A Map Generalization Model Based on Algebra Mapping Transformation

Tinghua Ai

Section GIS technology, Department of Geodesy
Faculty of Civil Engineering and Geosciences,
Delft University of Technology, the Netherlands

Faculty of Resource and Environment Science
Wuhan University, P. R. China
tinghua_ai@hotmail.com

Peter van Oosterom

Section GIS technology, Department of Geodesy
Faculty of Civil Engineering and Geosciences,
Delft University of Technology, the Netherlands
oosterom@geo.tudelft.nl

Abstract

From the point of view of mapping transformation, this paper presents a map generalization conceptual framework which regards generalization as two kinds of mapping procedures: spatial entity mapping and spatial relationship mapping. According to the number of changes in the participating entities, spatial entity mapping is classified as l-l, n-l, n-m mapping. Spatial relationship mapping is described as a composite relationship transformation of the components: topology, distance and orientation. The concept ***'spatial relationship resolution*** is introduced to describe spatial relationship related constraints. Based on the 9 intersection model, the cardinal direction model and the iso-distance-relationship model, the paper gives three sorts of relationship resolution representations for topological, distance and orientation relationship respectively. The behavior of the two mappings in map generalization is discussed and the spatial relationship abstraction obtains emphasis compared with the traditional generalization conceptual model.

Keywords map generalization, spatial relationship, spatial relationship resolution.

1. INTRODUCTION

The questions what the map generalization process is and how to describe the process are basic issues in the research field of generalization conceptual modeling. From different perspectives, related research gives various answers resulting in different solutions in conflict detection, operator classification, constraint analysis, workflow control, and generalized result evaluation. Based on the idea of "processing based on understanding", Brassel and Weibel (1988) gave a description dividing the map generalization into five steps: structure recognition, process recognition, process modeling, process execution, and data visualization. Supported by information theory, generalization could be considered as a process of information entropy transformation (Bjorke 1996, Weber 1980). According to this understanding, the original map is the information sender and the generalized map is the information receiver. The generalization reflects as the communication process of information coding and decoding with entropy reduction due to noise impact. From artificial intelligence viewpoint, generalization could be regarded as a problem solution finding (Ware and Jones 1998, Longergan 1999) to obtain the best solution from multiple candidates under the control of geometric, topological and semantic constraints. Some methods in AI and expert system field such as simulated annealing technology and hill climbing technology can be used in problems such as displacement decision or object selection (Ware and Jones 1998). As a complex processing system, map generalization involves multiple hierarchical analysis and multiple operation execution. Ruas and others think of generalization as an agent action process and try to use an agent method which is capable of controlling its own decision making and acting to resolve the problems in this complex system (Lamy,Ruas & Mackness 2000).

Supported by different theories and technologies, one understanding of the generalization process is able to solve some special problems and has advantages in some aspects over others. It is difficult and also not necessary to decide which generalization conceptual model is the best one. What we are interested in is the completeness degree of problem solution for one understanding. Usually integrated methods based on two or more understandings are required to solve one question in a complete generalization. In this field, one important trend is that the conceptual model requires formalized representation allowing the computer to understand and realize the process through data model and algorithm design. Based on the set mapping theory in relational algebra, we will present a map generalization conceptual model which regards generalization as two kinds of mapping procedures: spatial entity mapping and spatial relationship mapping. An outstanding characteristic of this model is the introduction of the spatial relationship abstraction. Traditionally the considered object in generalization focuses on spatial entity and most of the generalization operators are entity oriented.

Map generalization can be separated as database generalization and visualization generalization (Brassel and Weibel 1988, McMaster and Shea 1989, Peng 1995, Muller 1991). The first process focuses on data content abstraction from the point of view of lower resolution without consideration of data visualization. While the latter deals with such as graphic conflicts when a spatial object is represented as a symbol. The conceptual model in this paper deals with the database generalization and considers the geographic database as a data set containing spatial, attribute and temporal

GIS'01, November 9-10, 2001, Atlanta, Georgia, USA.

