

# Lunchmeeting GIS Technology

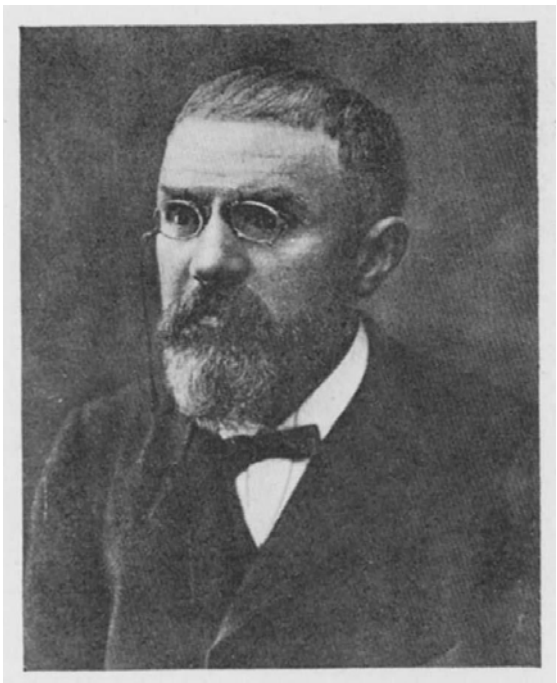
**Friday September 1st 12.00-14.00h**

RGI project 3D Topography

2 PhD's, 2 presentations:

- Friso Penninga (focus: data structure)
- Sander Oude Elberink (focus: data collection)

Check www.gdmc.nl/3dtopo !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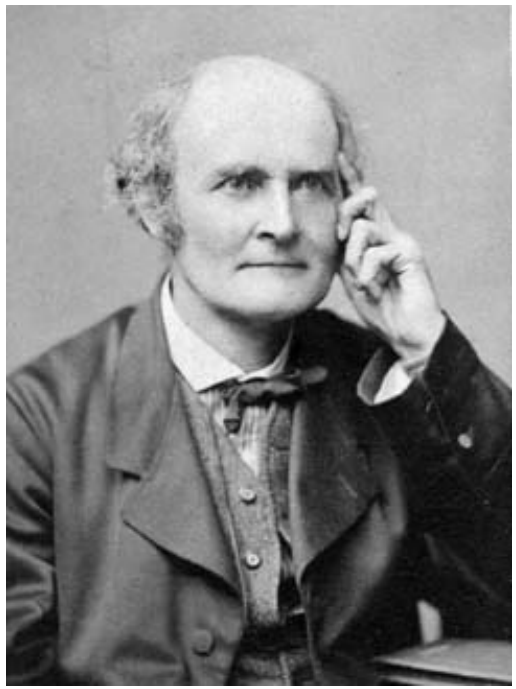
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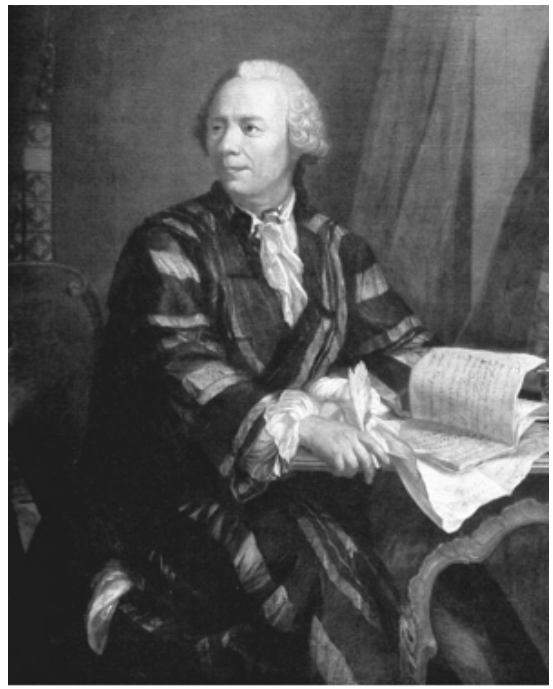

# = 3D GIS?

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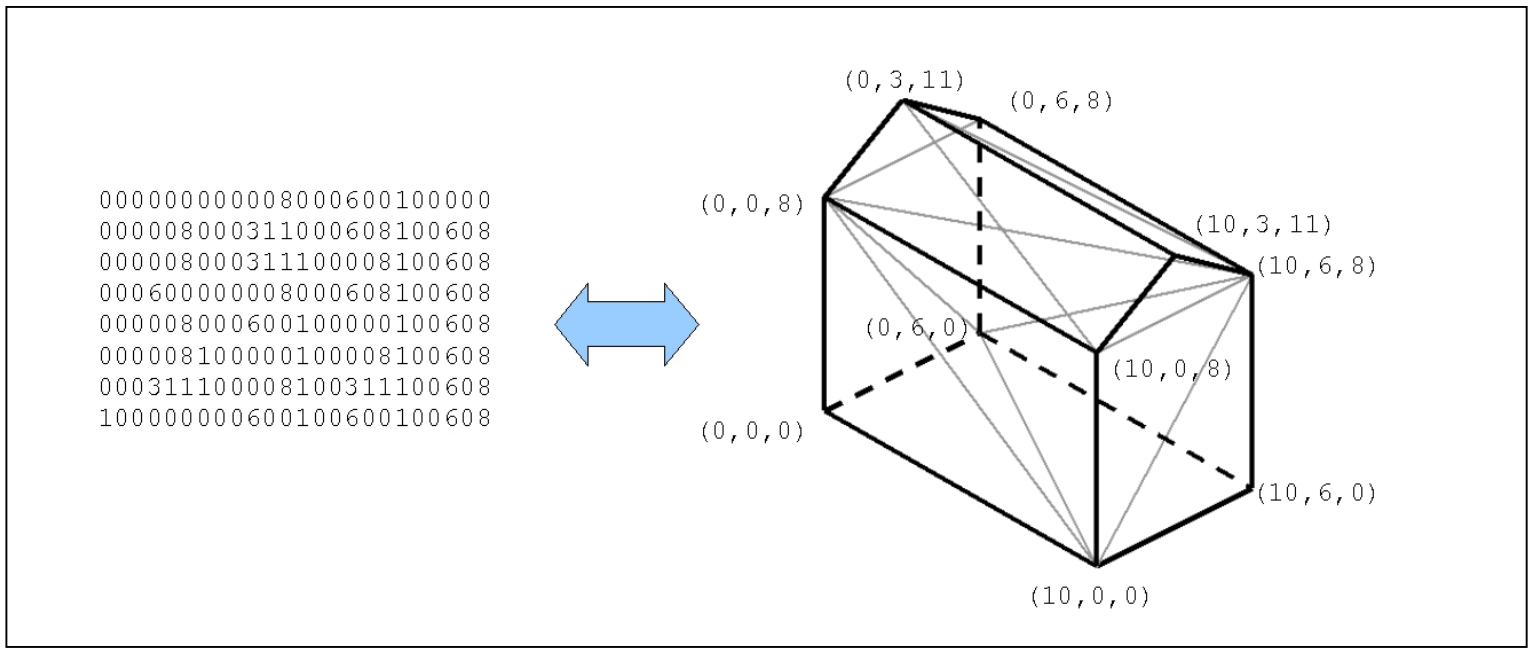
# Poincaré simplicial homology for 3D volume modeling




**Friso Penninga**
**Delft University of Technology,**
**section GIS Technology**

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# Outline

- Introduction
- PhD project history in a nutshell
- Basics of modelling approach
- Poincaré's formalism of simplicial homology
- Concept of Poincaré-based TEN structure
- Implementation → illustration of concept
- Conclusions & future research
- Discussion

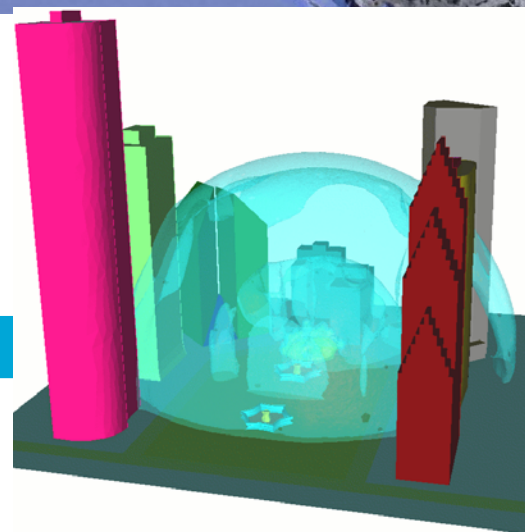
# Introduction:

## Need for 3D Topography

- Real world consists of 3D objects
- Objects + object representations get more complex due to multiple land use

Applications in:

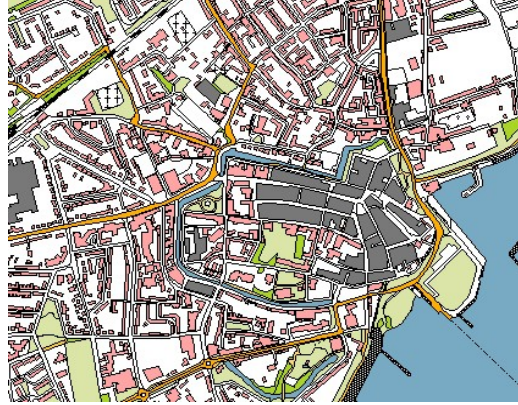
- Sustainable development (planning, analysis)
- Support disaster management



3D Topography: more than visualization!

