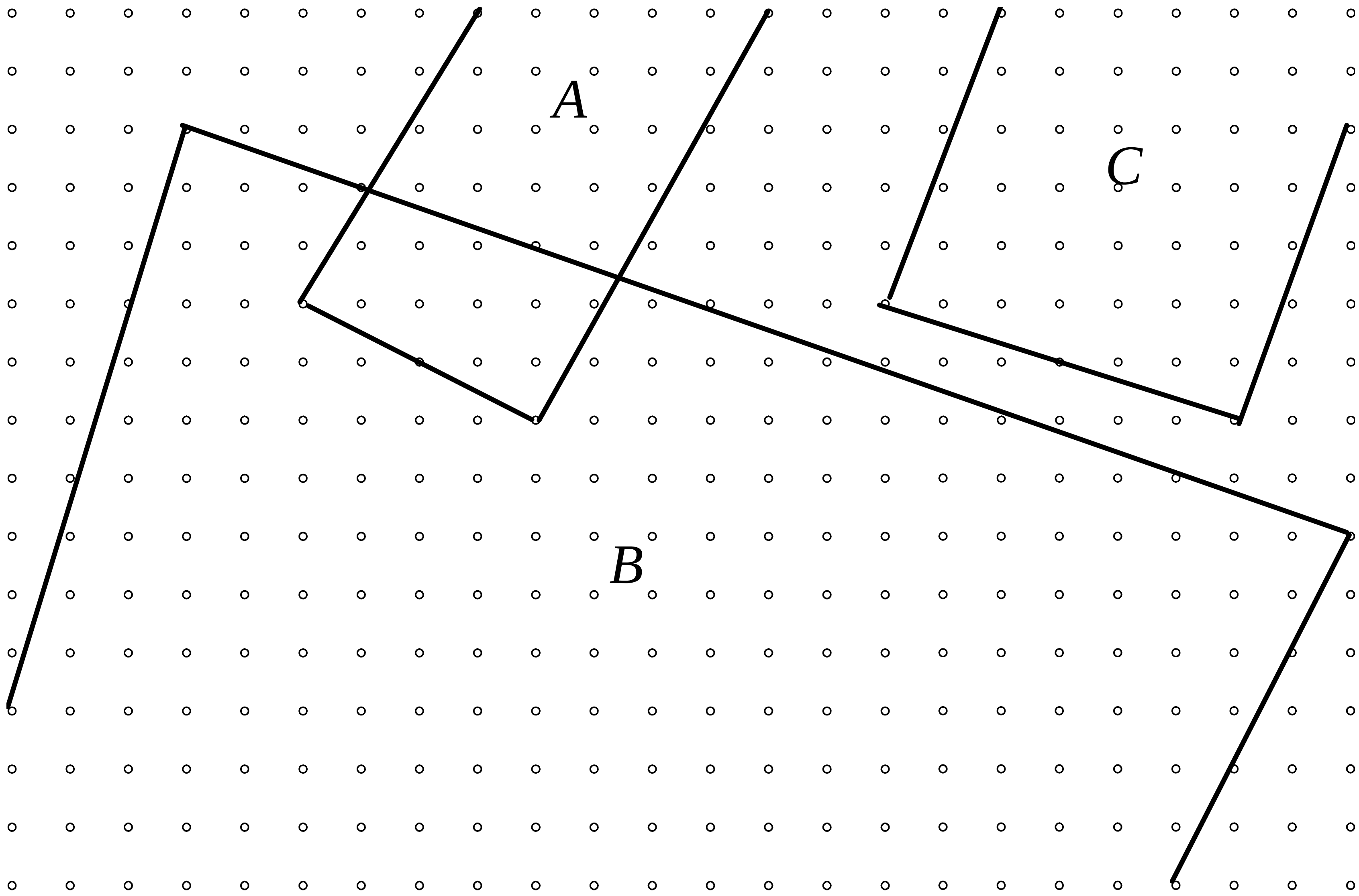


Tolerance in Calculation of Intersection

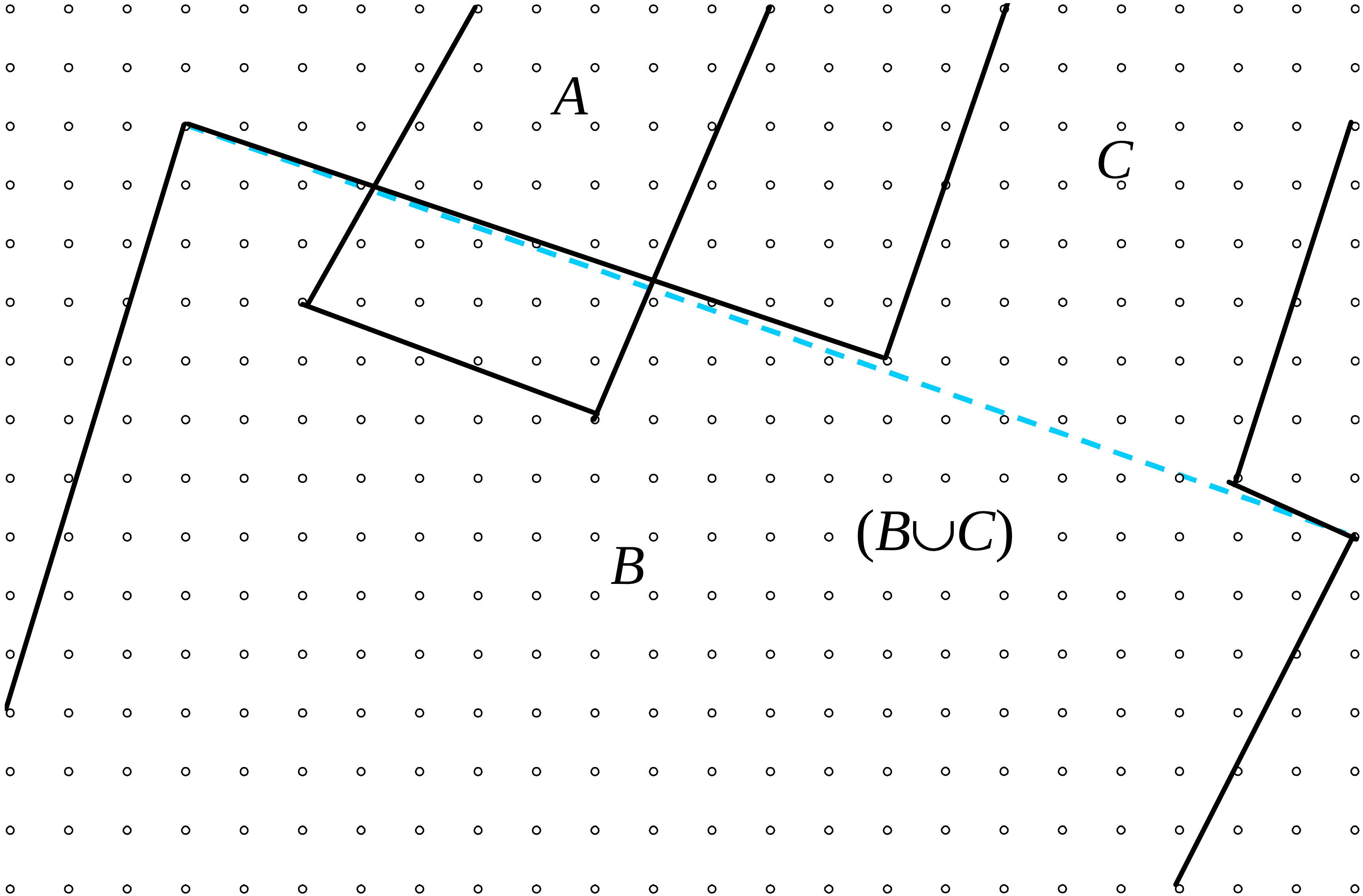


In almost all spatial data representations, the positions of points are represented rounded to the nearest grid point.