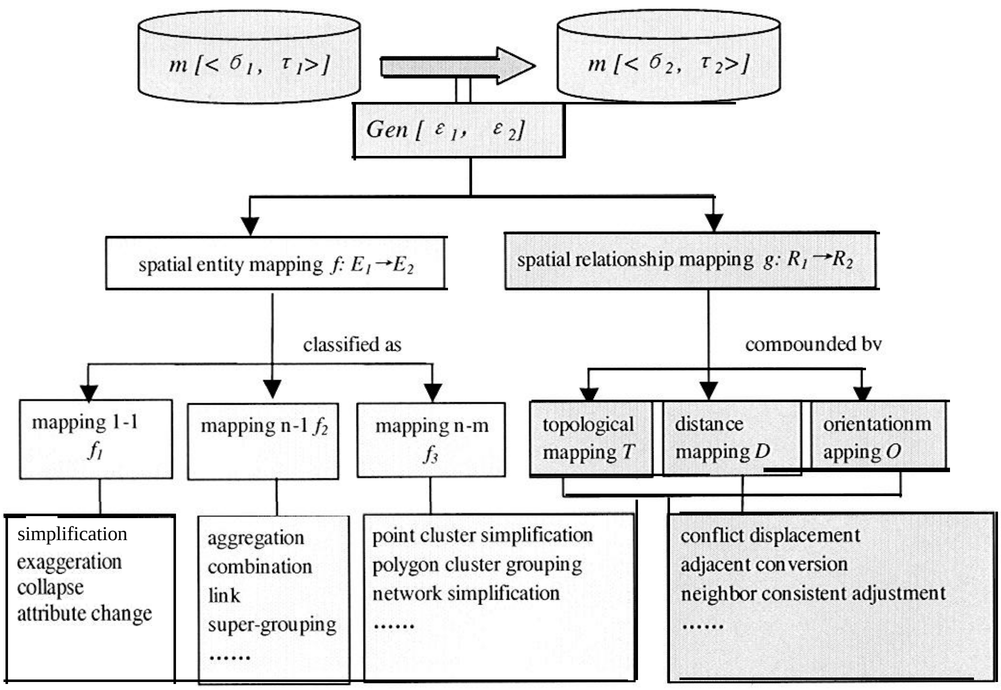


Figure 1. The map generalization conceptual model based on algebra mapping transformation.

information.

The remaining part of this paper is organized as follows. Section 2 presents the generalization conceptual model, which is based on mapping transformation. The concepts 'spatial relationship resolution' and 'spatial relationship abstraction' are discussed in section 3. The way the two mapping procedures behave in the generalization is discussed in section 4. Some future works are presented in section 5.

2. THE MAP GENERALIZATION CONCEPTUAL MODEL BASED ON MAPPING TRANSFORMATION

A geographic database contains two categories of information: spatial entities and spatial entity relationships. The original map can be represented as an entity set $E_{original} = \{ e_{original} \}$ and a relationship set $R_{original} = \{ r \mid r \in E_{original} \times E_{original} \}$. The generalized new map can be represented as an entity set $E_{new} = \{ e_{new} \}$ and a relationship set $R_{new} = \{ r \mid r \in E_{new} \times E_{new} \}$ (Wu 1997). The mapping transformation from set $E_{original}$, $R_{original}$ to E_{new} , R_{new} describes the map generalization process. This is the basic idea of the map generalization conceptual model based on set mapping. A more detailed description is presented below.

Each geographic data set M contains the sets E and R representing real space under the certain abstraction degree parameter ε, which depicts map representation resolution. State variable ε may be described as a two-element tuple (We do not consider temporal information here) $\varepsilon : (\sigma, \tau)$, where σ, τ stands for spatial resolution and attribute resolution respectively. Using $\varepsilon_1 < \varepsilon_2$ represents abstraction degree ε_1 (corresponding to the detailed map) less than ε_2 (corresponding to the simple map). Then the original map $M[\varepsilon_1]$ is represented as:

$$M[\varepsilon_1] = M[<\sigma_1, \tau_1>] : E_1 \cup R_1$$

The generalized map $M[\varepsilon_2]$ is represented as:

$$M[\varepsilon_2] = M[<\sigma_2, \tau_2>] : E_2 \cup R_2 , \quad Where\ \varepsilon_1 < \varepsilon_2$$