# Introduction:

## Research Goal



Develop a new topographic model to be realized within a robust data structure and filled with existing 2D, 2.5D and 3D data

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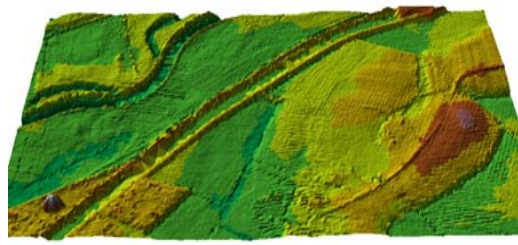

Data structure:

design / develop / implement a data structure that supports 3D analyses and maintains data-integrity

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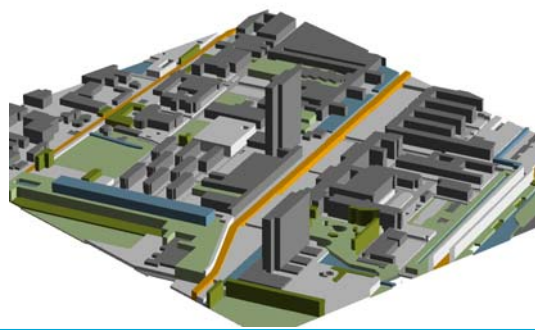

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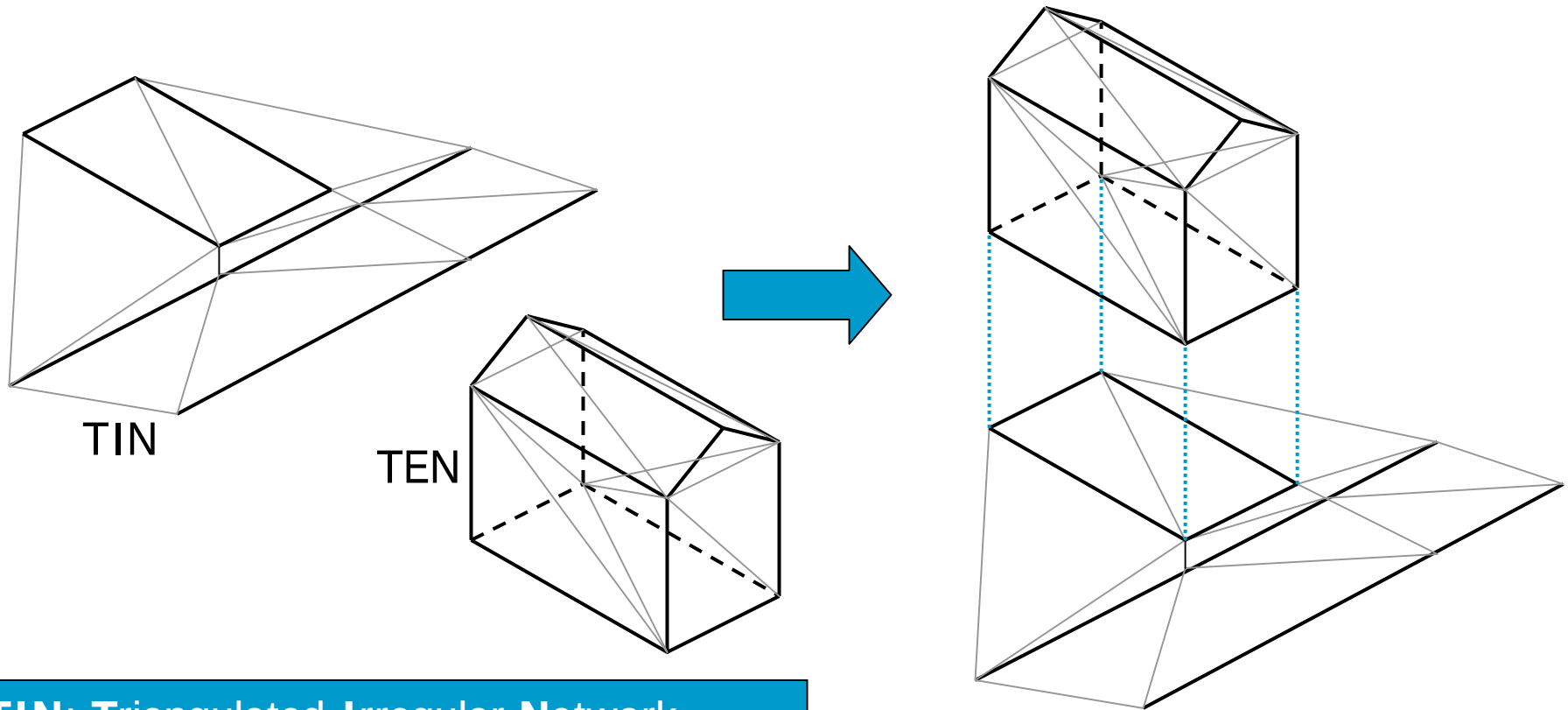
# History in a nutshell:
## First ideas

- July 2004: *"Realization of a three dimensional topographic terrain representation in an integrated TIN/TEN model"* (Agile 2005)

# Proposed data model:

## 2.5D TIN + 3D TEN

Model terrain in 2.5D, ‘glue’ 3D models on top or below 2.5D terrain



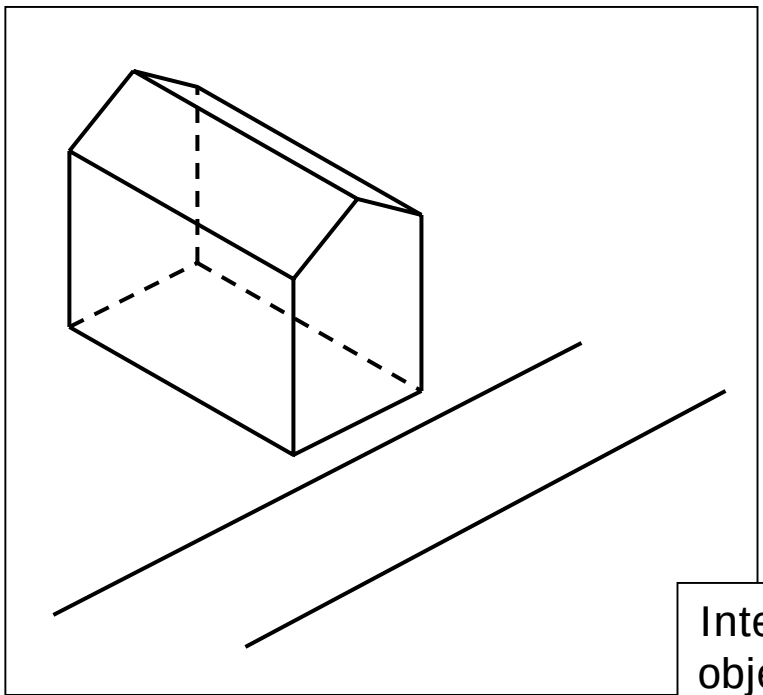

**TIN**: **T**riangulated **I**rregular **N**etwork

**TEN**: **Te**trahedronized Irregular **N**etwork

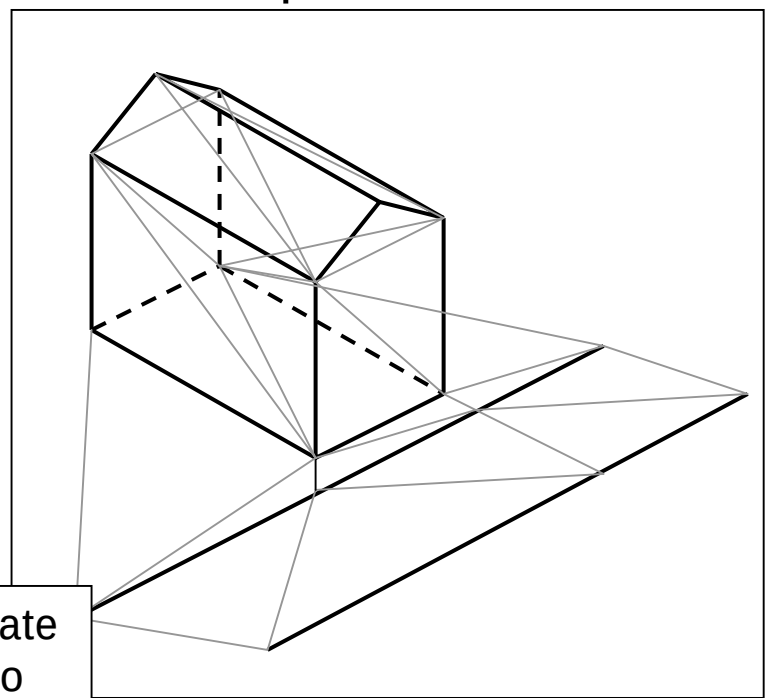
# Basic principles (3/3)