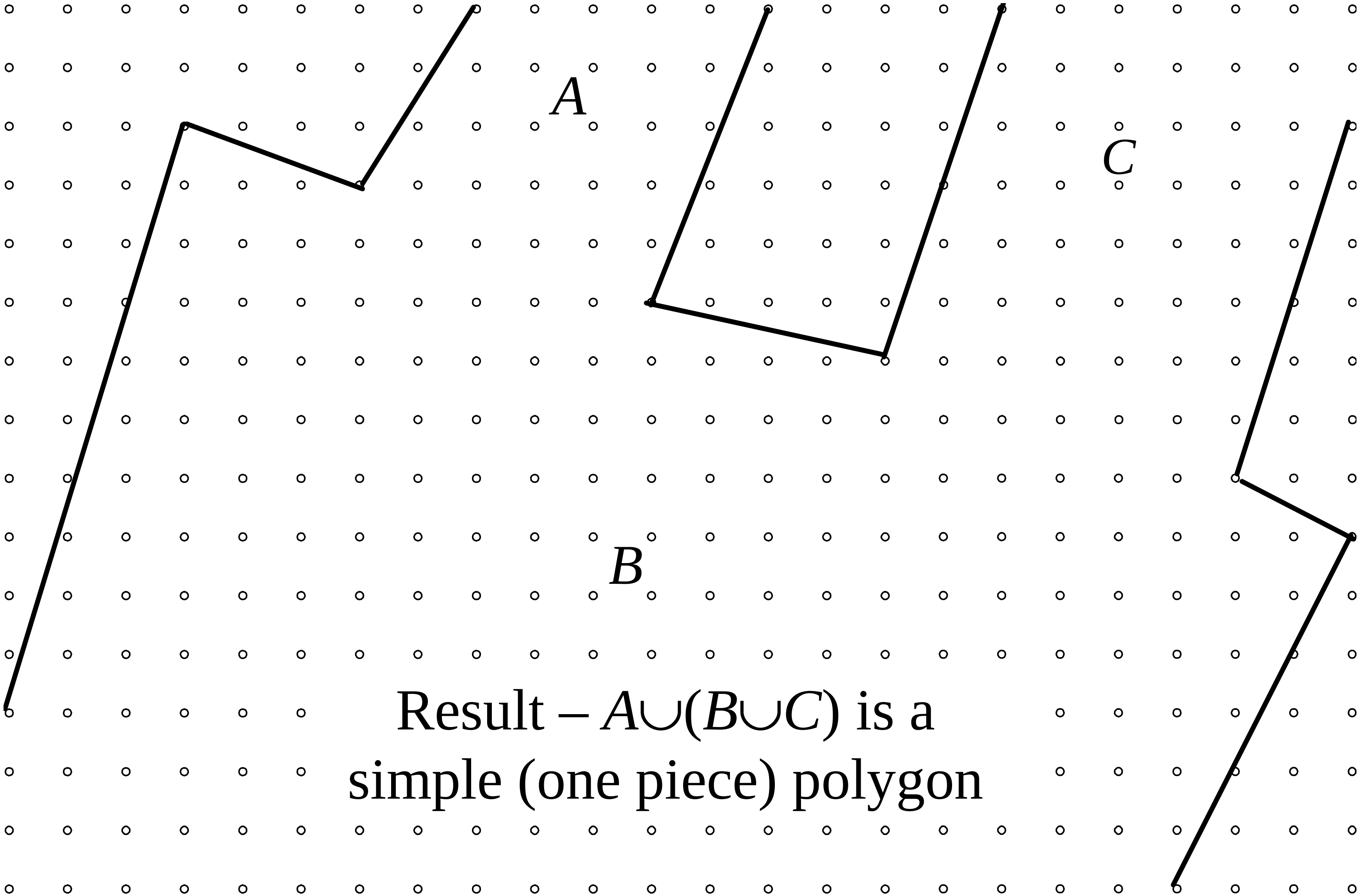
Associativity of Operations



Associativity of Operations

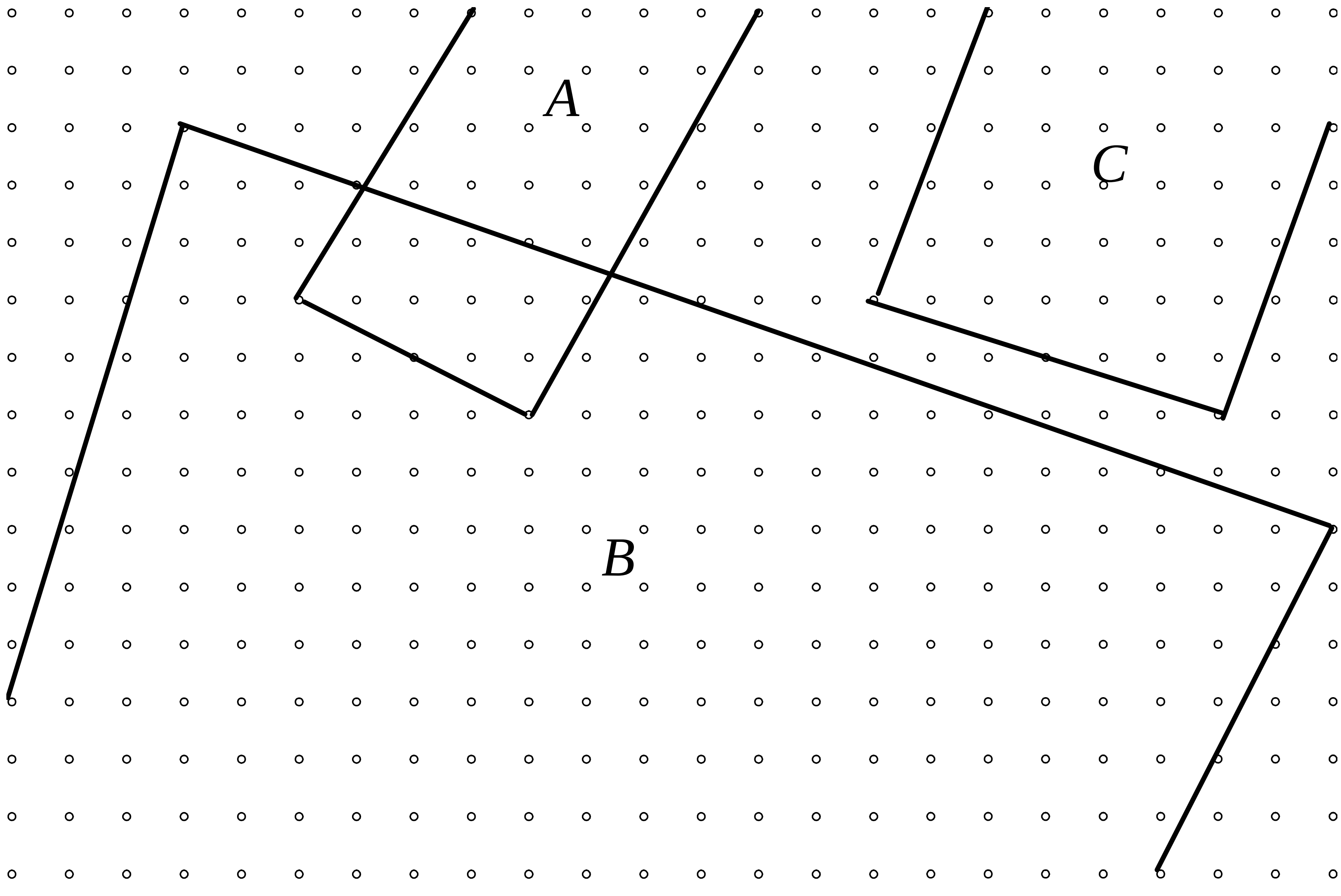


Associativity of Operations

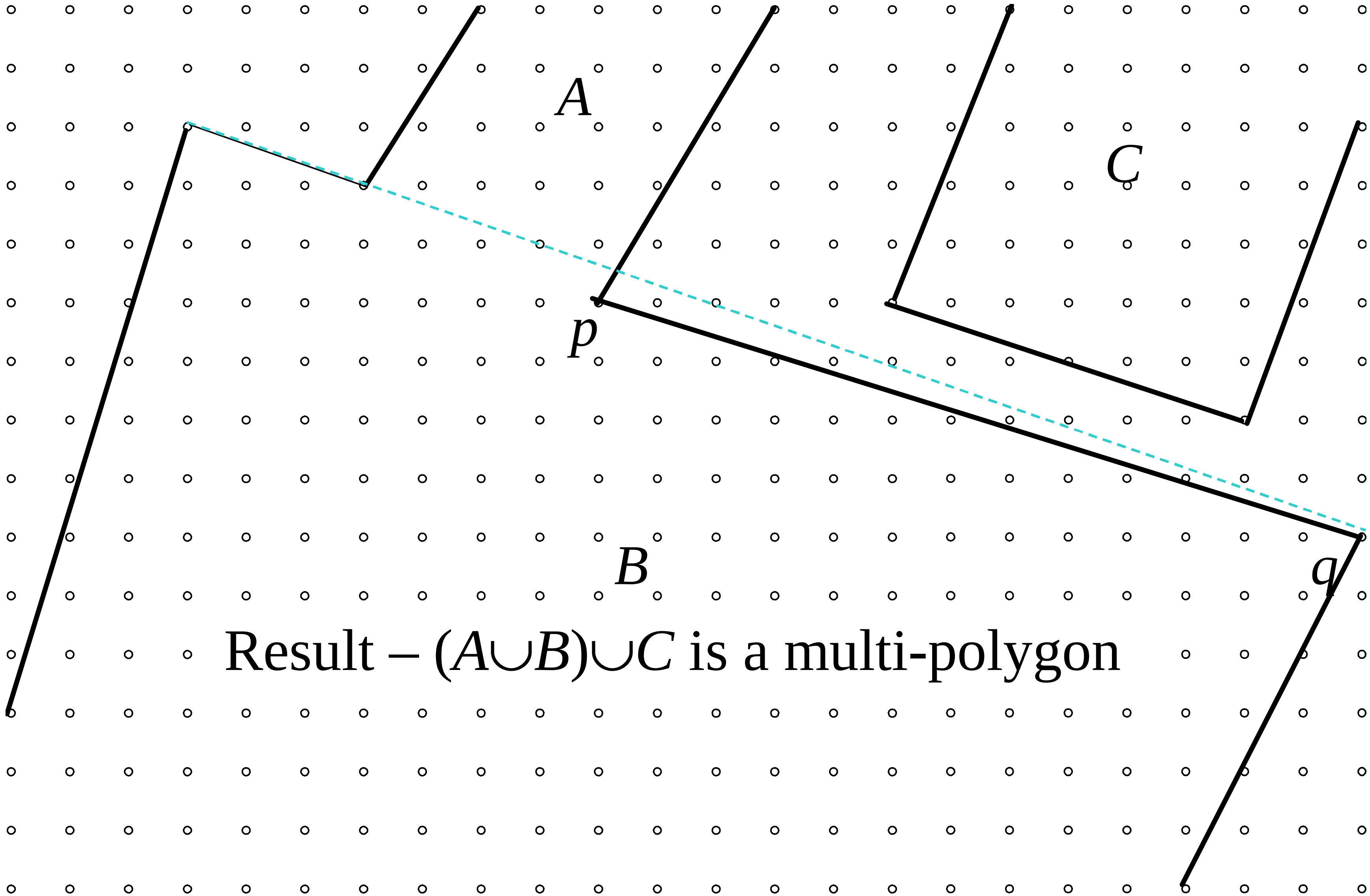


Result – $A\cup(B\cup C)$ is a simple (one piece) polygon

Associativity of Operations



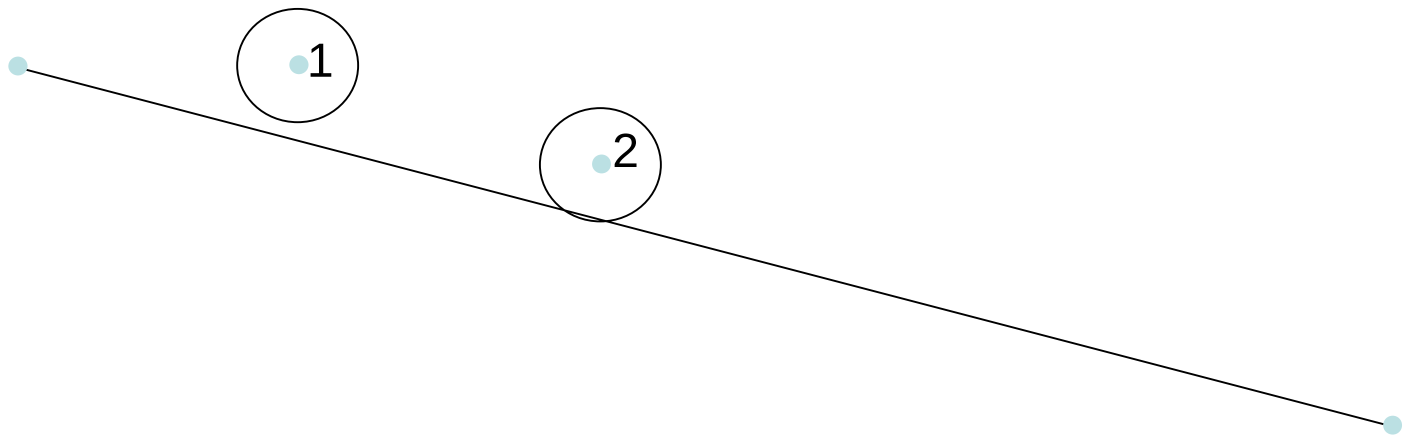
Associativity of Operations



Result – $(A \cup B) \cup C$ is a multi-polygon

It is common for the result of an operation to invalidate the result of earlier operations.

e.g. checking that no points are within a minimum distance of any line.



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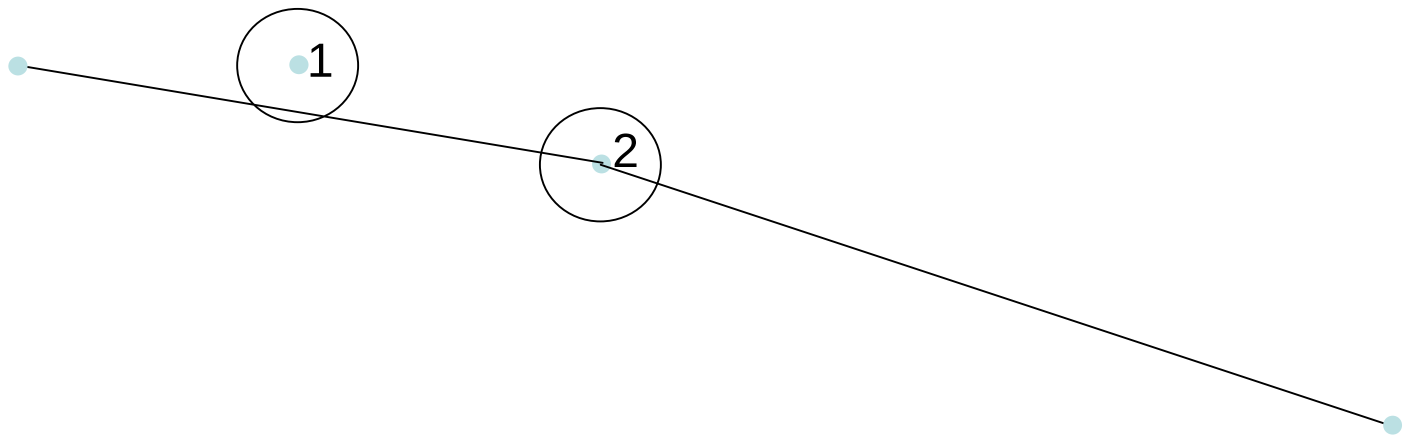
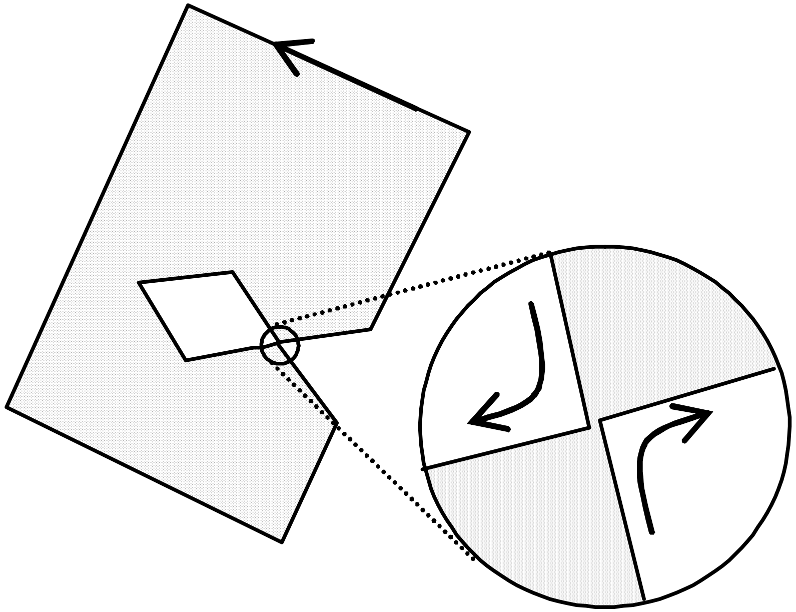
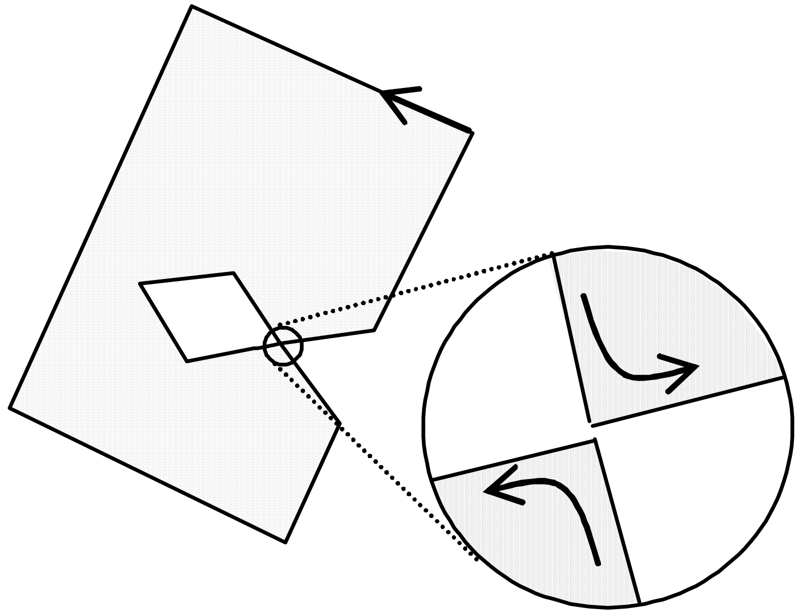