The generalization mapping can be represented as:

$$Gen[\varepsilon_1, \varepsilon_2] : M[\varepsilon_1] \rightarrow M[\varepsilon_2].$$

Mapping Gen can be separated as spatial entity mapping $f: E_1 \rightarrow E_2$, and spatial relationship mapping $g: R_1 \rightarrow R_2$. For spatial entity mapping f, based on the number of involved entity changes from the original and the generalized representation, we can divide it into three classes:

1-1 mapping: $e' = f_1(e);$

n-1 mapping : $e' = f_2(e_1, e_2, \ldots e_i)$

n-m mapping : $(e'_1, e'_2, \ldots e'_j) = f_3(e_1, e_2, \ldots e_i).$

For the spatial relationship mapping g, based on spatial relationship

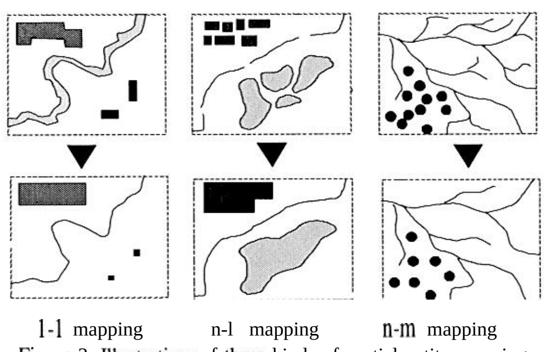


Figure 2. Illustrations of three kinds of spatial entity mapping

classification (Egenhofer 1991) we can consider it as the composite mapping of three independent spatial relationship mappings: topological relationship mapping T, distance relationship mapping D and orientation relationship mapping 0. There exists following representation: $r' = g(r) = T(r)D(r)O(r)$, where r is the spatial relationship. This generalization concept ual model may be depicted as in figure 1.

Spatial entity mapping and spatial relationship mapping change the information content in geographic database and describe a transformation. In generalization, there is the clean transformation going from high resolution to low resolution. In this sense, map generalization can be regarded as a special spatial mapping, an abstraction procedure.

Spatial entity mapping involves spatial information transformation and attribute information transformation respectively controlled under spatial resolution σ and attribute resolution τ In 1-1 mapping f_1 , the mapped entity is the same as the original one, but has different properties in geometric and semantic representation, see figure 2 left. If $f_1(e)=NULL$, it means that the spatial entity e has been removed from the database. Otherwise the image entity $f_1(e)$ still exists independently but with nature change, which may be the simplification of geometric shape, exaggeration of size to enhance existence, collapse conversion from polygon to skeleton line or collapse from polygon to center point, etc. In n-l mapping f_2 , the original entity does not remain independent and complete. It just acts as a part of a new mapped composite entity. This mapping reflects as object aggregation. According to two object class hierarchies between the basic elements and the aggregated object, *Is_A* and ***Part-of,*** the mapping can be further separated into aggregation of homogeneous spatial entities and amalgamation of heterogeneous spatial entities. In this mapping, both spatial adjacency and semantic adjacency have to be taken into account. For example, in land-use parcel aggregation, when parcels have a similar spatial distance to each other, those with a closer relationship in semantic hierarchical tree prefer to be aggregated first. In n-m mapping, it is cluster object oriented. The elements before and after mapping remain independent and complete. But because they are highly related to each other in spatial or semantic aspects, the cluster structure characteristics among them, such as spatial distribution, Gestalt nature, terrain landform feature, become the key consideration in the mapping. From the point of view of composite object, this kind of mapping can be thought of as a l-l mapping of a composite object, since entities participating in the mapping procedure make up a composite object. Considering the same resolution of mapping objects, the call of n-m mapping is more reasonable. The examples of this mapping could be resample