User works with features



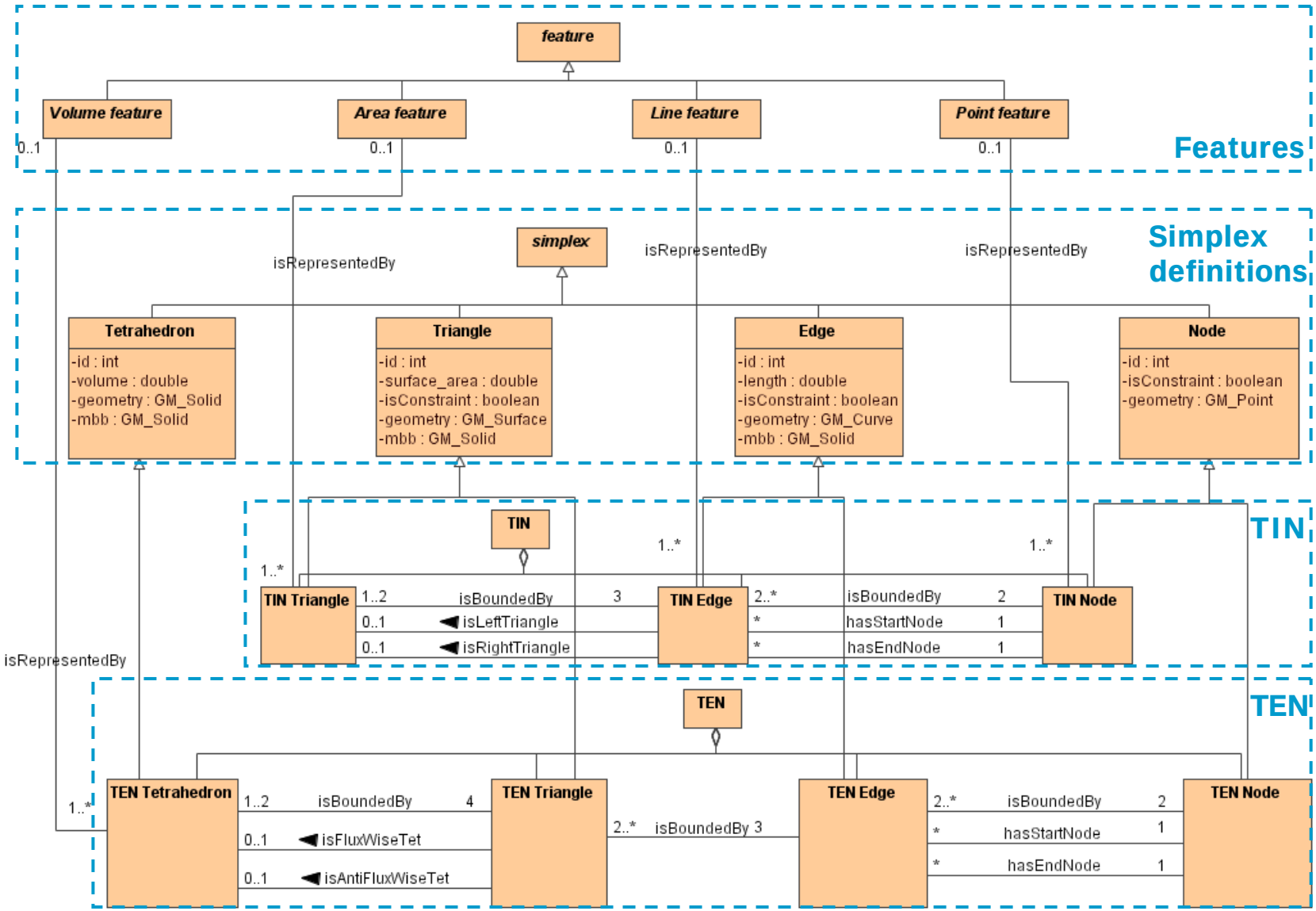
Internal representation



Interface: translate object model into set of constraints

Presentation sounding board, September 24 2004

# Feature-based Integrated TIN/TEN model in UML



# History in a nutshell:

## Continuation on hybrid approach

- July 2004: *"Realization of a three dimensional topographic terrain representation in an integrated TIN/TEN model"* (Agile 2005)
- September 2004: Linking TIN and TEN model (glue, stitch & nail)

# Connecting TIN+TENs
## Problems

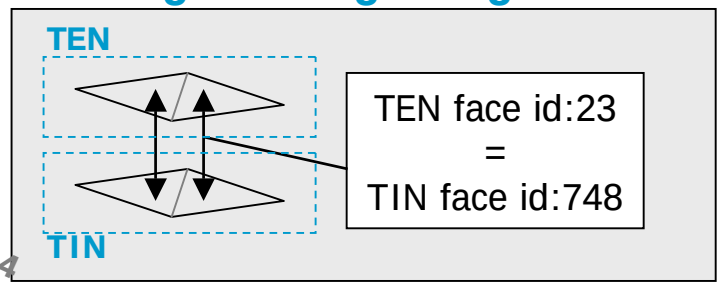
Presentation sounding board, September 24 2004

How to ensure identical faces or edges in TIN surface and TEN bottom?

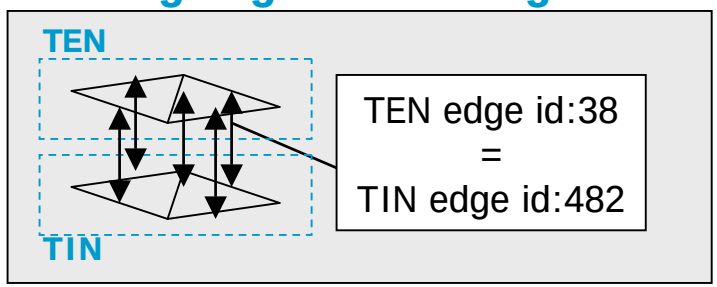
Might be problematic due to addition of Steiner points in triangulation / tetrahedronization (iterating process?)

Possible solution: addition of Steiner points in interior (common in mesh refinement, but rather unusual in GIS (nodes represent point measurents))

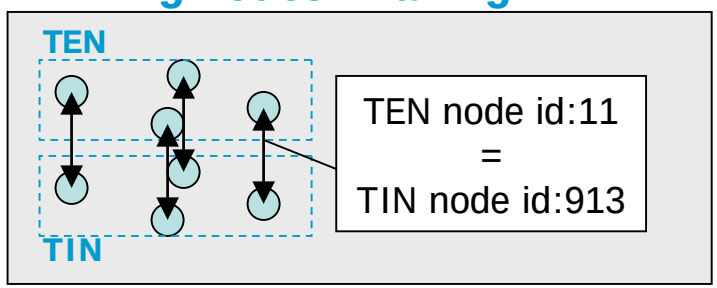
**Linking faces: 'glueing'**




**Linking edges: 'stitching'**




**Linking nodes: 'nailing'**



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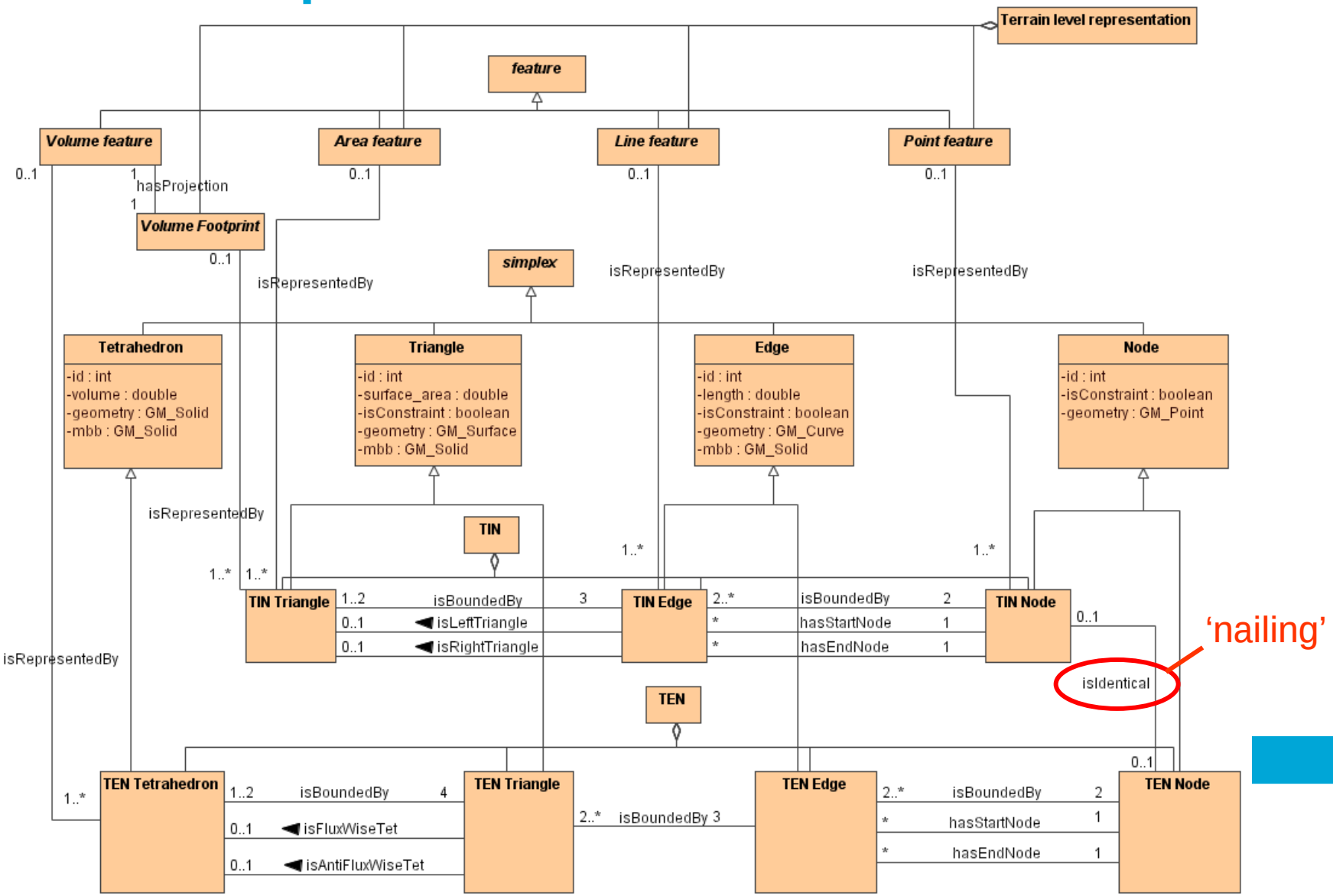
# Implementation:

## Conceptual model

Presentation sounding board, September 24 2004



# History in a nutshell:
## Switch to full 3D approach

- July 2004: *"Realization of a three dimensional topographic terrain representation in an integrated TIN/TEN model"* (Agile 2005)
- September 2004: Linking TIN and TEN model (glue, stitch & nail)
- Spring 2005: *"3D Topographic data modelling: why rigidity is preferable to pragmatism"* (Cosit '05)

# Proposed new approach

## Full 3D TEN model

Two fundamental observations:

- ISO19101: a feature is an 'abstraction of real world phenomena'. These real world phenomena have by definition a volume

- Real world can be considered to be a volume partition (analogous to a planar partition: a set of non-overlapping volumes that form a closed modelled space)

# Proposed new approach

## Why a full 3D approach?

'Abstraction of real world phenomena' (ISO19101):