Variation of Representation

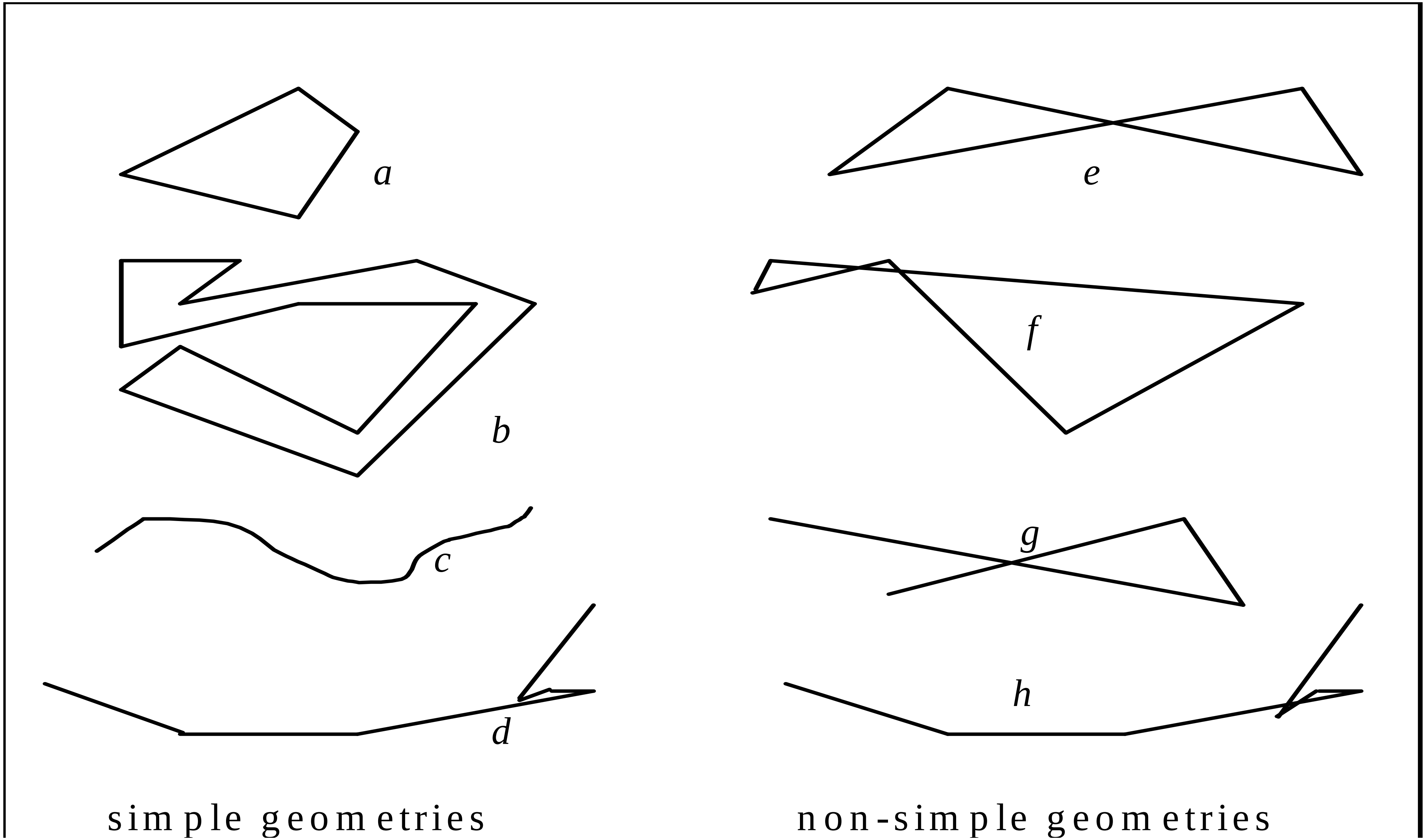


Represented as a polygon with a hole



Represented as a polygon with a continuous (one piece) outer boundary

Validity



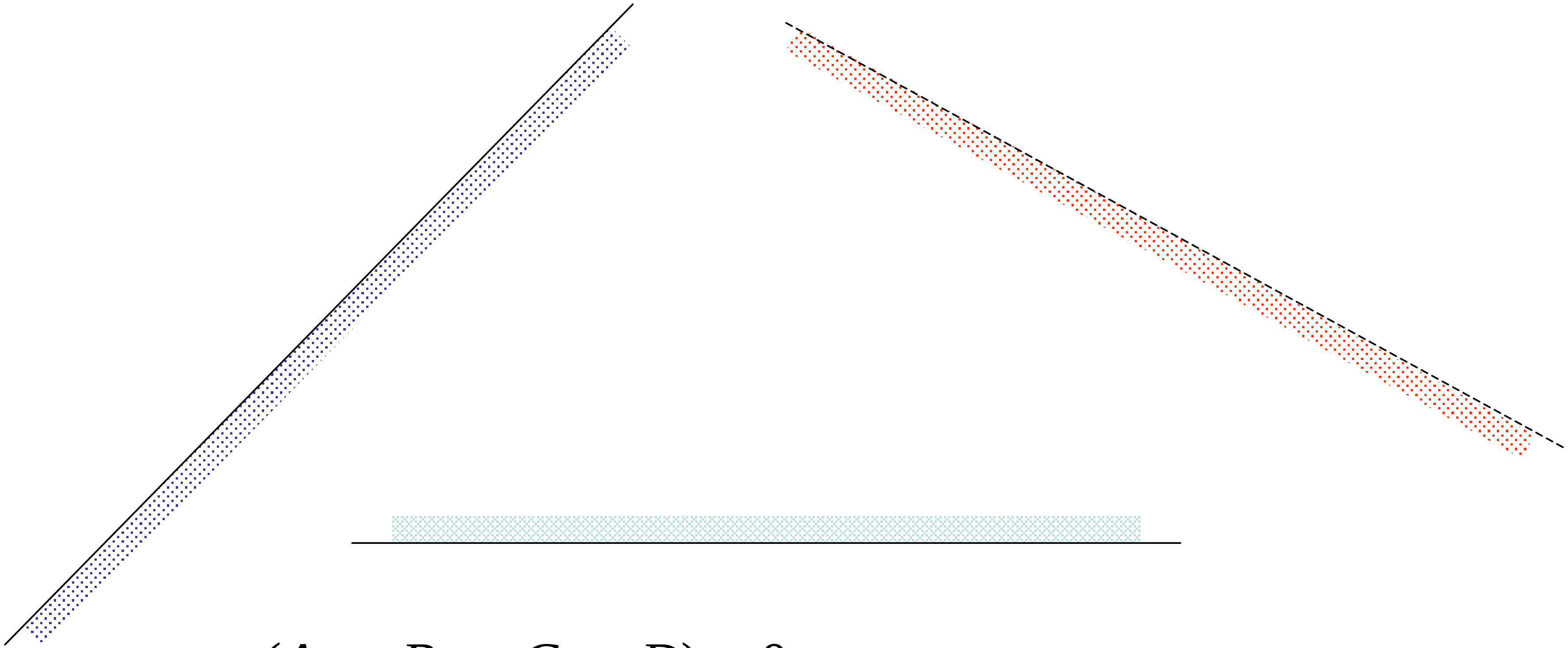
The Regular Polytope

- Definition
- Behaviour
- Connectivity
- Algebra

What Do We Want

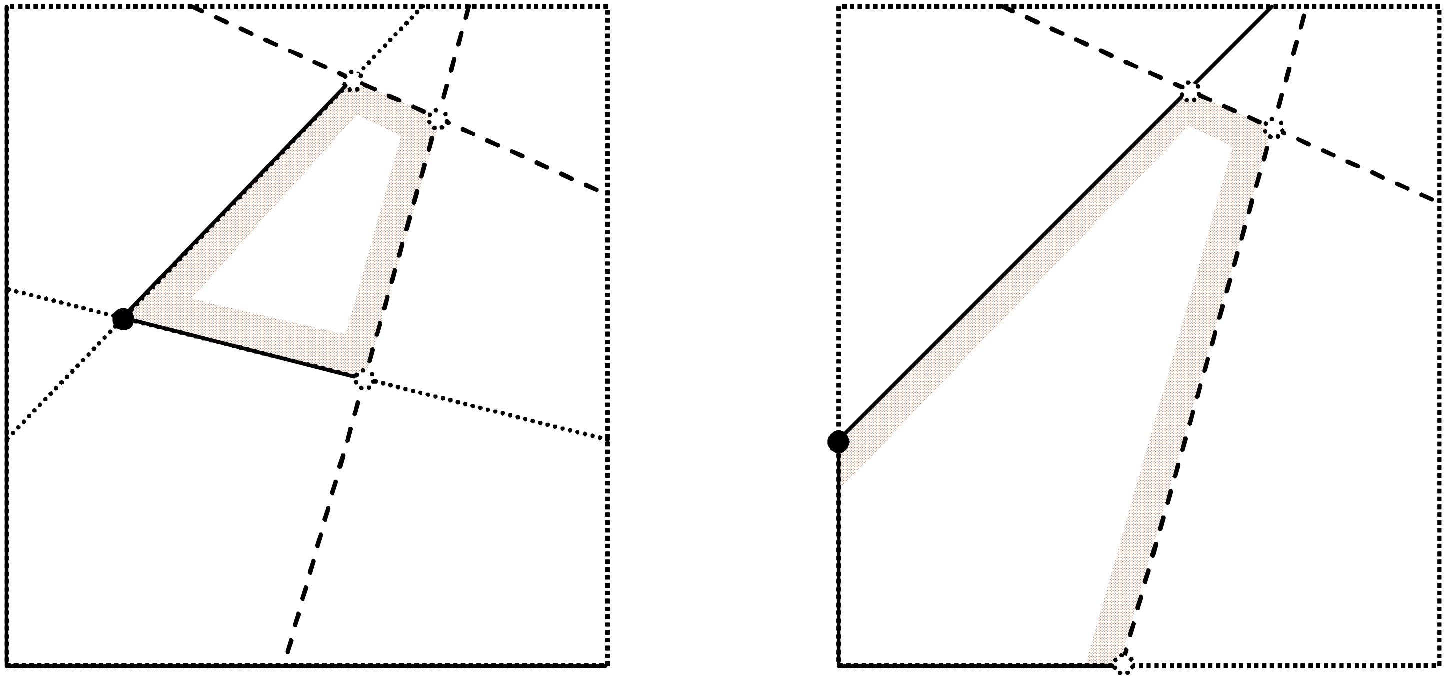
- Consistency of operations
- Reliable spatial data interchange
- Rigorous definitions of validity and equality
- Robustness of storage representation