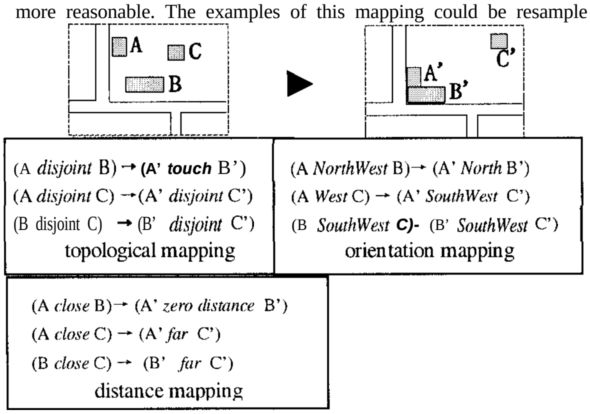


Figure 3. An illustration of spatial relationship mapping.

of resident point cluster, simplification of polygon cluster such as islands, lakes, buildings, etc., generalization of road network, simplification of street network and street blocks, abstraction of river drainage and abstraction of terrain contours, etc.

3. SPATIAL RELATIONSHIPSHIP RESOLUTION AND SPATIAL RELATIONSHIPSHIP ABSTRACTION

According to the relational algebra definition, spatial relationship is described as a Cartesian tuple of spatial entities. So the study object in spatial relationship mapping is a spatial entity pair rather than the spatial entity itself. It is different from the transformation of geometric space, such as affine transformation, in which the entity itself is oriented. The expression $e_1\, r\, e_2$ denotes spatial entity e_1 having relationship r with entity e_2 The mapping that the original relationship r is mapped as r' between e_1 and e_2 can be represented as follows:

$$e_1\, r'\, e_2 = g\,(e_1\, r\, e_2)$$

The three basic spatial relationships T, D, 0 have different semantic descriptions. However the way how to combine them to get an integrated description for spatial cognition has not been resolved by now. So we can not find the mathematical function of mapping g just like an affine transformation which can be represented as one matrix to integrate the three independent transformations: translation, rotation and scaling. In the content of spatial relationship representations, one may ask whether topological, orientation and distance relationship are really basic relationship elements? Is there aother type of relationship,which need to be added? Although spatial relationship research meets challenges, we can give qualitative discussions for spatial relationship mapping.

Unlike general spatial relationship mapping, the relationship mapping contained in generalization is the transformation from detailed state to abstract state. In this sense, we call it spatial relationship abstraction. Displacement operation is a typical spatial relationship mapping in generalization, through entity position adjustment to resolve spatial relationship conflicts. In this procedure the operated object is a spatial entity pair rather than an independent entity.

In the gepgraphic model, we have spatial resolution σ, attribute resolution τ as well as temporal resolution. Peng (1995) classified spatial resolution as spatial size resolution, spatial feature resolution and spatial distance resolution. These resolutions aim at spatial entity abstraction. Spatial relationship abstraction is also based on resolution change. So a new resolution concept, ***spatial relationship resolution*** has to be built. Spatial relationship resolution (abbreviated as **SRR** later on) is defined as the minimum identifiable semantic description of spatial relationship. Spatial relationship representations, including topological, orientation and distance relationship, have different similarity to each other. It means that some relationships are close to each other while others are far away. The close relationships can be further grouped as a higher level semantic description. So the spatial relationship description is a hierarchical tree structure. The SRR describes the hierarchical level, represented as the node depth in the tree structure. The SRR description depends on the model of spatial relationship representations. Next we will give three methods of constructing a relationship hierarchical tree for topological, distance and orientation relationship respectively.

3.1 Topological relationship resolution

The nine intersection representation of topological relationship (Egenhofer 1991,1995) can get 2^9=512 sorts of relationships between two spatial objects. However, tie valid number of relationships is less than this number after meaningless relationships are removed. Among the remailing relationships, according to the steps of changing one state to another state, Egenhofer and Mark (1995) built the conceptual neighbohood graph of topological relationship. The connection between neighbor representations results in a network to describe adjacency relationship of topological relationship (note the saying relationship of relationship), as shownin figure 4 top. In this mode,, the less steps from one relationship to another relationship, the closer similarity between them is. Based an this made,, we can construct the hierarchical tree to representthe semantic level ofa topological relationship According to certain cognition standards, some neighbor relationships in the neighbor network are grouped into high level description. As shown in figure 4, original 19 relationships between line and area can be grouped as 4 high level