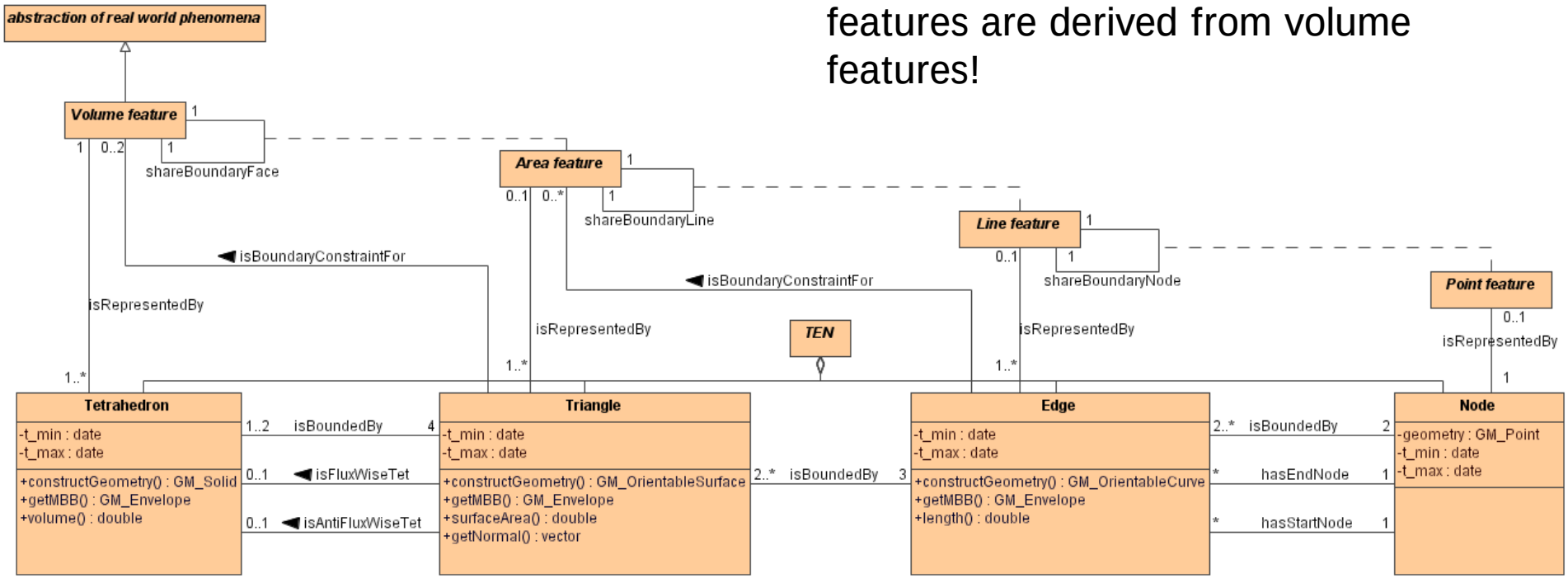
- until now:
  abstraction (simplification) → less dimensional representation
- when using meshes:
  simplification is in subdivision into easy-to-handle parts
  (analogously to Finite Element Method for solving Partial Differential Equations)

As a result: model only volume features in a volume partition (but handle polygon features as association class, derived from volume features, as faces mark transition between two volumes: for instance wall, earth surface)

# Further research

## UML model

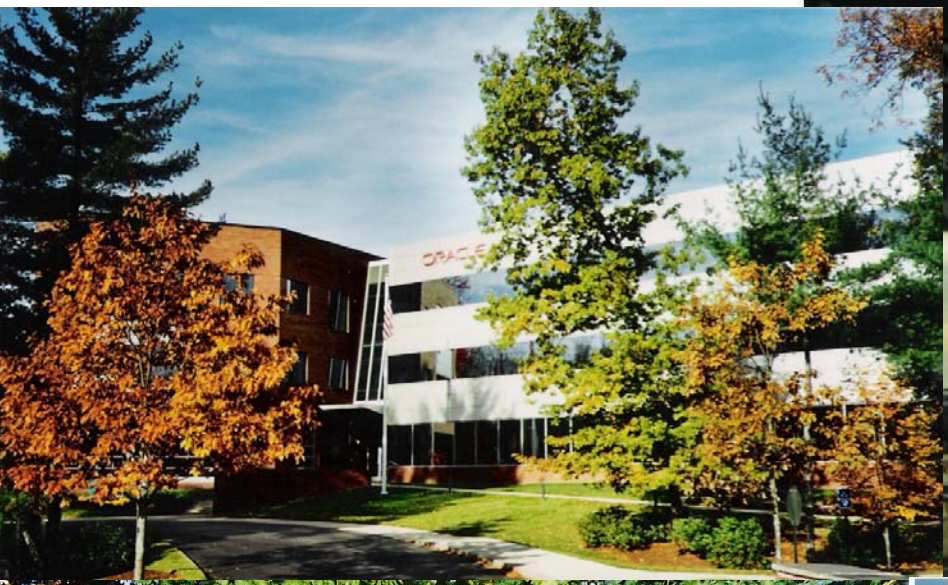
Note: area, line and point features are derived from volume features!



# History in a nutshell:
## Introduced to Poincaré

- July 2004: *"Realization of a three dimensional topographic terrain representation in an integrated TIN/TEN model"* (Agile 2005)
- September 2004: Linking TIN and TEN model (glue, stitch & nail)
- September 2005: *"3D Topographic data modelling: why rigidity is preferable to pragmatism"* (Cosit '05)
- November 2005: one week visit to Oracle Spatial Development Centre

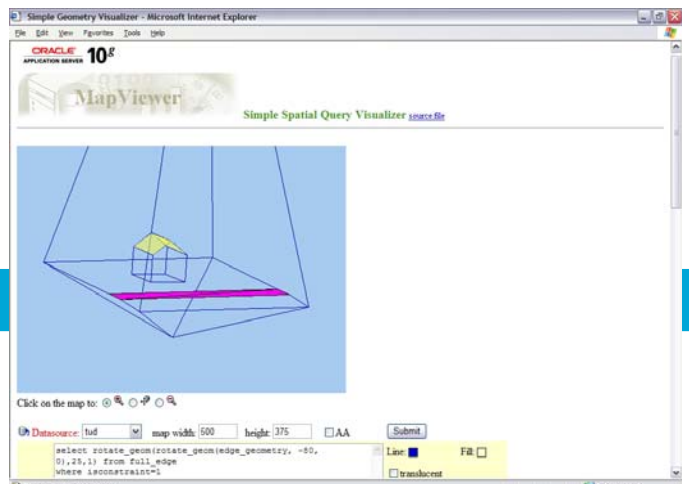
# Oracle visit






# Oracle visit

- Discussions on 3D
- First experiences ‘toy’ dataset: 56 tetrahedrons, 120 triangles, 83 edges, 20 nodes
- John Herring suggests Poincaré
- ‘Hands-on’ session
- Visualisation: Oracle MapViewer + function **`rotateGeom`**

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# History in a nutshell:
## Poincaré-based approach

- July 2004: *"Realization of a three dimensional topographic terrain representation in an integrated TIN/TEN model"* (Agile 2005)
- September 2004: Linking TIN and TEN model (glue, stitch & nail)
- September 2005: *"3D Topographic data modelling: why rigidity is preferable to pragmatism"* (Cosit '05)
- November 2005: one week visit to Oracle Spatial Development Centre
- July 2006: *"A Tetrahedronized Irregular Network Based DBMS approach for 3D Topographic Data Modelling"* (SDH'06)
- September 2006: *"Updating Features in a TEN-based DBMS approach for 3D Topographic Data Modelling"* (GIScience'06)

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# Basics of modelling approach:
## Fundamental concept (1/2)

- Physical world objects have by definition a volumetric shape: there are no such things as point, line or area features! (*only point, line and area representations at a certain level of generalization*)

- The real world can be considered as a volume partition: a set of non-overlapping volumes that form a closed modeled space. As a consequence, objects like 'earth' or 'air' are explicitly part of the real world and thus have to be modeled.

# Basics of modelling approach

## Fundamental concept (2/2)

No area features at all?

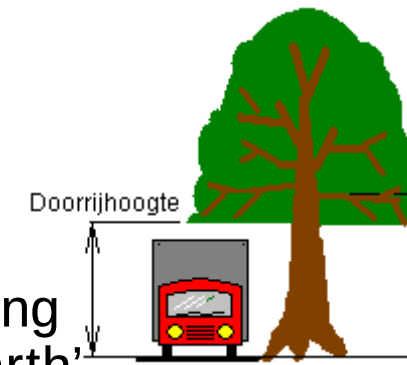
→ yes, but as derived features: area features (e.g. ‘wall’,’roof’) mark transition between volume features

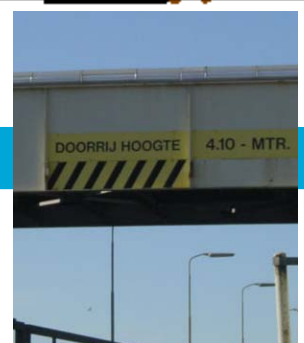
For instance:

- ‘Earth surface’ is transition between ‘earth’ and ‘air’
- ‘Wall/roof’ is transition between ‘house’ and ‘air’

But how to model for instance a road?

- Road is a volume (in which cars can drive without hitting something: ‘tunnel in the sky’). Transition between ‘earth’ and ‘road’ can be drawn with ‘road’ colors



# Basics of modelling approach

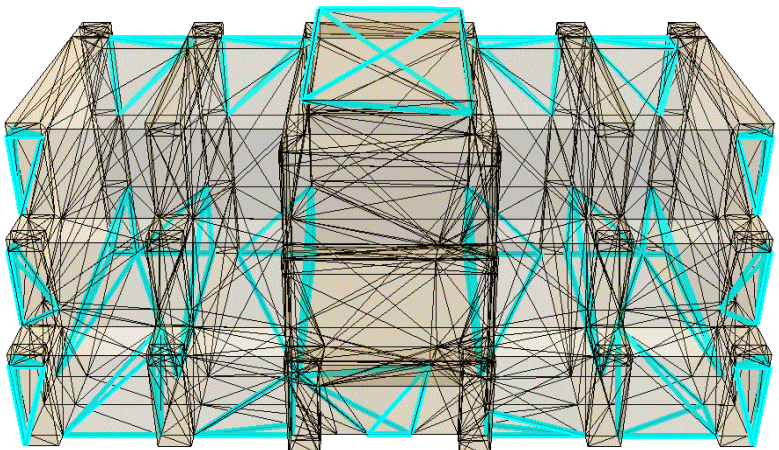
## TEN approach (1/2)

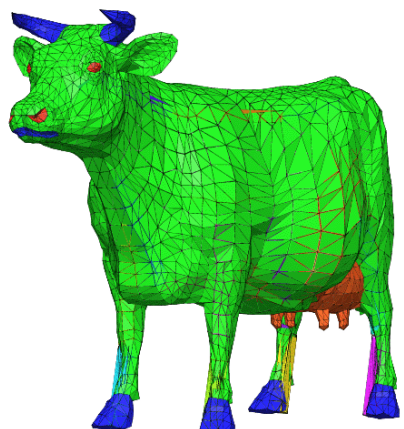
Important: data consistency and data analysis