Half Space



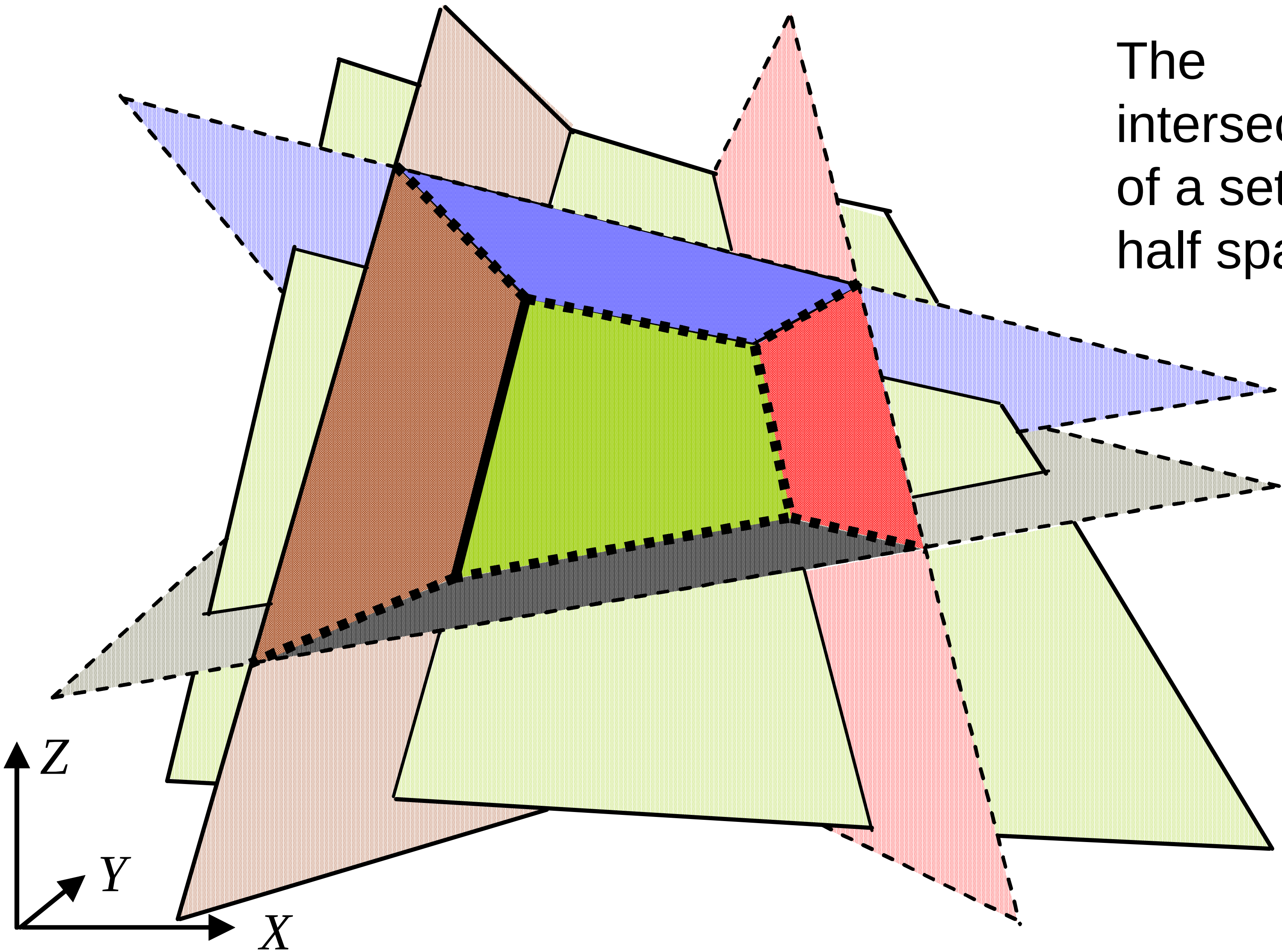
$(Ax + By + Cz + D) > 0$ or
$[(Ax + By + Cz + D) = 0$ and $A > 0]$ or
$[(By + Cz + D) = 0$ and $A=0$ and $B>0]$ or
$[(Cz + D) = 0$ and $A=0, B=0$ and $C>0]$,

Convex Polytopes



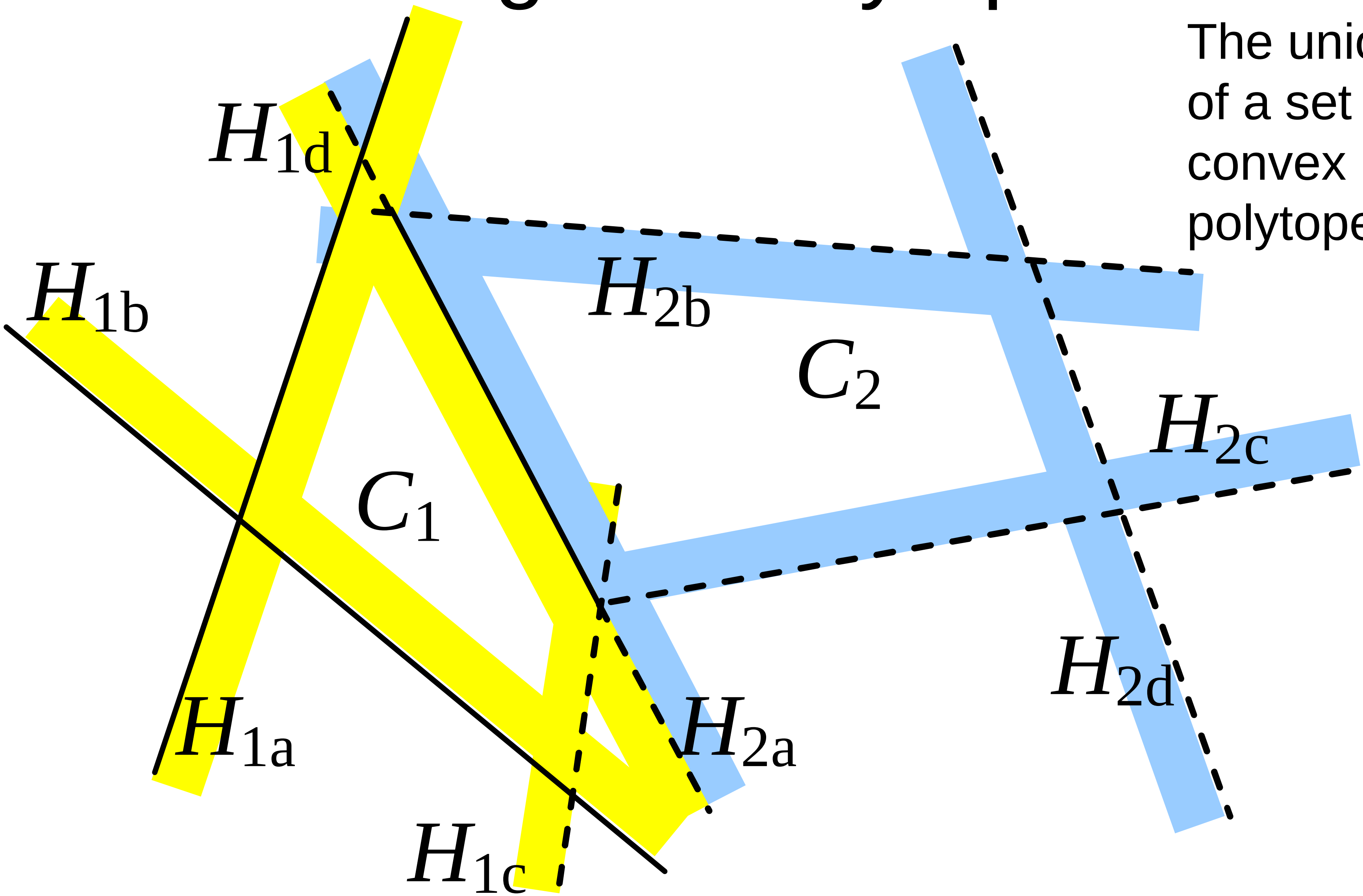
$(A_i x + B_i y + C_i z + D_i) > 0$ or
$[(A_i x + B_i y + C_i z + D_i) = 0$ and $A_i > 0]$ or
$[(B_i y + C_i z + D_i) = 0$ and $A_i=0$ and $B_i>0]$ or
$[(C_i z + D_i) = 0$ and $A_i=0$, $B_i=0$ and $C_i>0]$,

Convex Polytope



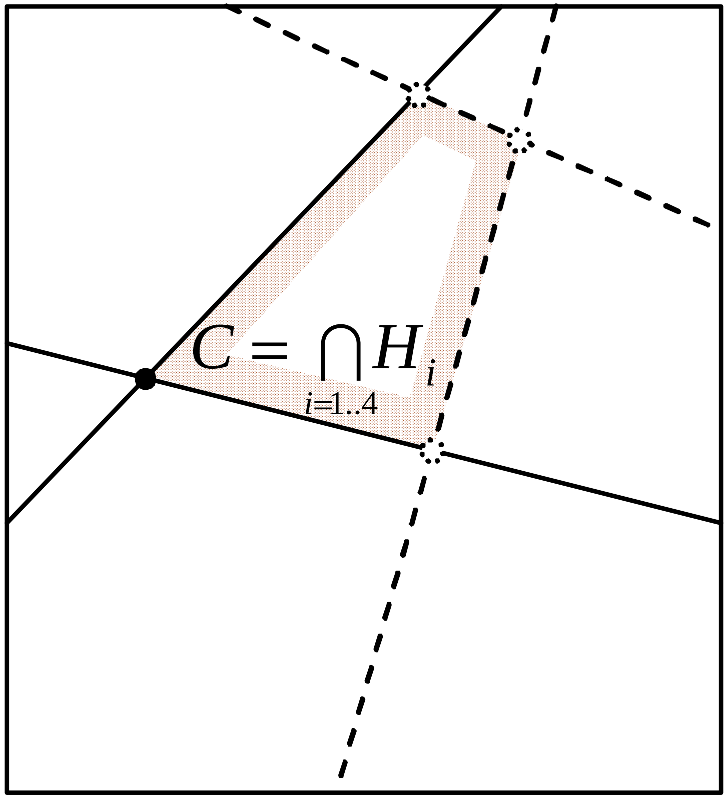
The intersection of a set of half spaces