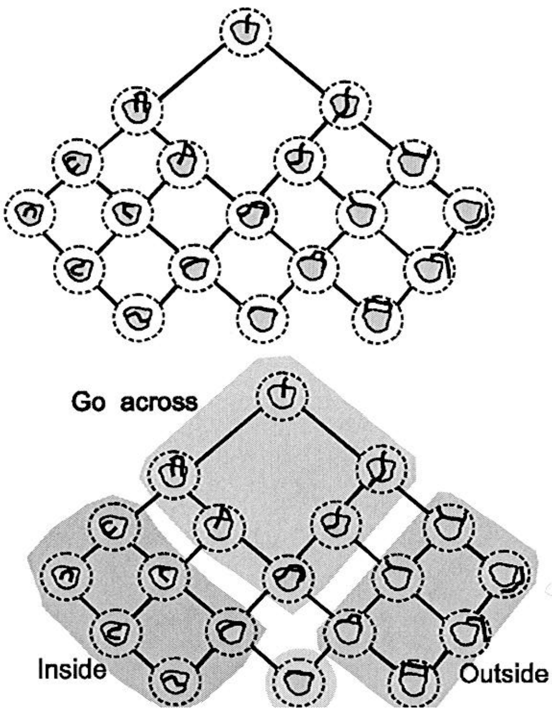


Figure 4. 19 topological relationships between line and area object (top) can be mapped as 4 descriptions (bottom) with resolution decrease.(The top graphic is fromEgenhofer 1992)

relationships:*inside*, *outside*, *go across* and on *boundary*. Within each group, the relationship is no longer to be distinguishedfrom each other under the tower resolution standard. For some purpose, 4 distinguished relationship representation arc enough. Under control of this resolution we can execute spatial relationship mapping to get abstract representation.

Another grouping of detailed topological relationships into high level (more common) relationships is given byClementini, Di Feliceand van Oosterom (1993).

3.2 Distance relationship resolution

Compared with topological relationship research, distance relationship research is less active and has few achievements in qualitative description. Absolute quantitative representation of how far between two objects is able to usethe sentence of Euclidean distance. But in distance relationship representation, what it means for A to be near to B depends not only on their absolute positons (and the metric distance between them), but also on their relative sizes and shapes, the position of other objects, the frame of reference (Hernandez and Clementini, ,995). The context environment plays an important role in distance relationship representation. We give a method based an Voronoi diagram (VD) to represent distance representation, and based on this representation the distance relationship resolution will be discussed

Each spatial entity in scene environment has a certain influence region surrounding to which this object is closer than any other objects. Of course, the influence region has to consider the existence of neighbor objects. Assuming that the space is isotropic, then we can use the Voronoi diagram partitioning of the area, acting as the spatial entity influence region. The boundary of VD cell polygon equally partitions space between two neighbor left/right entities. The VD partitioning can be thought of as the result of each entity equally competing outward for growth range If two VD cells share a common boundary, we can say that the two entities belonging to the cell polygons are adjacent, evenif their metric distance is far. Based on this idea, we canuse relationship between VD cells to represent the distance relatimship between spatial entities. Making use of the adjacency transmitting property, we define a variable *adjacent degree to* describe distance relationship and use the next algorithm to obtain the *adjacency degree value* of all objects withrespect to object *A*.

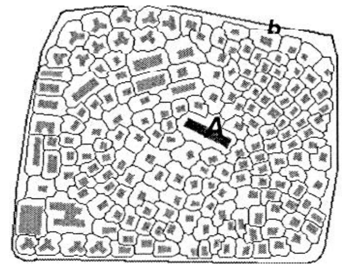


Figure 5. Building polygon cluster and Voronoi diagram

1> Let A itself adjacency *degree* 0, and initiate other object adjacencydegree-1;
2> Initiate *A* belonging to active object set. Initiate variable *degree-count 0;*
3> Repeat next steps until active object set NULL;
- 3.1> Find all adjacent objects of active object set based on VD cell extending search:
- 3.2> Ignore those adjacent objects with adjacency degree other than -1;
- 3.3> *degree-count* add 1 and assign the value into each valid adjacent object;
- 3.4> Empty active object set and let valid adjacent objects belong to active object set;