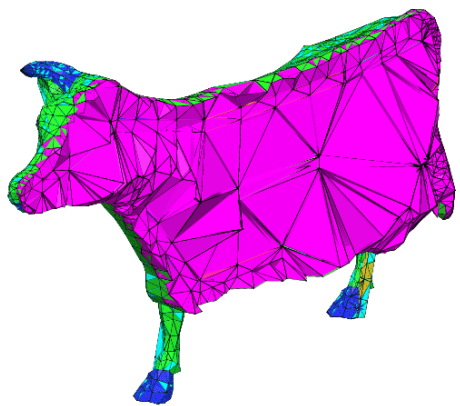
→ TEN structure well-suited:
well-defined, only convex volumes, flat faces

In 3D (complex) shapes → subdivide in (many) tetrahedrons:

TEN is based on points, line segments, triangles and tetrahedrons:
simplexes (‘simplest shape in a given dimension’)







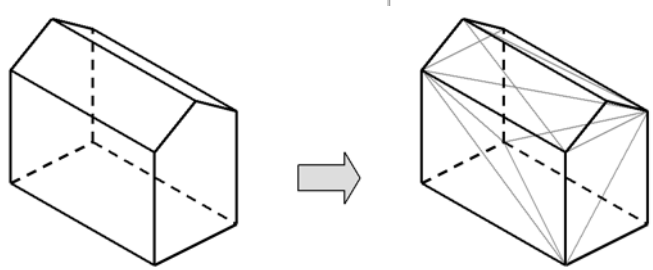
# Basics of modelling approach

## TEN approach (2/2)

Drawbacks:

- Complexity (but hide it from user...)
- Storage requirements:

| Building as polyhedron | Building as TEN |
|---|---|
| (1 volume) | 8 tetrahedrons |
| 7 polygons | 24 triangles |
| (15 edges) | 25 edges |
| (10 points) | 10 nodes |



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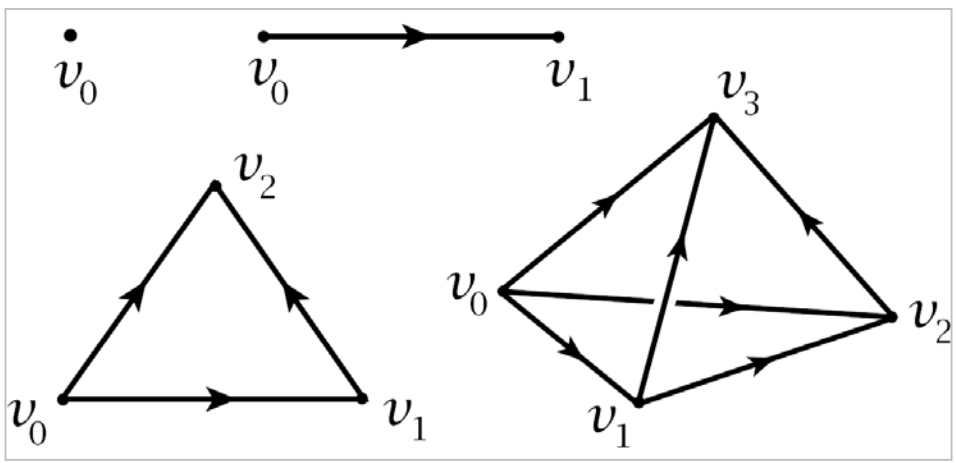
# Poincaré simplicial homology (1/5)
## Definition simplex

*Jules Henri Poincaré (1854-1912)*

Solid mathematical foundation:

A $n$ -simplex $S_n$ is defined as smallest convex set in Euclidian space $R^m$ of $n+1$ points $v_0, \ldots, v_n$ (which do not lie in a hyper plane of dimension less than $n$)

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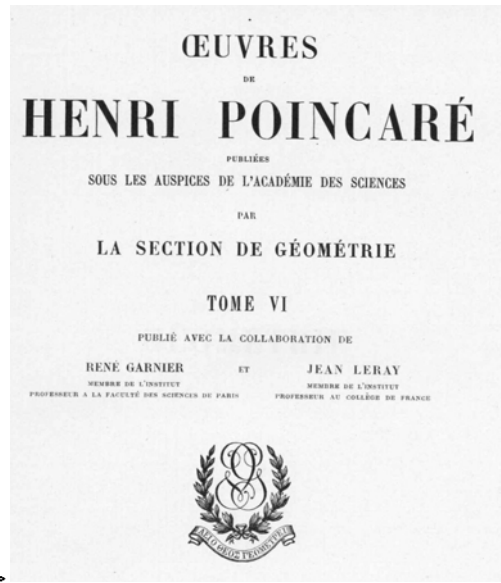
# Poincaré simplicial homology (2/5)
## Boundary operator

The boundary $\partial$ of simplex $S_n$ is defined as sum of *(n-1)* dimensional simplexes (note that 'hat' means skip the node):

$$\partial S_n = \sum_{i=0}^{n} (-1)^i < v_0, ..., \hat{v}_i, ..., v_n >$$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TOME VI
PUBLIÉ AVEC LA COLLABORATION DE
RENÉ GARNIER ET JEAN LERAY

remark: sum has n+1 terms

$$S_1 = <v_0, v_1>$$
$$S_2 = <v_0, v_1, v_2>$$
$$S_3 = <v_0, v_1, v_2, v_3>$$

$$\partial S_1 = <v_1> - <v_0>$$
$$\partial S_2 = <v_1, v_2> - <v_0, v_2> + <v_0, v_1>$$
$$\partial S_3 = <v_1, v_2, v_3> - <v_0, v_2, v_3> + <v_0, v_1, v_3> - <v_0, v_1, v_2>$$

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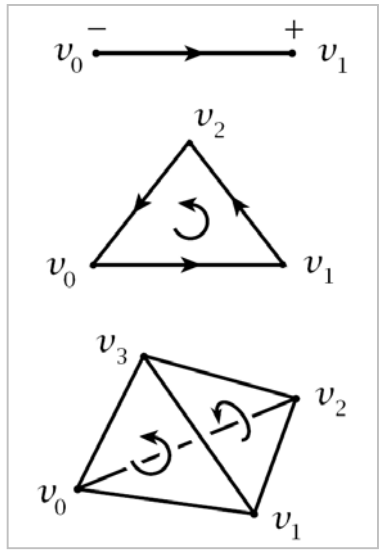
# Poincaré simplicial homology (3/5)

## Simplex construction

$$S_n \text{ has} \binom{n+1}{p+1} \text{faces of dimension } p \text{ with } (0 \leq p < n)$$

- 2D: this means that triangle ($S_2$) has 3 edges ($S_1$) and 3 nodes ($S_0$)
- 3D: this means that tetrahedron ($S_3$) has 4 triangles ($S_2$), 6 edges ($S_1$) and 4 nodes ($S_0$)

# Poincaré simplicial homology (4/5)
## Orientation of boundaries

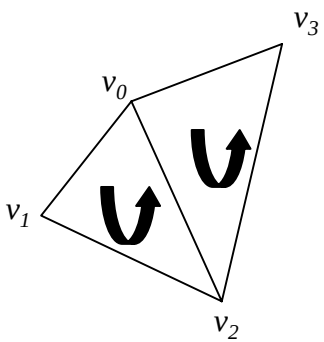



With (n+1) points, there are (n+1)! permutations of these points. In 3D for the 4 simplexes this means 1, 2, 6 and 24 options ($S_0$ obvious):

- For $S_1$ the two permutations are $<v_0, v_1>$ and $<v_1, v_0>$ (one positive and one negative $<v_0, v_1> = - <v_1, v_0>$ )
- For $S_2$ there are 6: $<v_0,v_1,v_2>$, $<v_1,v_2,v_0>$, $<v_2,v_0,v_1>$, $<v_2,v_1,v_0>$, $<v_0,v_2,v_1>$, and $<v_1,v_0,v_2>$. First 3 opposite orientation from last 3, e.g. $<v_0,v_1,v_2> = - <v_2,v_1,v_0>$. counter clockwise (+) and the negative orientation is clockwise (-)
- For $S_3$ there are 24, of which 12 with all normal vectors outside (+) and 12 others with all normal vectors inside (-)!