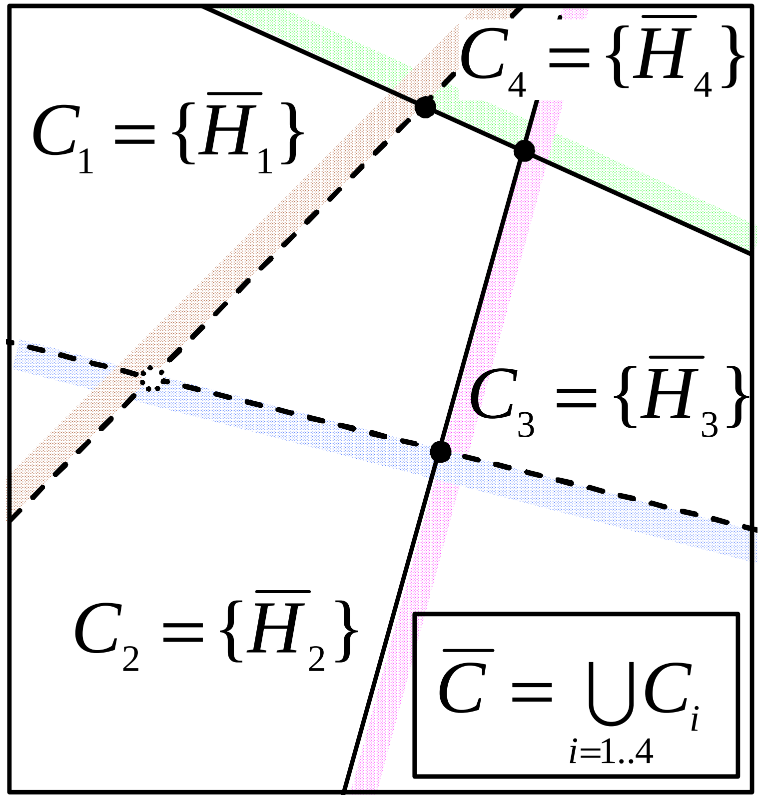
Regular Polytope



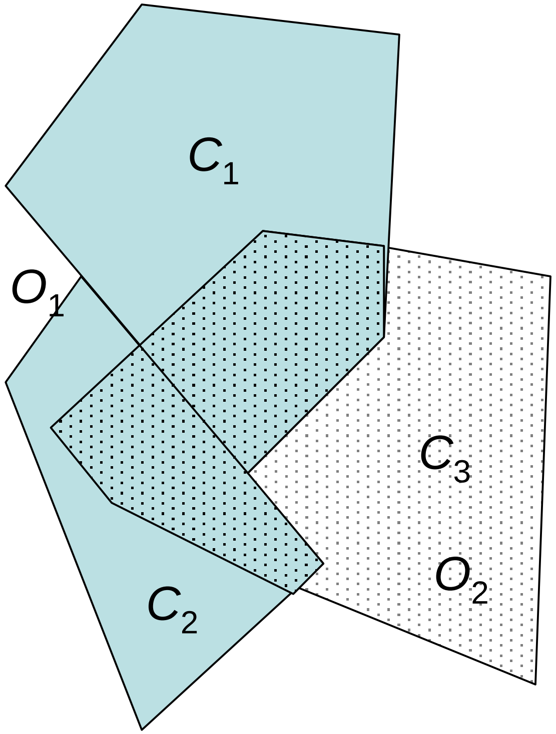
The union of a set of convex polytopes

Complement of a Convex Polytope





Union and Intersection of Regular Polytopes

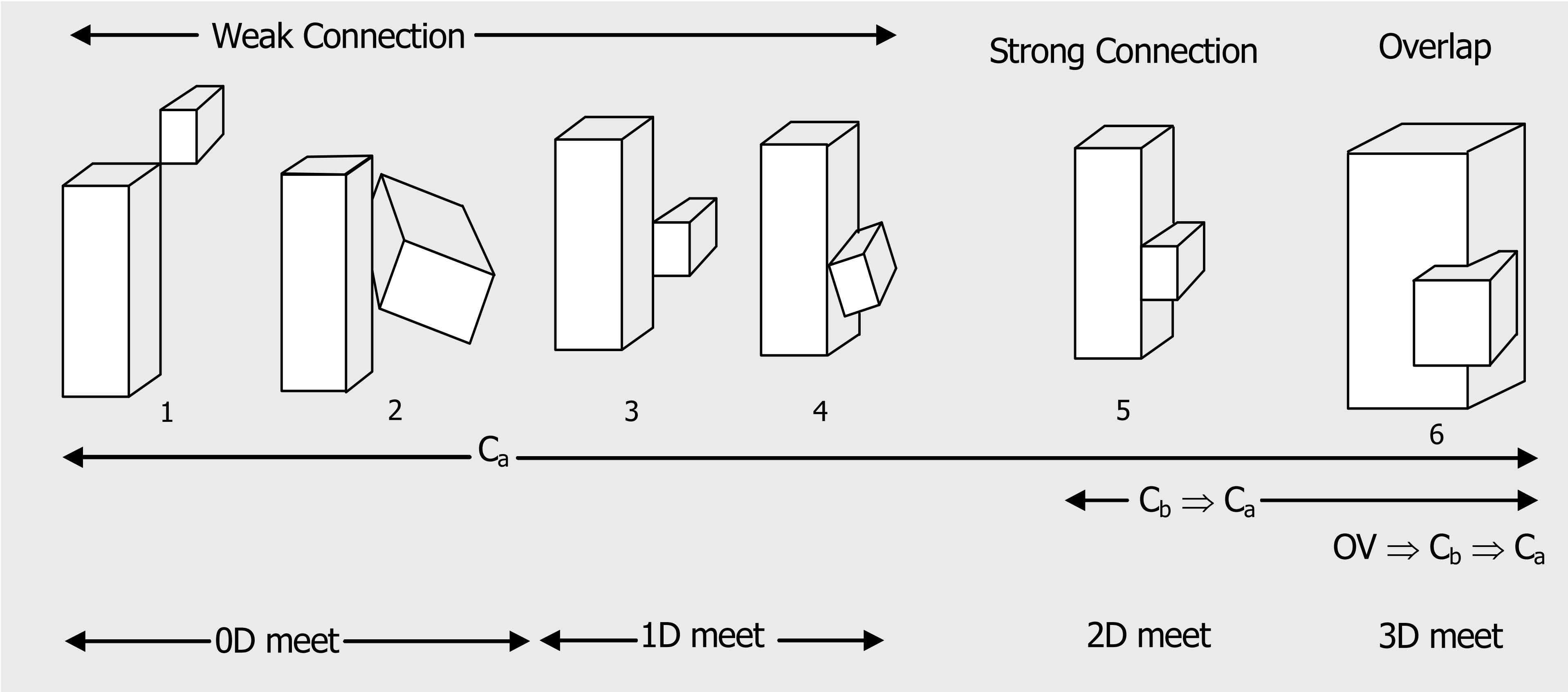


$$O_1 = \{C_1, C_2\},\ O_2 = \{C_3\}$$

$$O_1 \cup O_2 = \{C_1, C_2, C_3\}$$

$$O_1 \cap O_2 = \{C_1 \cap C_3, C_2 \cap C_3\}$$

Connectivity



Interpretation of Regular Polytopes

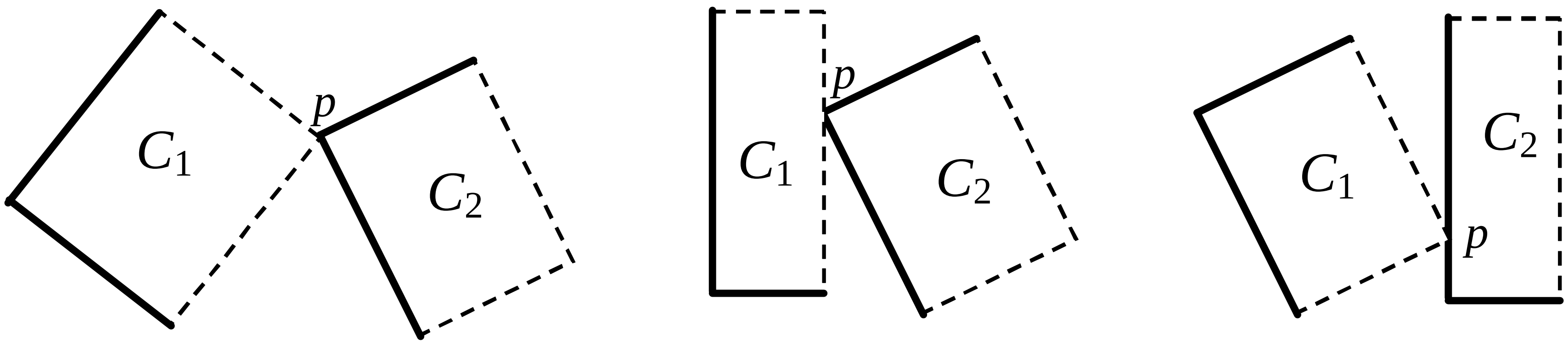
$$Ax + By + Cz + D : 0$$

Interpretation of (x, y, z)	Topological space	Metric space	C_a	C_b
Floating Point	y	n?	?	?
Integer	y	y	y	not satisfactory
Dr-Rational	y	y	y	y

Domain-Restricted Rational Numbers

- A rational number *r* is defined as *P*/*Q*, where P,Q are integers.
- It is possible to avoid the problems caused by gridded representations by letting *P* and *Q* get arbitrarily large. (But they can get very large indeed).
- This dr-rational approach limits the size of *P* and *Q*, and thus is a gridded representation, but preserves the rigour.

C_a Connectivity

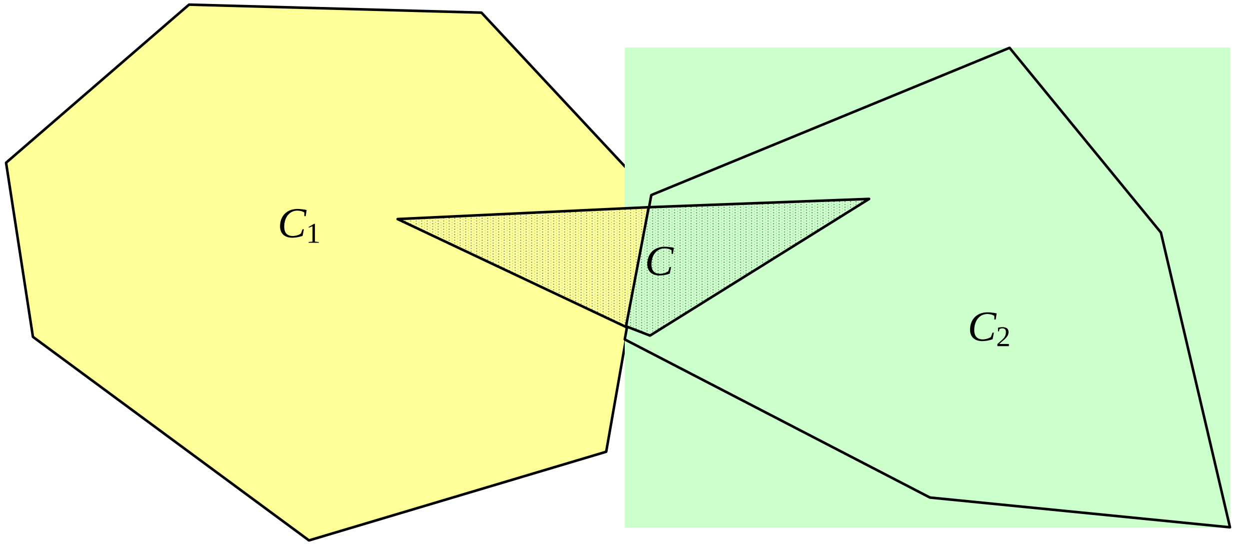


Requires the concept of “pseudo-closure”.

$$(Ax + By + Cz + D) \geq 0$$

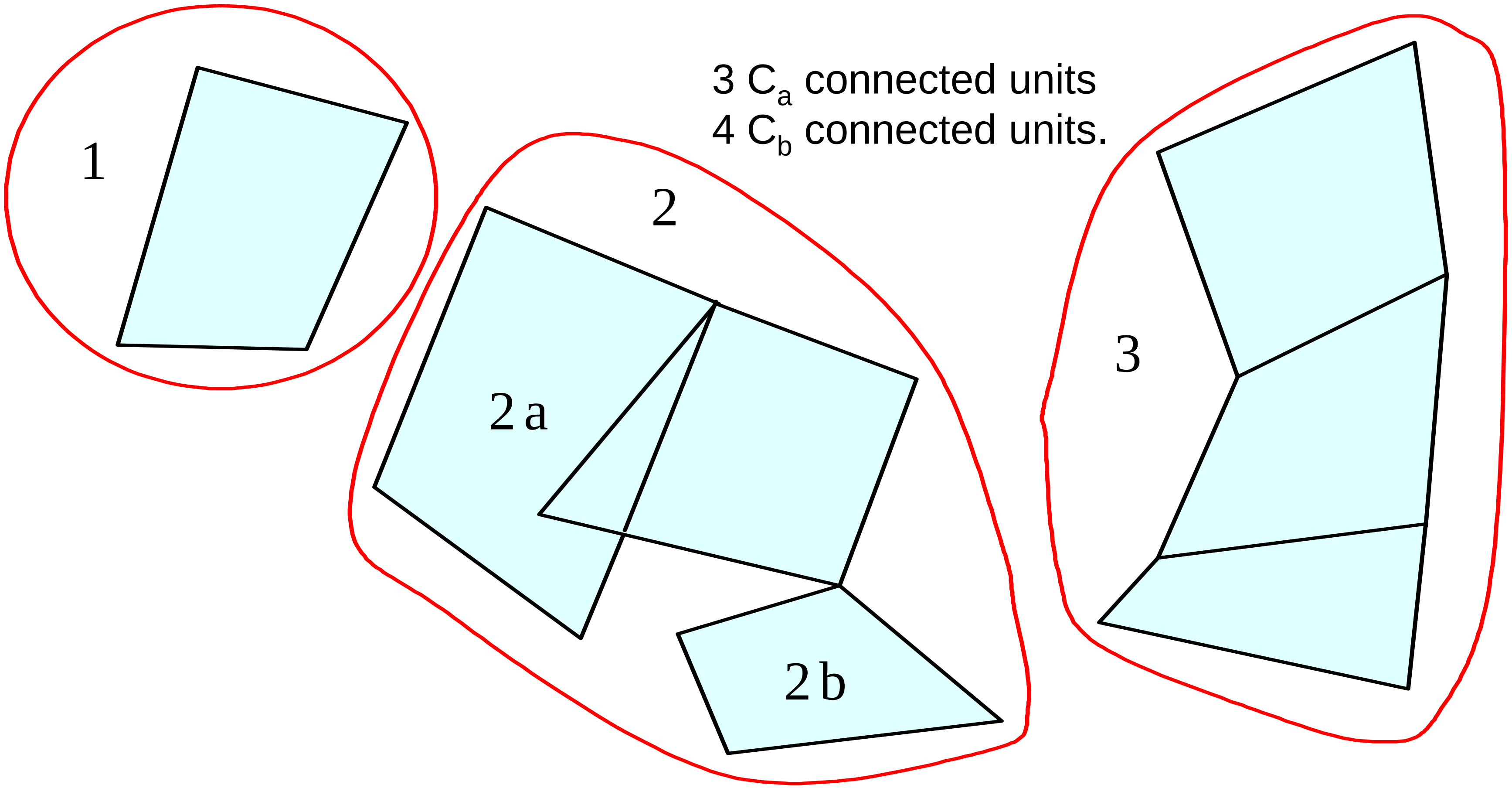
Polytopes are C_a connected if their pseudo-closures overlap.