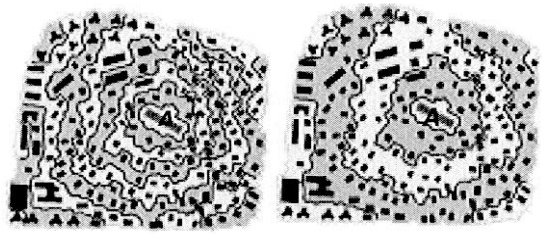


Figure 6, The iso-distance-relationship contour with respect to the center object A, left with interval adjacency degree value 1 unit and right with 2 unit.

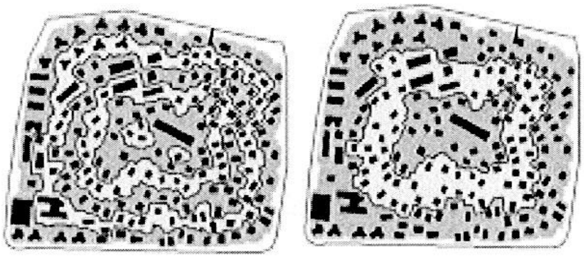


Figure 7. The iso-distance-relationship contour with respect to the boundary b, left with interval adjacency degree value 1 unit and right with 2 unit.

Next we select and connect parts of VD cell boundaries getting the contour line which separates objects with *adjacency degree n* from those with *adjacent degree n+1*, getting the result as shown in figure 6 left. The objects within the loop between two neighbor contour lines have the same *adjacent degree* with respect to object A. So we call this kind of contour line the iso-distance-relationship contour, just like the altitude contour of terrain representation. The smaller value of *adjacency degree* is, the closer distance relationship two objects have to each other. Obviously this contour is different from the iso-distance contour which is represented as progressive circle buffers with the same center and increasing radius. The iso-distance-relationship model considers the context environment and spatial distribution. An object far away in metric distance, may possibly have very low *adjacency degree* and close distance relationship with the reference object. The reference object could be an object set rather than a single object. For example, the objects adjacent to outside boundary as shown in figure 7.

Having the iso-distance-relationship model, we can now discuss the distance relationship resolution. In terrain contour representation, we use altitude intervals expressing resolution, the smaller interval altitude, the higher resolution. In the same way, we adopt the *adjacency degree* interval describing the distance relationship resolution. Selecting one from every two neighbor contour lines gets the distance relationship representation whose resolution reduces half as shown in figure 6 right and figure 7 right. Higher resolution corresponds to more grades in distance relationship representation. For example in the representation of 1 unit *adjacency degree* interval we use the following semantic expression containing 7 grades:

(very close, close, medium close, medium, medium far, far, very far)

But for the representation of 2 unit interval *adjacency degree*, the relationships will be grouped into an updated semantic description with such as following 3 grades: *(close, medium, far).*

Based on this model, if the object moves within the loop, it does not change its distance relationship with the reference abject. In case of considering the distance relationship only, it is not necessary to execute relationship mapping to correct the distance relationship. But if the object moves across the loop, distance relationship mapping is required to correct the destroyed distance relationship. The lower resolution is, the wider loop exists and the chances of destroying original relationship are less. The way how resolution impacts mapping generalization will be discussed in section 4.

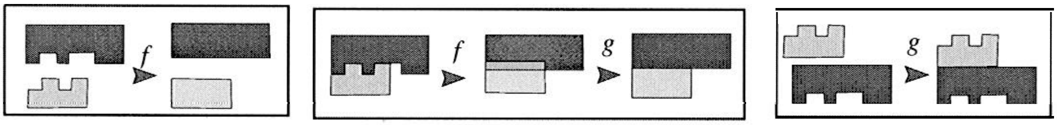
3.3 Orientation relationship resolution

hank (1992,1996) presented two methods of cardinal orientation direction representation, one based on triangular areas and another based on projection. Here we give the description of the orientation relationship resolution based on this cardinal direction model. The semantic description of cardinal direction has hierarchical properties. We have 4 distinguished direction relationships: *north, west, south, east* with each covering 2 π /4 sector range. Further separating, we can get 8 direction relationships: *north, northeast, east, southeast, south, southwest, west, northwest* with each covering 2 π /8 sector range. The separation can goon and get more detailed direction relationship descriptions. The angle range of one direction covering is able to be defined as an orientation relationship resolution.

