EXTENSION OF SPATIAL OPERATIONS FOR MULTI-DIMENSIONAL GIS

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ABSTRACT

Extension of spatial operations to support 3D and temporal objects is an essential requirement toward a multi-dimensional GIS. Current approach is extending each spatial operation to a new multi-dimensional space with the least increase in complexity and speed based on the characteristics of the operation at hand as well as the destination space. Such operation- and dimension-dependent view points have led to technical solutions which can not be generalized. This research hypothesizes that studying spatial operations via their dimension-independent properties can bridge the artificial gap created in implementing a spatial operation for static 2D, static 3D, moving 2D and moving 3D objects. Algebraic structures – and specifically category theory – play the role of an integrating framework for the research.

1. INTRODUCTION

A multi-dimensional GIS must have support to apply a spatial operation on objects of different dimensions, e.g. static 2D, moving 2D, static 3D and moving 3D. Current approaches, mostly in computational geometry, suggest finding a technical solution to extend each spatial operation to each multi-dimensional space concentrating on the least increase in complexity and speed (De Berg, Kreveld et al. 2000, Ledoux 2008, Mostafavia, Gold et al. 2003, CGAL Website).

While there are a number of advantages in having efficient operations in terms of complexity and speed, their operation- and dimension-dependent view points cause losing the connections exist in reality between different dimensions and led to technical solutions which can not be generalized (Frank 2007, Rezayan, Frank et al. 2005); To extend each spatial operation to each multi-dimensional space, a new research is required to find out a technical solution (Karimipour 2008, Karimipour, Delavar et al. 2008). It means that to have a, say 3D GIS package, the spatial operations which have already been implemented in 2D must be re-implemented one by one in 3D, each of which via a different technical solution. If the destination space changes, for example to moving 2D, the same effort must be done, again. The situation is even worse when combination of some already implemented spaces is required! “It is not likely to achieve multi-dimensional counterparts of all of the already implemented 2D spatial operations in this haphazard way in the near future” (Karimipour, Delavar et al. 2008).

Toward achieving a general solution, this research investigates possibility of providing an integrated framework for all of the desired multi-dimensional spaces in GIS. The approach is studying spatial operations via their dimension-independent properties. If some spatial operations are implemented in 2D by combination of the elements of this integrated framework, then they can further be extended altogether to higher dimensions through a general mapping between 2D and the desired higher dimensional spaces. Algebraic structures, which are studied in mathematics, play the role of the integrating framework in this research. More specifically, category theory is used to concentrate on those properties of operations which are independent of the objects they are applied to.

2. ALGEBRAIC VIEW TO SPATIAL OPERATIONS IN MULTI-DIMENSIONAL SPACES

A spatial operation in reality has a conceptual meaning, independent of the space: “People do not think about the types of the values when doing an operation; they do the same for adding things, independent of what is added” (Karimipour, Delavar et al. 2008). However, during implementation, current approaches ignore such properties and suggest operation- and dimension-dependent solutions. For example, while the concept of Euclidean distance between two points is unique, current implementations have different variants based on the type of the points, i.e. 2D or 3D, static or moving, etc. (Karimipour, Delavar et al. 2008).

This research investigates possibility of providing a framework in which spatial operations are described via concentrating on their dimension-independent properties. To do this, an abstract view is required which is provided by algebra (Frank 1999, Loeckx and Markus 1996). "Algebra discusses the structure of operations, ...[which are] properties of operations that are independent of the objects the operations are applied to" (Frank 2007, p. 57). Especially, category theory is used, because it "gives a very high level abstract viewpoint: instead of discussing the properties of individual objects we directly address the properties of the operations" (Frank 2007, p. 66).

3. CATEGORICAL APPROACH TO EXTEND SPATIAL OPERATIONS FOR MULTI-DIMENSIONAL GIS

A *category* C consists of a class of objects and a class of morphisms, which are functions between objects, with composition and identity properties as follows (MacLane and Birkhoff 1999, Lawvere and Schanuel 2005):

$$\forall A \in C \quad \exists e_A : A \to A \quad \ni [\forall f : A \to B, g : B \to C \Rightarrow e_A.f = f, g.e_A = g]$$

$$\forall f : A \to B, g : B \to C \quad \exists h : A \to C \quad \ni h = f.g$$

$$\forall f : A \to B, g : B \to C, h : A \to C \Rightarrow (f.g).h = f.(g.h) = f.g.h$$

As it is shown in Figure 1, a *functor* between two categories associates objects and morphisms from one category to another such that identity and composition are preserved (MacLane and Birkhoff 1999, Lawvere and Schanuel 2005):

$$F(e_A) = e_{F(A)}$$
$$F(f.g) = F(f).F(g)$$

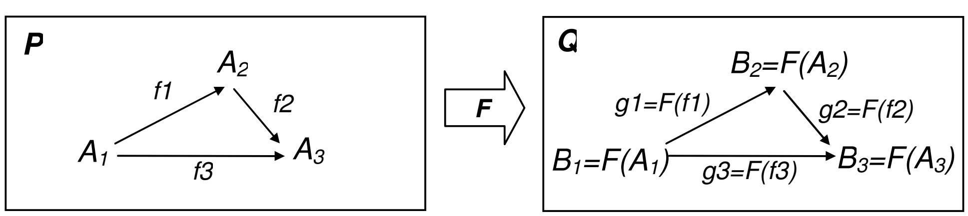

Figure 1. Functor F transforms the first category P to the second category Q

The research believes that the desired multi-dimensional spaces in GIS can be represented as categories. Then having implemented a spatial operation in the category of static 2D space, it can be extended to the categories of other multi-dimensional spaces using the relevant functors. Note that, the functor is independent of the operation, so extensions of all operations form a category to another are done in the same manner.

The main concern of this research is on mathematical validation of the conceptual framework and investigation of its implementation issues, so performance is not a key factor for evaluation of the results.

4. Future works

The implementation feasibility of the theoretical idea of the research has been tested for a number of case studies, including 2D/3D static and moving convexhull (Karimipour 2008), 2D static and moving voronoi diagrams and point in polygon (Karimipour, Delavar et al. 2005; 2006; 2008). We are using the same approach to implement more complex cases such as 2D/3D static and moving Delaunay Triangulation.

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