C_b Connectivity



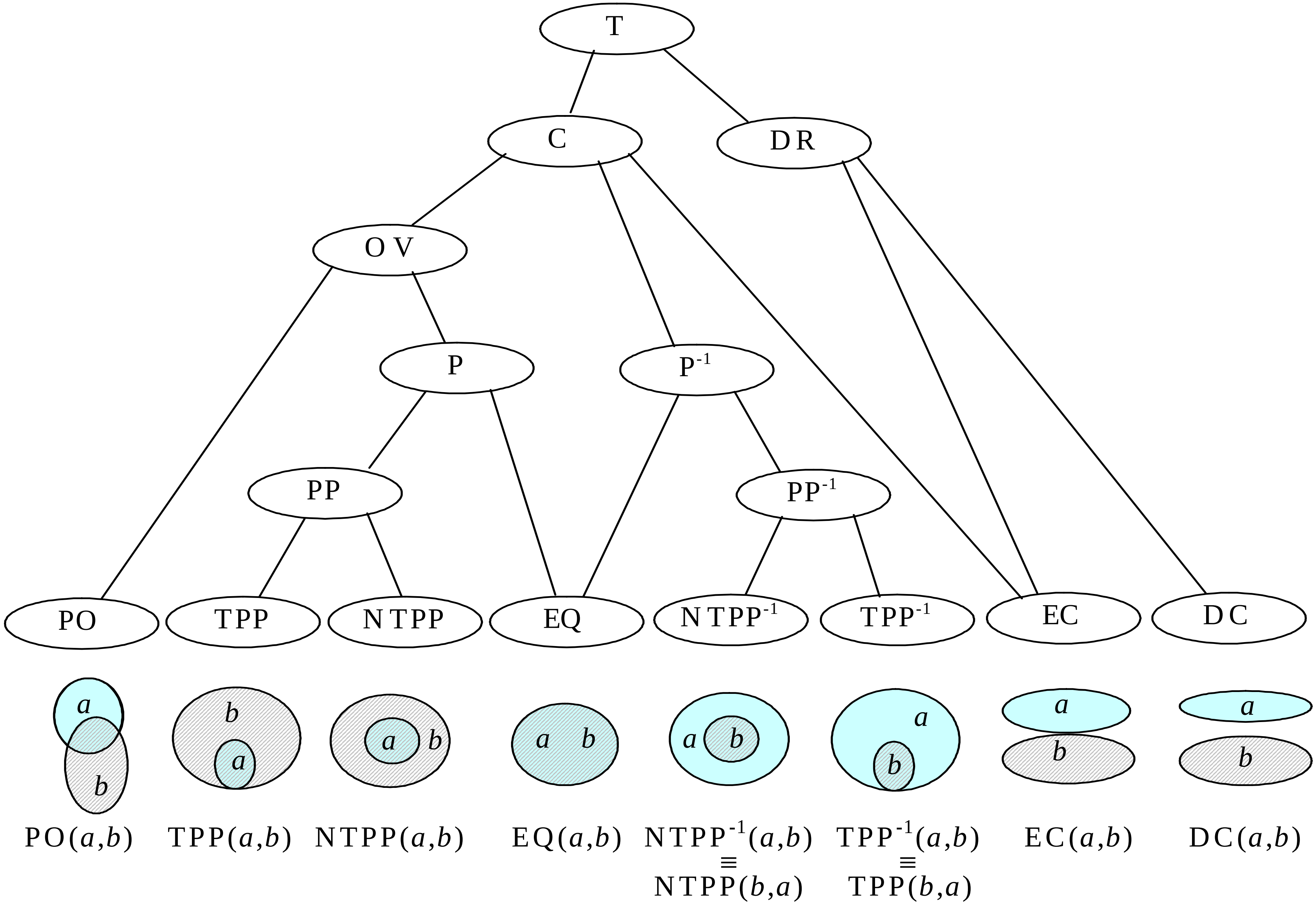
Two Convex Polytopes are Cb connected if it is possible to place a convex polytope entirely within their union, such that it intersects each convex polytope.

Connectivity Between Regular Polytopes and Within Regular Polytopes

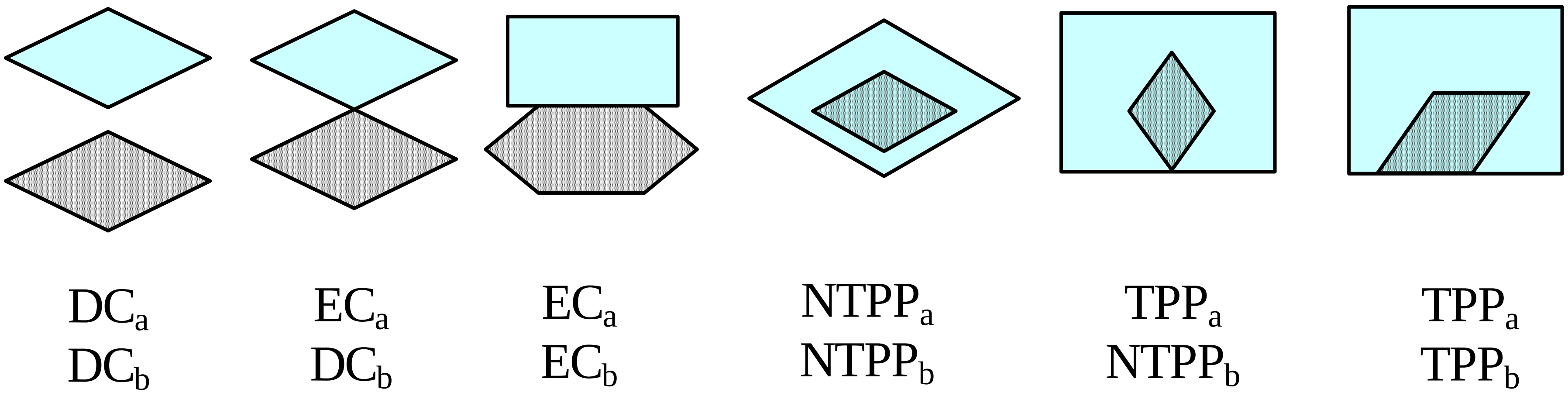


3 C_a connected units
4 C_b connected units.

The RCC Relations



Weak and Strong Connectivity



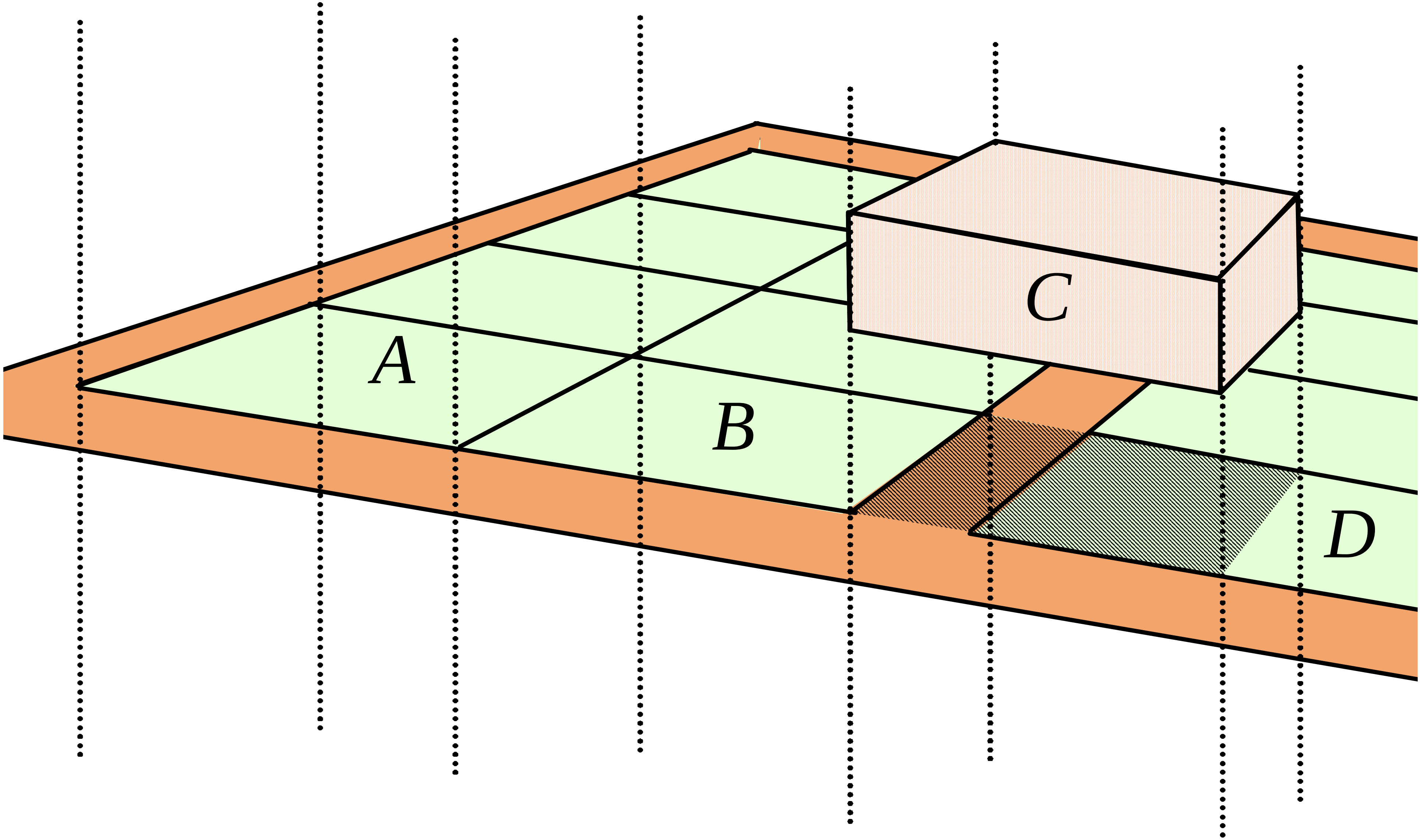
RCC Theory admits any definition of connectivity.
Here we have implemented weak (C_a) and strong (C_b) forms.