# Poincaré simplicial homology (5/5)

## Simplicial complexes

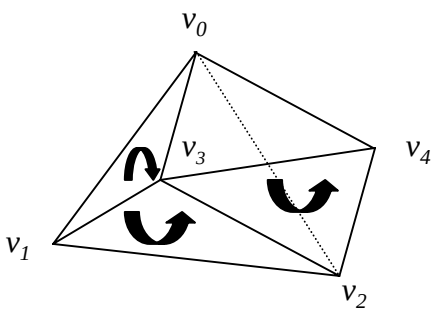



$$S_{21} = < v_0, v_1, v_2 > \text{ and } S_{22} = < v_0, v_2, v_3 >$$

$$C_2 = < v_1, v_2 > - < v_0, v_2 > + < v_0, v_1 >$$

$$+ < v_2, v_3 > - < v_0, v_3 > + < v_0, v_2 >$$

$$= < v_1, v_2 > + < v_0, v_1 > + < v_2, v_3 > + < v_3, v_0 >$$




$$S_{31} = < v_0, v_1, v_2, v_3 > \text{ and } S_{32} = < v_0, v_2, v_4, v_3 >$$

$$C_3 = < v_1, v_2, v_3 > - < v_0, v_2, v_3 > + < v_0, v_1, v_3 >$$

$$- < v_0, v_1, v_2 > + < v_2, v_4, v_3 > - < v_0, v_4, v_3 >$$

$$+ < v_0, v_2, v_3 > - < v_0, v_2, v_4 >$$

$$= < v_1, v_2, v_3 > + < v_0, v_1, v_3 > - < v_0, v_1, v_2 >$$

$$+ < v_2, v_4, v_3 > - < v_0, v_4, v_3 > - < v_0, v_2, v_4 >$$

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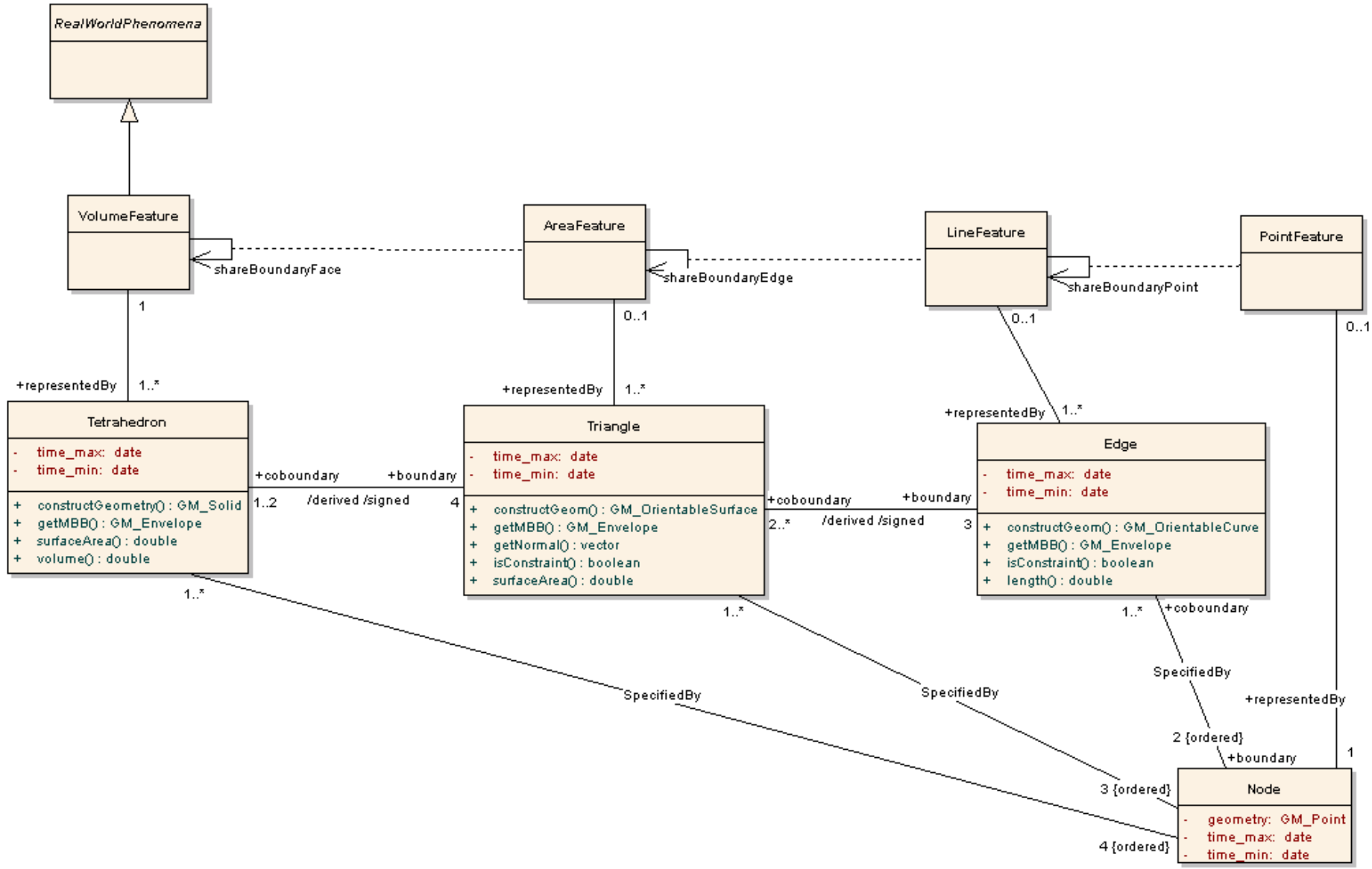
# TEN approach using simplicial homology

## First ideas

- Describe all simplexes by their vertices
- Store tetrahedrons and vertices in table
- Derive triangles and edges (views) using the boundary operator
- Use signed and oriented simplexes

In UML this looks like...


RealWorldPhenomena
VolumeFeature
AreaFeature
LineFeature
PointFeature
shareBoundaryFace
shareBoundaryEdge
shareBoundaryPoint
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0..1
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+ getNormal() : vector
+ isConstraint() : boolean
+ surfaceArea() : double
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3
Edge
- time_max: date
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4 {ordered}
Node
- geometry: GM_Point
- time_max: date
- time_min: date

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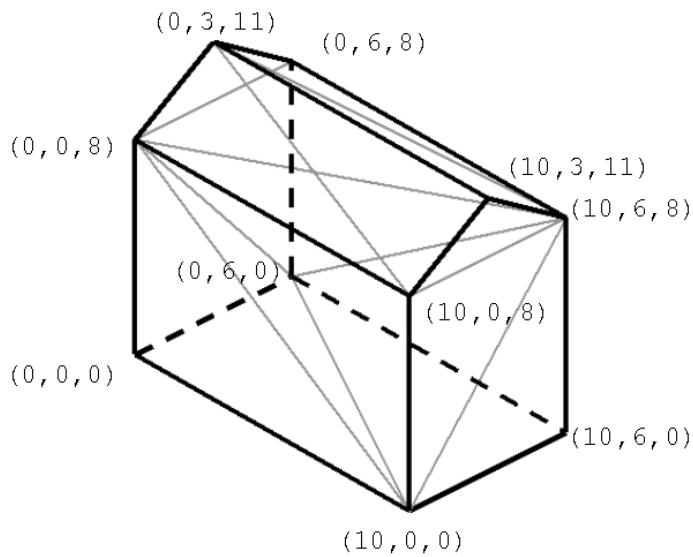
# TEN approach using simplicial homology

## Current ideas

- Encode vertex coordinates
- Describe all simplexes by their coded vertices
- Store tetrahedrons in table
- Derive triangles, edges and nodes (views) using the boundary operator
- Use signed and oriented simplexes

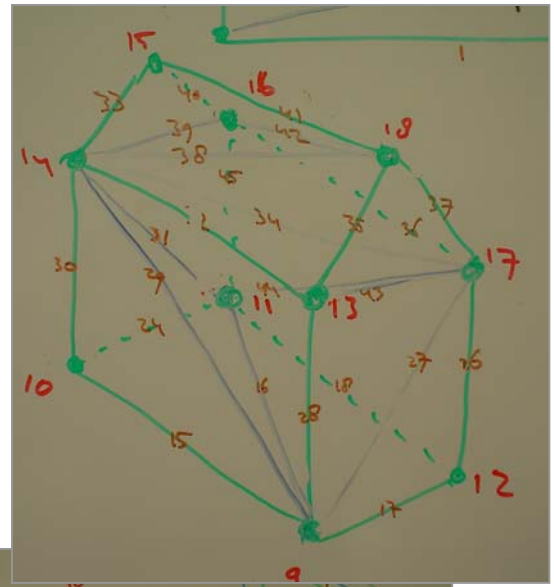
```
(00,00,00)(00,00,08)(00,06,00)(10,00,00)
(00,00,08)(00,03,11)(00,06,08)(10,06,08)
(00,00,08)(00,03,11)(10,00,08)(10,06,08)
(00,00,08)(00,06,00)(00,06,08)(10,06,08)
(00,00,08)(00,06,00)(10,00,00)(10,06,08)
(00,00,08)(10,00,00)(10,00,08)(10,06,08)
(00,03,11)(10,00,08)(10,03,11)(10,06,08)
(00,06,00)(10,00,00)(10,06,00)(10,06,08)
```



# Outline

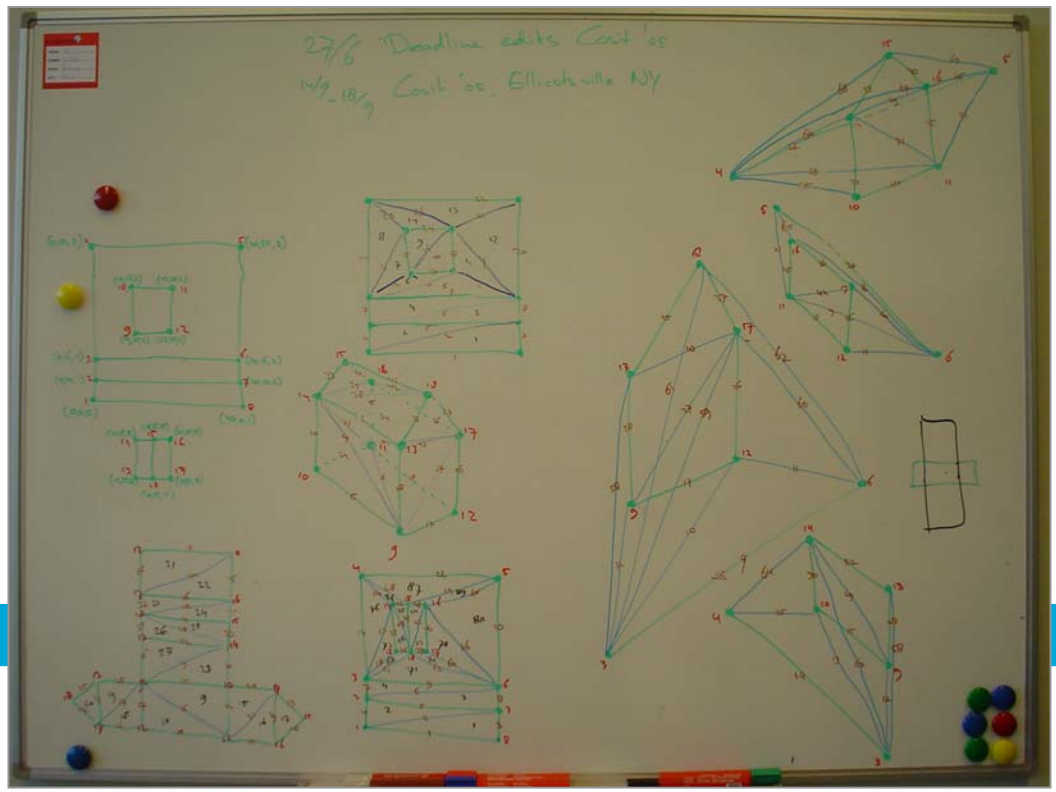
- Introduction
- PhD project history in a nutshell
- Basics of modelling approach
- Poincaré's formalism of simplicial homology
- Concept of Poincaré-based TEN structure
- Implementation → illustration of concept
- Conclusions & future research
- Discussion