Implementation Issues

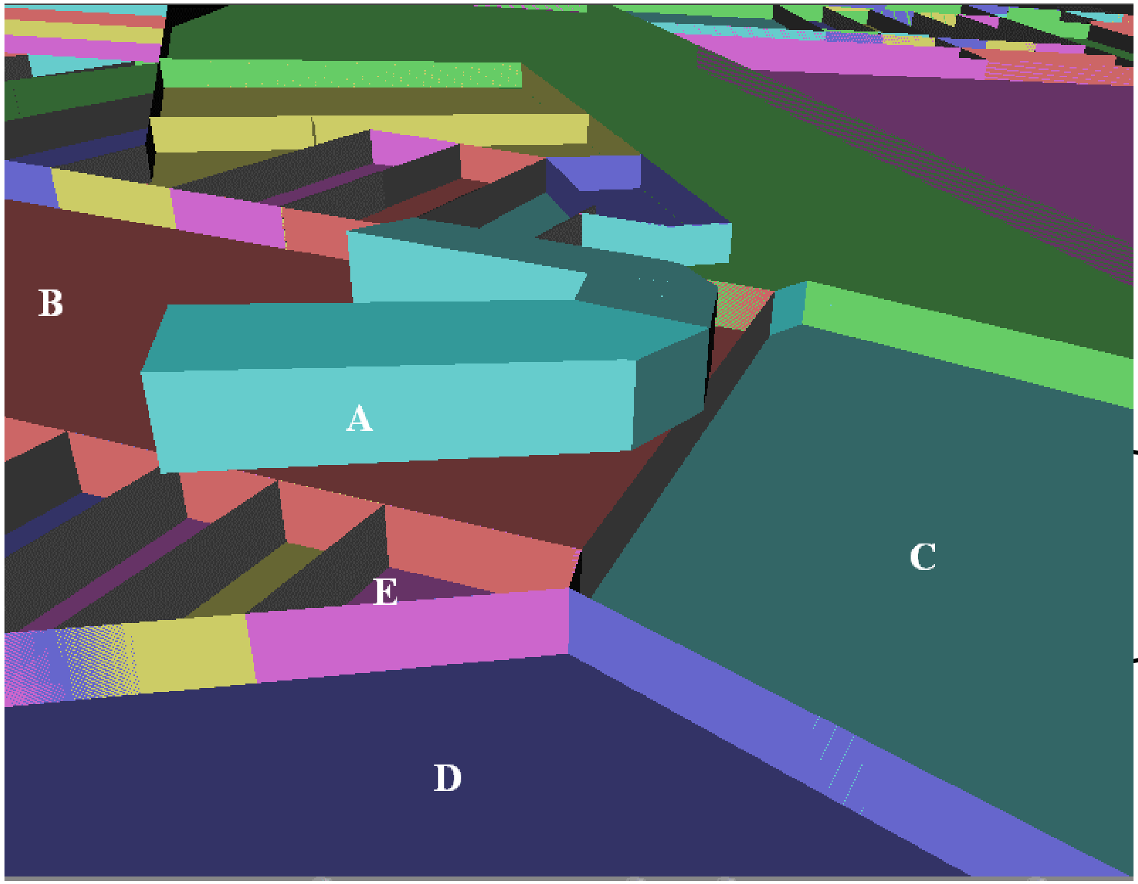
- Data Models
- Topological Encoding
- Java Classes and Methods
- Results

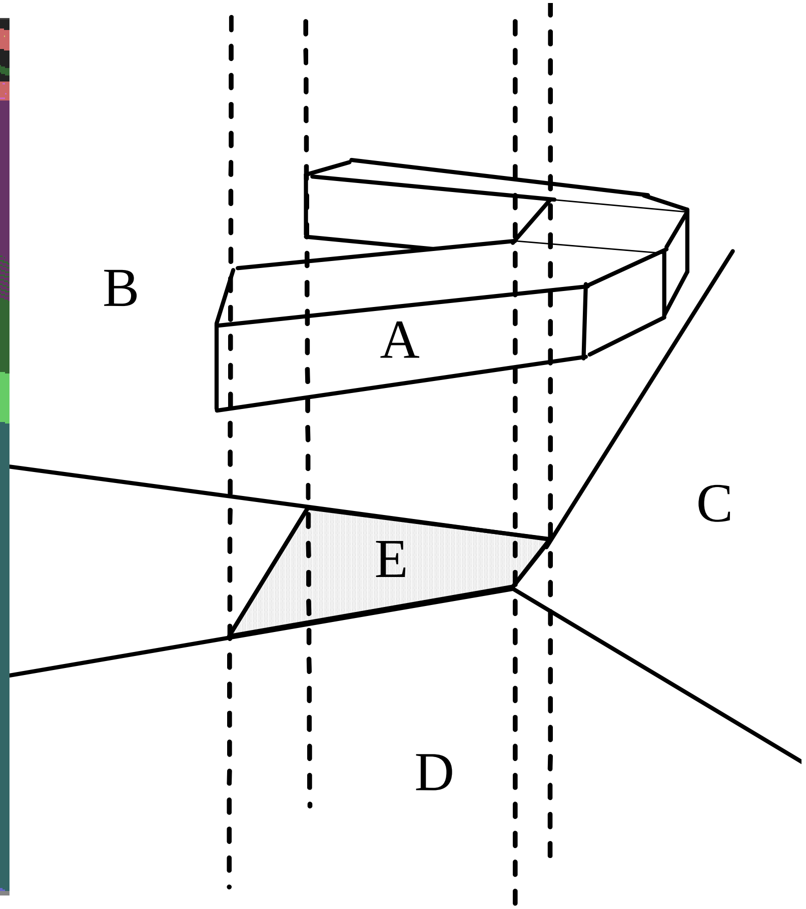
To be discussed at 3D geoinfo07 this Thursday at 15:30.

Mixing 2D and 3D Cadastre

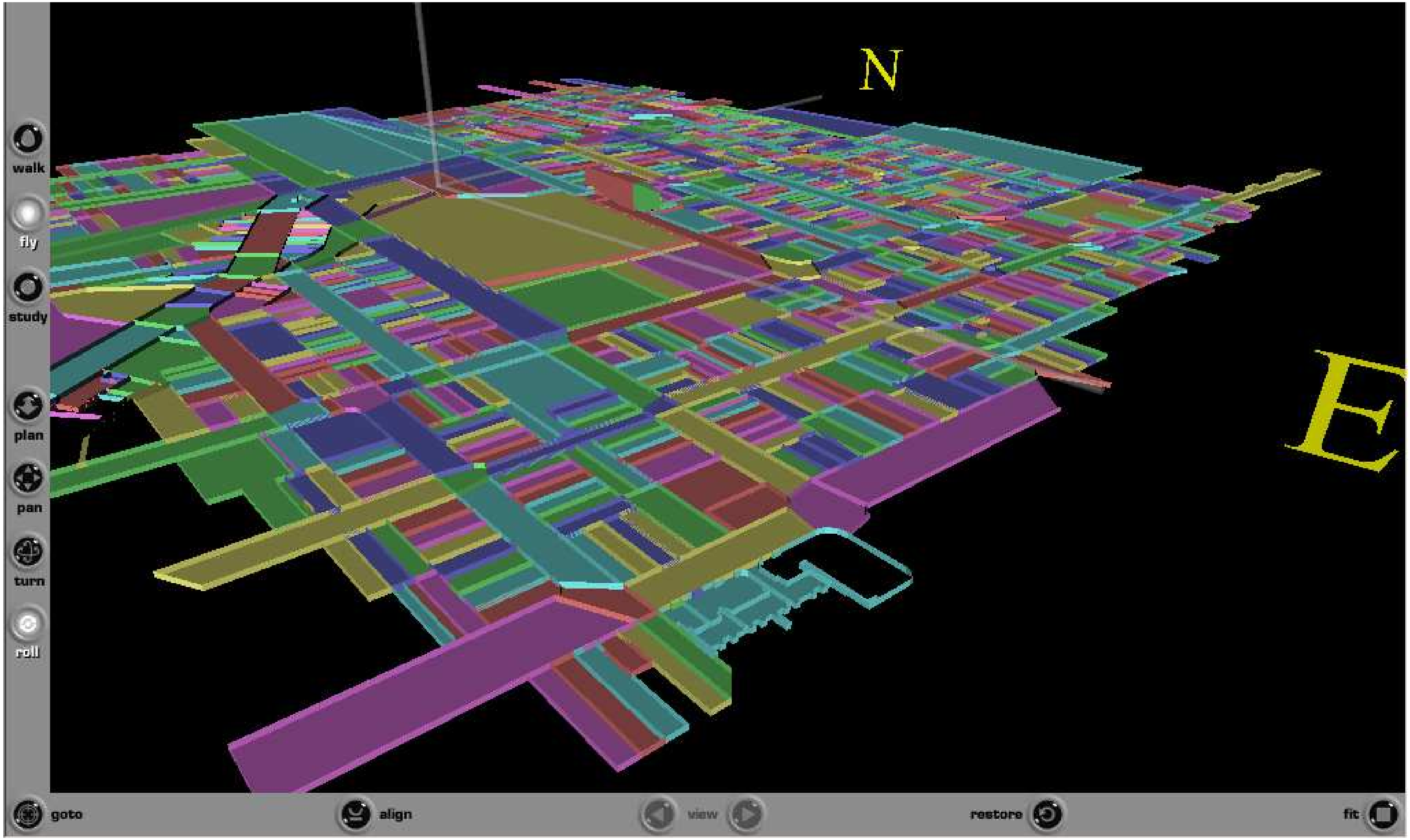


2D and 3D Example





Data


N
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study
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pan
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goto
align
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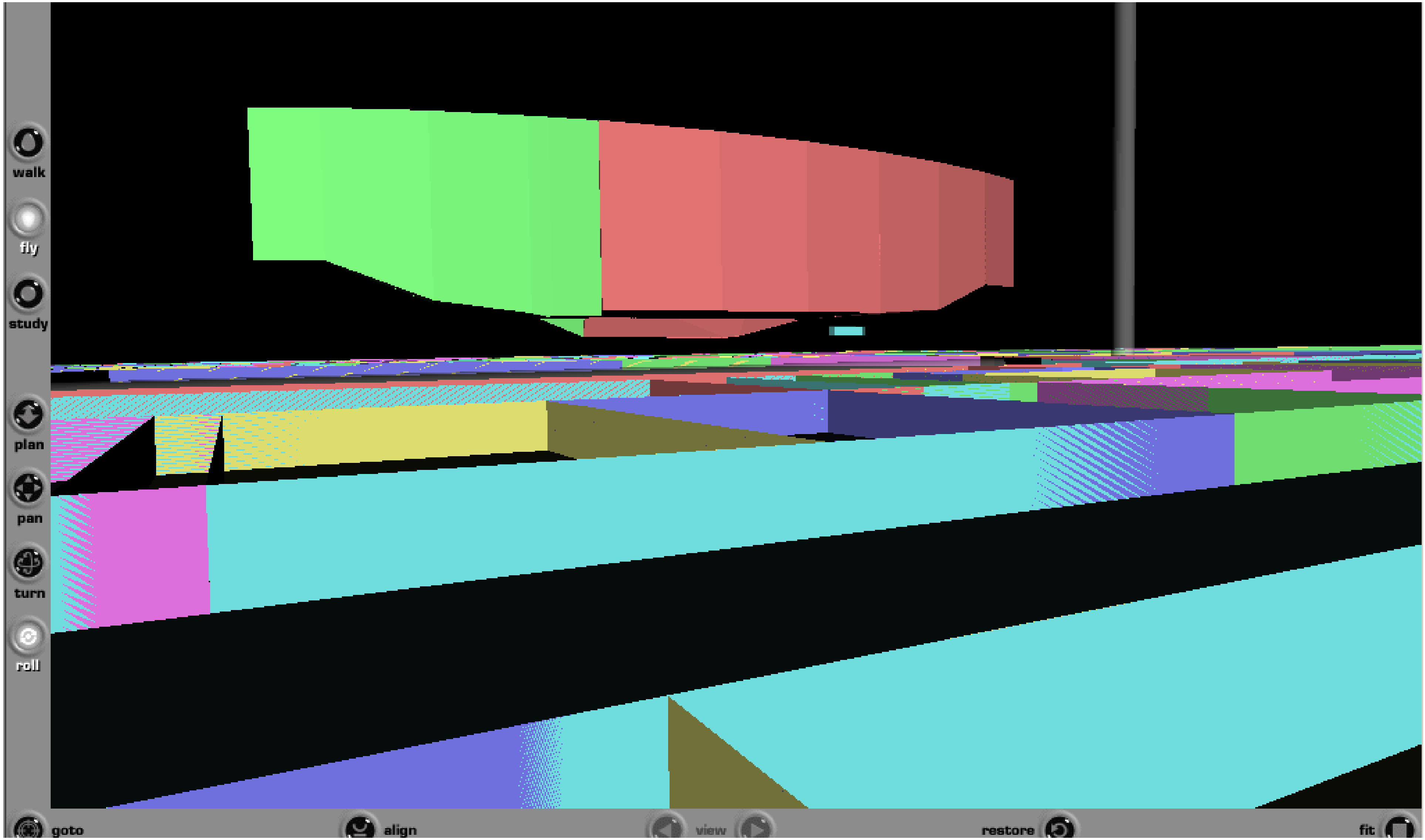
restore
fit

Data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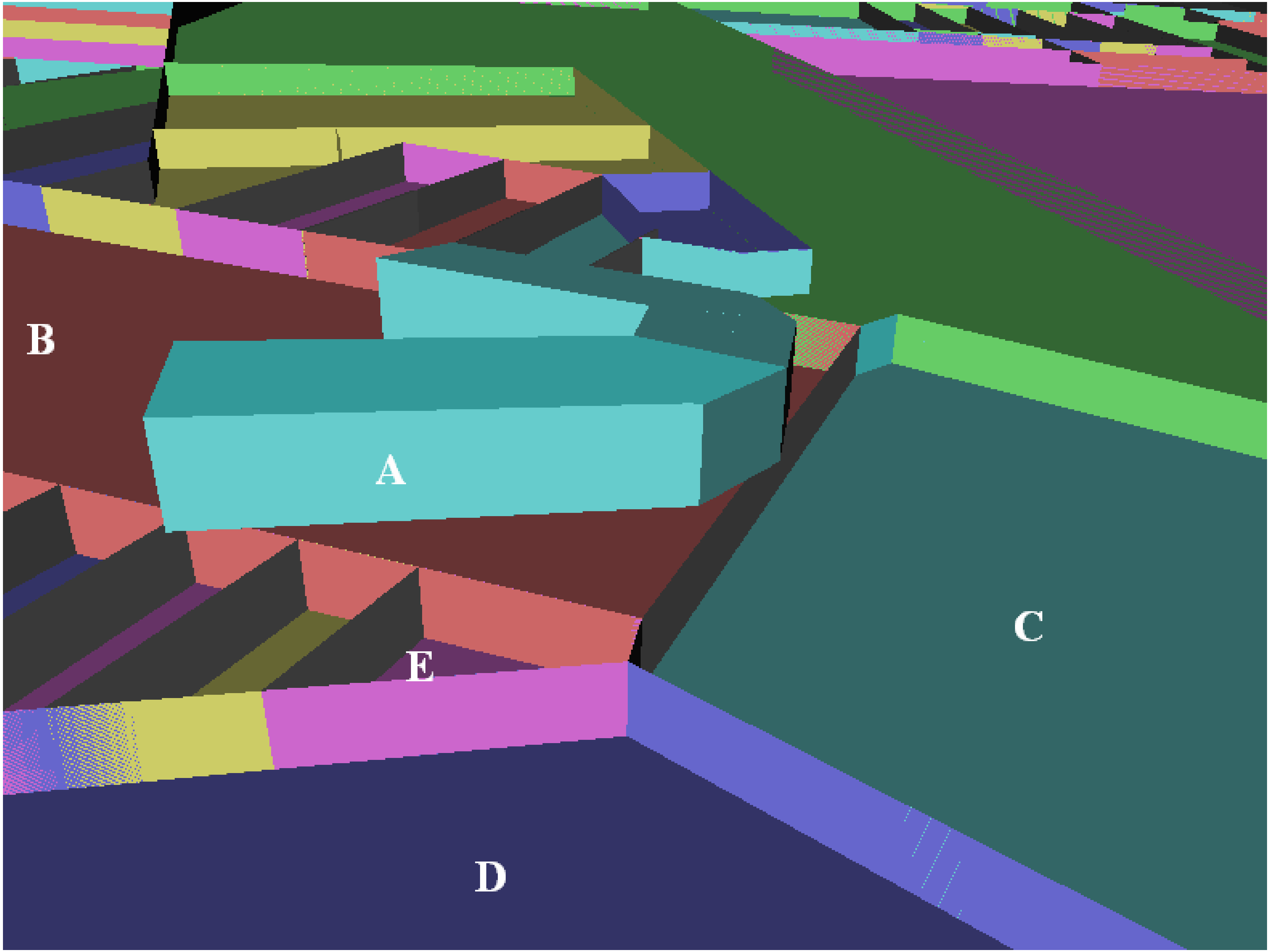

B
A
C
E
D

Caution

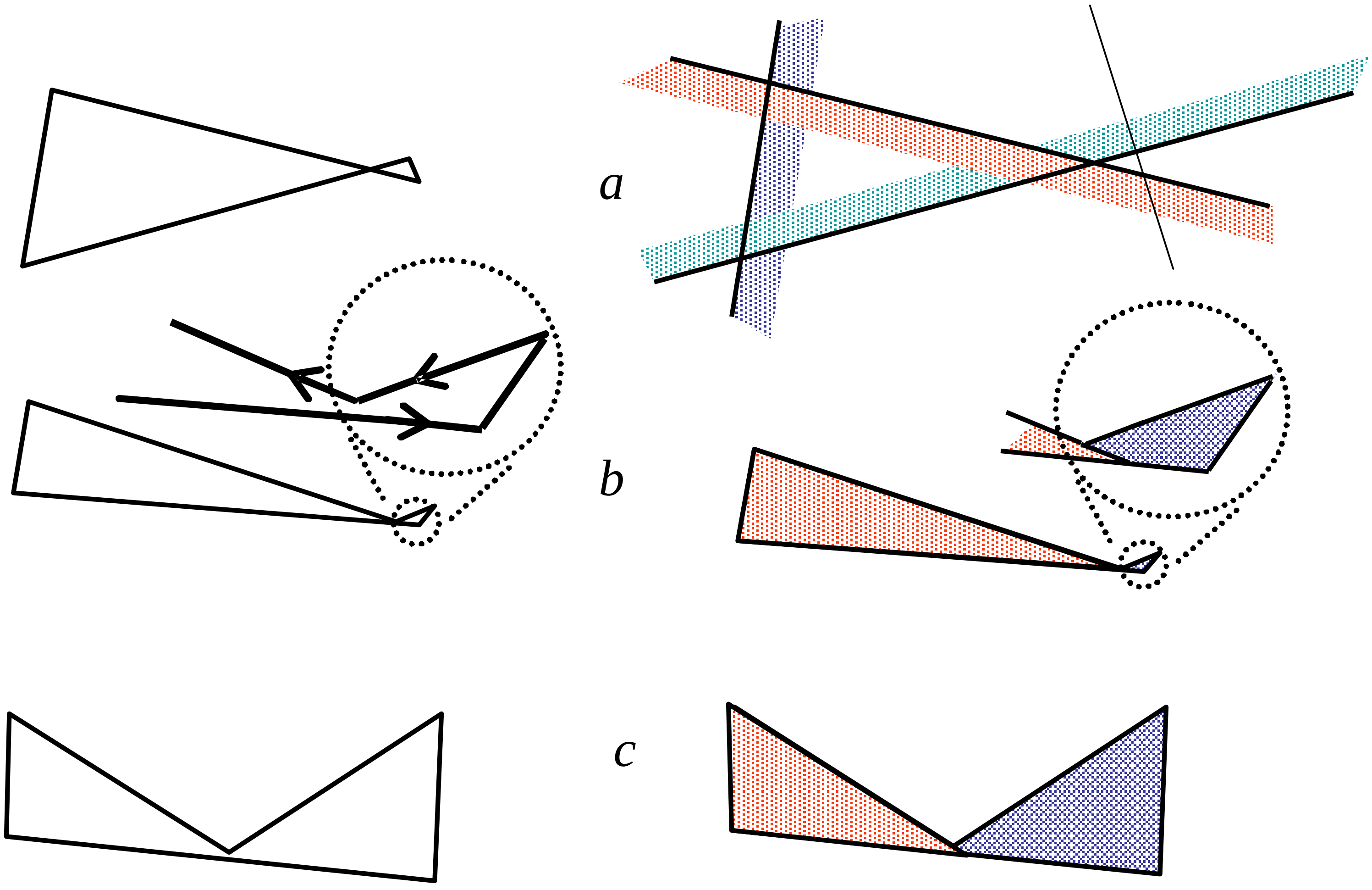
This does not say that the regular polytope representation is intrinsically more accurate than conventional representations

BUT

Once the features have been encoded, any operations between them are correct, and thus there can be no failures such as non-associativity.

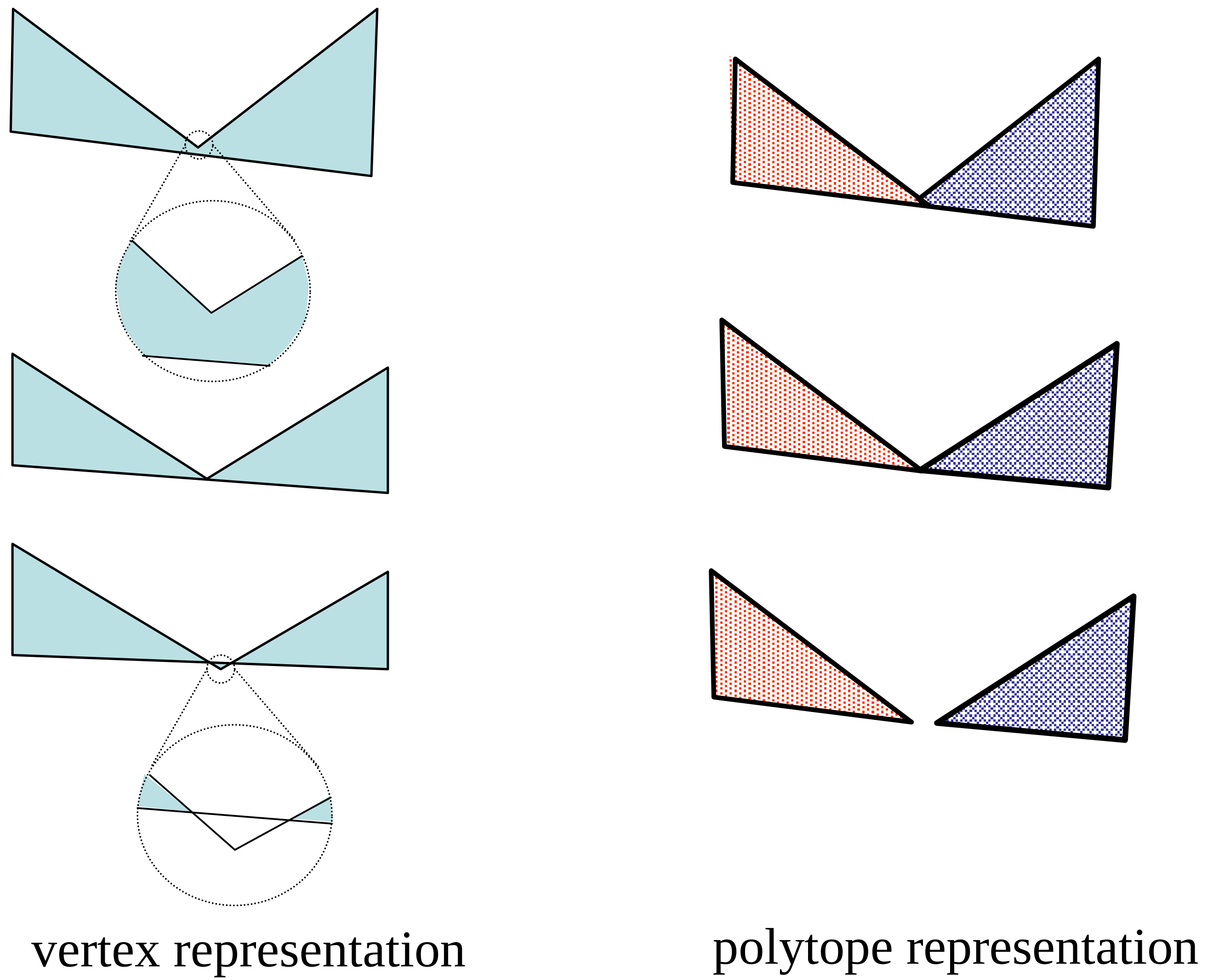
Equality can be determined correctly.

Validity of Regular Polytopes



vertex representation polytope representation

Validity of Regular Polytopes



Conclusions

- A rigorous implementation is feasible.
- The approach is applicable to Cadastral data.
- Some more effort is justified in optimisation of the algorithms.
- Although more storage is required than in conventional representations, this is not significant.

Future Research

- Applicability to Topography
- Lower dimension objects
- Optimisation
- Non-linear boundaries
- Spatial data interchange

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