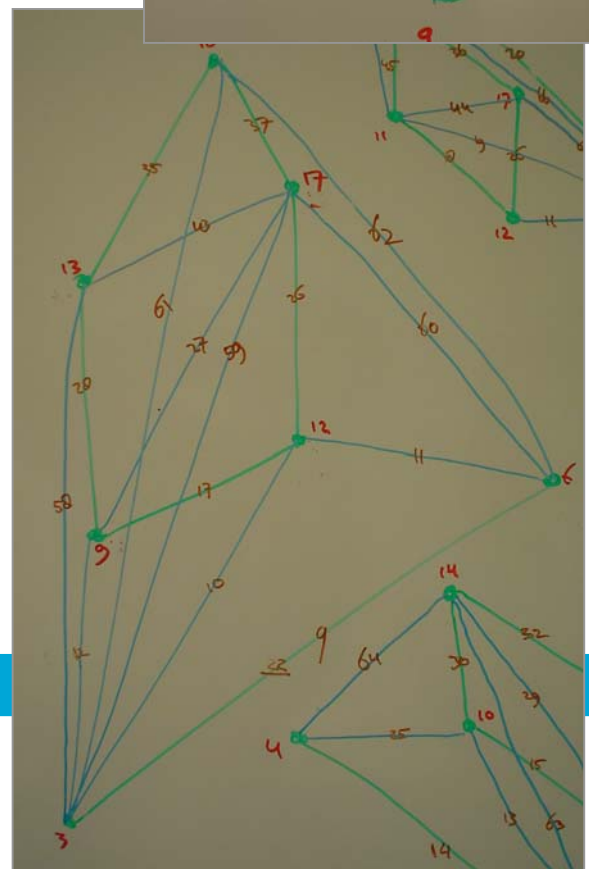
# Implementation
## Tetrahedronization



At this moment: tetrahedronization by hand

‘Toy’ dataset: 56 tetrahedrons, 120 triangles, 83 edges, 20 nodes





# Implementation

## Loading data in tetrahedron table

```
CREATE TABLE tetrahedron( tetcode NVARCHAR2(100));
```

```
LOAD DATA INFILE 'data/miniset.data' APPEND
INTO TABLE tetrahedron
fields terminated by ' '
( tetcode )
```

Result: table with tetrahedrons

```
01402501201403501202203501201403501800 3
01402501202203501202202501202202501800 3
01402501201402501801403501802202501800 3
02203501201403501802203501802202501800 3
01402501202203501201403501802202501800 3
01402501801403501802202501801802502100 3
01403501801803502102203501801802502100 3
01403501802203501802202501801802502100 3
02005500000000001004001001204000001100 1
02005500000000001000001001104001001200 1
02005500000010011040016012040010012004
02005500000010011000016011040016012004
02005500000016011040016012022025012001
02005500000016011014025012022025012001
02005500000016011014025012014035012001
02005500000016011000050012014035012001
```

# Implementation
## Fixing orientation tetrahedrons

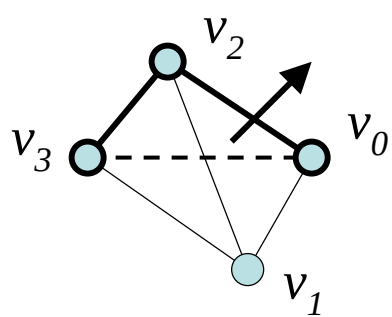
Objective: all tetrahedrons oriented outwards

→ each boundary triangle appears two times: 1x pos., 1x neg.

```
CREATE OR REPLACE PROCEDURE tettableoutwards
(…)
checkorientation(codelength,currenttetcode,bool);
IF (bool = 0) THEN
   permutation12(codelength,currenttetcode,newtetcode);
   UPDATE tetrahedron SET tetcode=newtetcode WHERE CURRENT OF tetcur;
(…)
END;
```

`checkorientation` : angle normal vector and vector to opposite point

`permutation12` : $(v_0,v_1,v_2,v_3) \rightarrow (v_1,v_0,v_2,v_3)$



# Implementation

## Deriving boundary triangles

Applying boundary operator: $\partial S_n = \sum_{i=0}^{n} (-1)^i < v_0, \ldots, \hat{v}_i, \ldots, v_n >$

Procedure:

```
CREATE OR REPLACE PROCEDURE deriveboundarytriangles(
    (…)
   a := (SUBSTR(tetcode,1,3*codelength));
   b := (SUBSTR(tetcode,1+3*codelength,3*codelength));
   c := (SUBSTR(tetcode,1+6*codelength,3*codelength));
   d := (SUBSTR(tetcode,1+9*codelength,3*codelength));
   id := (SUBSTR(tetcode,1+12*codelength));
   ordertriangle(codelength,'+'||b||c||d||id, tricode1);
   ordertriangle(codelength,'-'||a||c||d||id, tricode2);
   ordertriangle(codelength,'+'||a||b||d||id, tricode3);
   ordertriangle(codelength,'-'||a||b||c||id, tricode4);
     (…)
```

Example:

```
01402501201403501202203501201403501800 3

a : 014025012
b : 014035012
c : 022035012
d : 014035018
id: 003
```

see next slide for results of **ordertriangle**

Note: triangles inherit ID of object which is represented by the tetrahedron of which they are a boundary!

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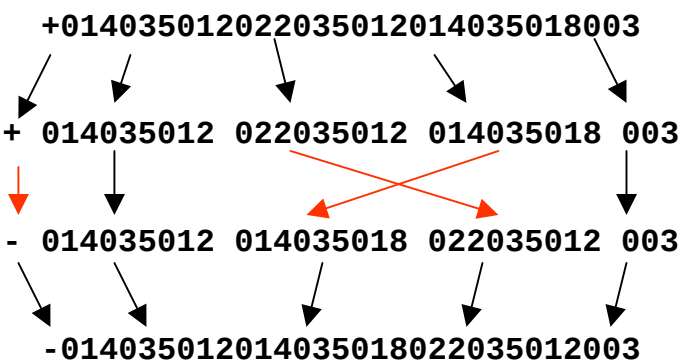
# Implementation
## Ordering triangles

Objective: gain control over which permutation is used

**`ordertriangle`** rewrites in form <a,b,c> such that a<b<c




$<v_0,v_1,v_2>$ -

$<v_0,v_2,v_1>$ +

$<v_2,v_0,v_1>$ -

$<v_2,v_1,v_0>$ +

$<v_1,v_2,v_0>$ -

$<v_1,v_0,v_2>$ +

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# Implementation

## Creating view triangle

```
CREATE OR REPLACE VIEW triangle AS
   SELECT deriveboundarytriangle1(3,tetcode) tricode FROM tetrahedron
   UNION ALL
   SELECT deriveboundarytriangle2(3,tetcode) tricode FROM tetrahedron
   UNION ALL
   SELECT deriveboundarytriangle3(3,tetcode) tricode FROM tetrahedron
   UNION ALL
   SELECT deriveboundarytriangle4(3,tetcode) tricode FROM tetrahedron;
```

Four functions: first gives first boundary, etc.

Result: #triangles = 4 * #tetrahedrons

Every triangle appears two times: once with sign +, once with sign – (and NOT in a permutated form → due to **`ordertriangle`**!)

# Implementation

## Creating view constrainedtriangle

```
CREATE OR REPLACE VIEW constrainedtriangle AS
   SELECT t1.tricode tricode FROM triangle t1
   WHERE NOT EXISTS (SELECT t2.tricode FROM triangle t2 WHERE t1.tricode =
    t2.tricode*-1);
```

Well, not every triangle appears two times:

A constrained triangle is a boundary between two objects

→ two different id's → two different triangle codes!

Example: in -1,7,2,-7,-3,1 the constrained triangles are 2 and -3

# Implementation

## Creating views edge, constrainededge

In current implementation edges are undirected en do not inherit object id's (as no application for this is identified at the moment)

```
CREATE OR REPLACE VIEW edge AS
   SELECT DISTINCT deriveabsboundaryedge1(3,tricode) edcode FROM triangle
   UNION
   SELECT DISTINCT deriveabsboundaryedge2(3,tricode) edcode FROM triangle
   UNION
   SELECT DISTINCT deriveabsboundaryedge3(3,tricode) edcode FROM triangle;
```

All boundary edges from constrained triangles are constrained edges:

```
CREATE OR REPLACE VIEW constrainedtriangle AS
   SELECT t1.tricode tricode FROM triangle t1
   WHERE NOT EXISTS (SELECT t2.tricode FROM triangle t2 WHERE t1.tricode =
t2.tricode*-1);
```

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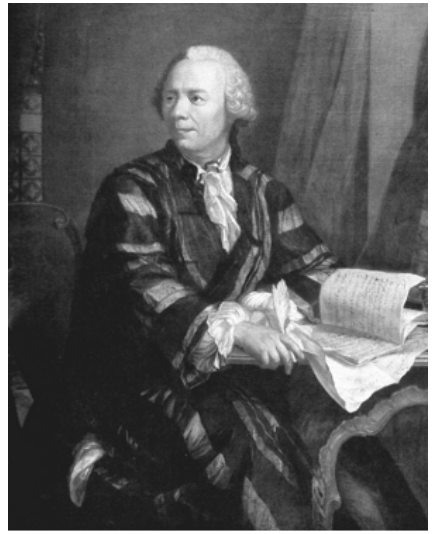
# Implementation
## Validating the structure (1/3)



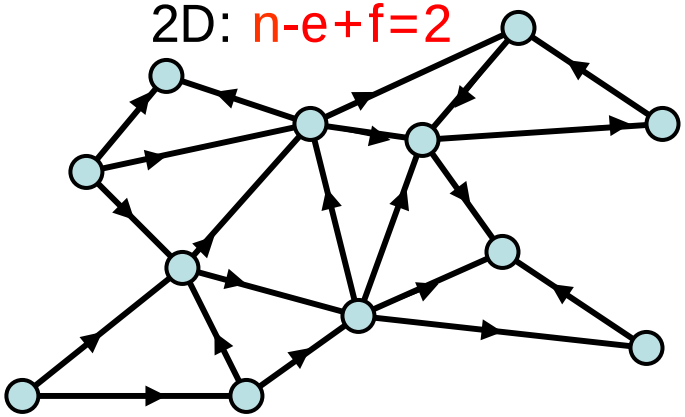
*Leonard Euler, 1707-1783*

After creating node view the structure can be validated:

3D Euler-Poincaré

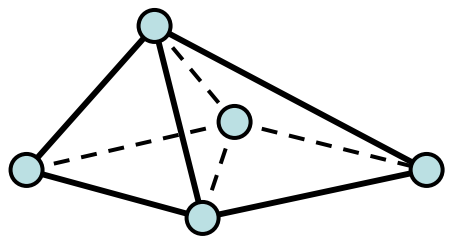
$$N - E + F - V = 0$$

2D: n-e+f=2



(12-21+11=2)

3D: n-e+f-v=0



(5-9+7-3=0)

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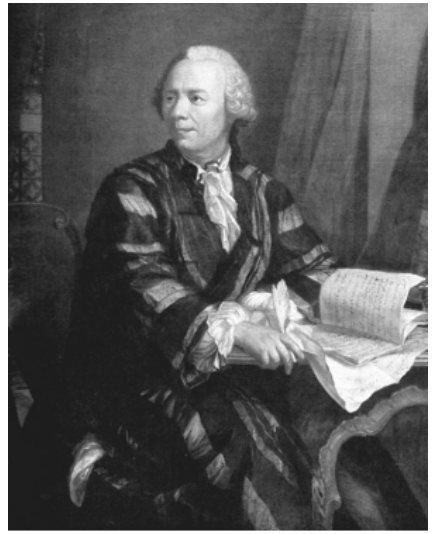
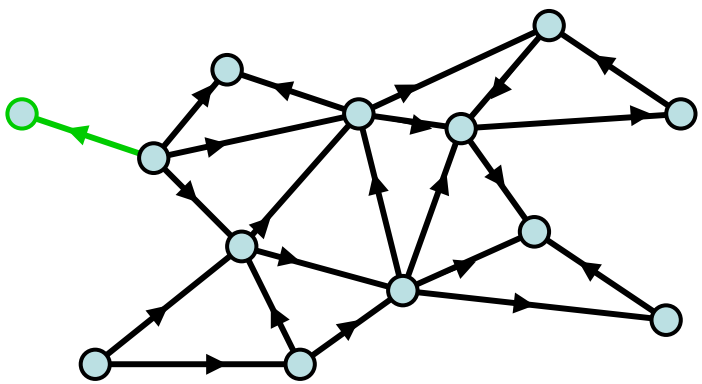
# Implementation
## Validating the structure (2/3)



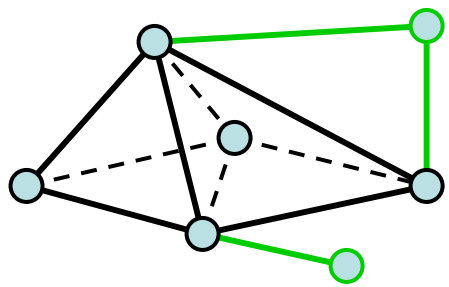
*Leonard Euler, 1707-1783*

Limitations:

3D Euler-Poincaré holds for all simplicial complexes, including complexes build up of simplexes of different dimension (i.e. dangling edges and faces are allowed)



(13-22+11=2)



(7-12+8-3=0)

# Implementation
## Validating the structure (3/3)

3D Euler-Poincaré for current dataset:

$$N - E + F - V = 0$$

20 –83 + 224 –57 <> 0 !

→ View triangle contains duals, need to be excluded from count:

```
> select count(*) from tetrahedron;
  COUNT(*)
----------
        56
> select count(*) from triangle;
  COUNT(*)
----------
       224
> select count(*) from edge;
  COUNT(*)
----------
        83
> select count(*) from node;
  COUNT(*)
----------
        20
```

```
SELECT COUNT(DISTINCT ABS(removeobjectid(3,tricode)))
   INTO numtri FROM triangle;
```

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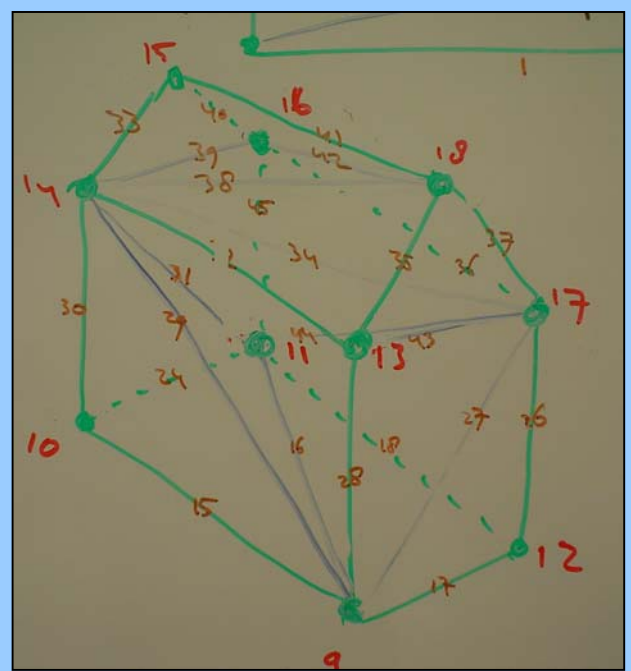
# Implementation

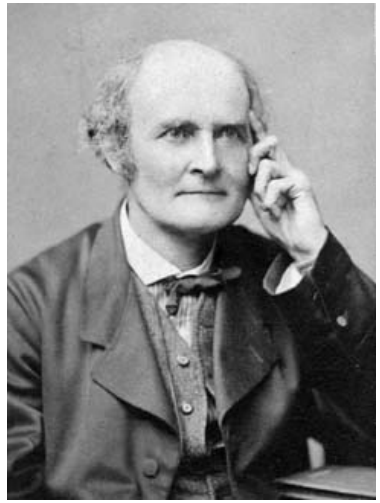
## Query and analysis (1/2)

Query: boundary triangulation of building (object ID = 3)

```
SQL> select tricode from constrainedtriangle where getobjectid(3,tricode)=3;
TRICODE
--------------------------------------------------------------------------------
-0140250120140250180220250180003
-0140250180180250210220250180003
-0180250210180350210220350180003
+0140250120220250120220250180003
-0140250120140350120140350180003
+0180250210220250180220350180003
-0140250120220250120220350120003
+0140250120140250180140350180003
+0140250120140350120220350120003
-0140350180220350120220350180003
+0140350120140350180220350120003
+0140350180180250210180350210003
+0220250180220350120220350180003
+0140350180180350210220350180003
-0220250120220250180220350120003
-0140250180140350180180250210003
16 rows selected.
Elapsed: 00:00:00.09
```



# Implementation
## Query and analysis (2/2)



*Arthur Cayley, 1821-1895*

Volume of house, surface of boundary of house:

**`simplexvolume()`** implements Cayley-Menger determinant

Cayley-Menger determinant gives the volume of a simplex in *j* dimensions.

*j=2:* $$-16\,\Delta^2 = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & c^2 & b^2 \\ 1 & c^2 & 0 & a^2 \\ 1 & b^2 & a^2 & 0 \end{vmatrix}$$

*j=3:* $$288\,V^2 = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & d_{12}^2 & d_{13}^2 & d_{14}^2 \\ 1 & d_{21}^2 & 0 & d_{23}^2 & d_{24}^2 \\ 1 & d_{31}^2 & d_{32}^2 & 0 & d_{34}^2 \\ 1 & d_{41}^2 & d_{42}^2 & d_{43}^2 & 0 \end{vmatrix}$$

(with *a,b,*c and $d_{ij}$ length of simplex edges)

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# Implementation
## Performance

Indexing:

- Primary index: sort coded simplexes
- Secondary index: R-tree on tetrahedrons, using **`gettetrahedronmbb(), gettrianglembb()`** etc.

Coding:

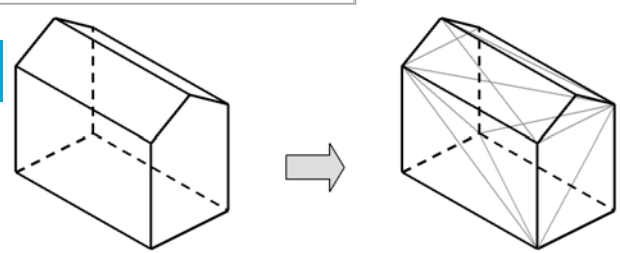
- More work on encoding coordinates: bitwise interleaving, ...

# Implementation
## Final thougth

Polyhedron vs. TEN:

does a TEN really require that much more storage space than polyhedrons?

| Building as polyhedron | Building as TEN |
|---|---|
| (1 volume) | 8 tetrahedrons |
| 7 faces | (24 triangles) |
| (15 edges) | (25 edges) |
| (10 points) | (10 nodes) |



# Conclusions & future research

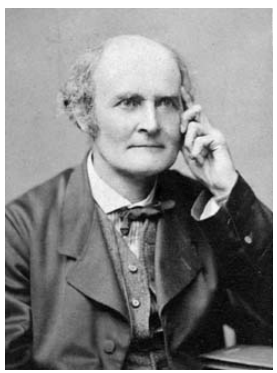
Result:

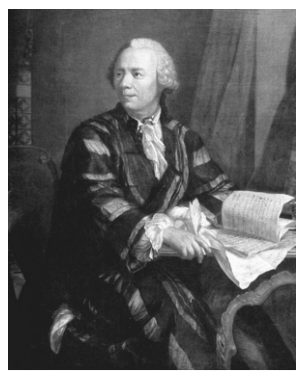
- Topological 3D (TEN) data structure, stored in one single-column table (!)
- with advantages of TEN, but not its drawbacks (?)
- based on a solid theoretical foundation (100 years old math)

Future research ideas:

- Inclusion of incremental constrained Delaunay tetrahedronization
- Test with real data (Den Bosch case?)
- Compare to Calin/Oracle11 polyhedron approach
- History (?)

# Discussion
# (with the living…)





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