4. SPATIAL RELATIONSHIP ABSTRACTION BEHAVIOR IN MAP GENERALIZATION

The reason of generalization exists in the smaller scale representation not satisfying constraints and so abstraction processing is needed to adjust it. The constraints of generalization are usually statements related to spatial, attribute and temporal resolution. Weibel and Dutton(1998) gave 4 types of distinguished constraints: graphical, topological, structural, and Gestalt. From the point of view of mapping transformation, the constraints can be categorized as spatial entity associated constraints and spatial relationship associated constraints. The latter relates to not only topological relationship which appears in Weibel and Dutton's classification but also to distance relationship and orientation relationship. Its statement format usually reflects as "remaining spatial relationship unchanged" or "avoiding the appearance of undistinguishable relationship". The comparison of spatial relationship quality has to be based on *SRR* just like the equality judgment between two float numbers, in which the considered precision should be predefined. Under a certain resolution, according to the category of destroyed constraints, corresponding spatial entity mapping and spatial relationship mapping are required. From one state to another state during map generalization, the relationship between spatial entities may have changed in a strict sense. But under a low resolution, the cognition neglects most of tic small changes thinking they remain original state. Only for those distinct relationship changes, post-processing is required to adjust the spatial position. Post-processing is usually called displacement in traditional map generalization. Based on the spatial mapping model in this paper, displacement is just one of the concrete forms of spatial relationship mapping, used to satisfy a constraint. In this section wc will focus on spatial relationship mapping and give three cases of behavior in generalization.

We use some algebra notations of section 2. In map generalization algebra system $<E, R, E', R', f, g>$, E, R, E', R' respectively denote



A	B	C
$e_i \to f(e_i)$, $e_j \to f(e_j)$, $f(e_i)\ r'\ f(e_j) = e_i\ r\ e_j$ No relationship mapping	$e_i \to f(e_i)$, $e_j \to f(e_j)$, $f(e_i) r' f(e_j) \neq e_i\ r\ e_j$ Require relationship mapping g : $g(f(e_i) r' f(e_j)) \to f(e_i) r f(e_j)$	No entity mapping, Require mapping g: $g(e_i\ r' e_j) \to e_i\ r\ e_j$

Figure 8. Behavior of two mappings in map generalization

the spatial entity set, the spatial relationship set of the original map, and the spatial entity set, the spatial relationship set of the new map. f, g denote spatial entity mapping and spatial relationship mapping respectively. For two original entities e_i, $e_j \in E$, there is the relationship $r \in R$, $e_i\ r\ e_j$. After mapping $e_i \to f(e_i) \in E'$, $e_j \to f(e_j) \in E'$, the new mapped entities have the new relationship $r' \in R'$, $f(e_i)\ r'\ f(e_j)$. The spatial relationship mapping: $e_i\ r\ e_j \to g(e_i\ r\ e_j) = f(e_i) r' f(e_j)$ carries out the relationship conversion from r to r'.

4.1 Spatial entity against constraint

Spatial entity representation before generalization destroys the constraints associated with such as size resolution, feature resolution or attribute class resolution. Spatial entity mapping f is required to abstract and get simple representation which respects resolution requirements. If the spatial relationship r' between abstracted entities equals to the original r, $f(e_i) r' f(e_j) = e_i\ r\ e_j$, and the relationship r or r' is not against relationship constraint, then relationship mapping is not necessary, see figure 8 A. Otherwise $f(e_i) r' f(e_j) \neq e_i\ r\ e_j$, it means that the constraint "remaining original spatial relationship" is destroyed, and the relationship mapping g is required to convert r' to r, just like the example in figure 8 B, in which relationship mapping g corrects *overlap* relationship between two simplified buildings returning to original relationship *touch*.

4.2 Spatial relationship against constraint

The spatial entities satisfy the constraints of independent representation, but the relationship representations of entity pairs have the problem of being too detailed to be distinguished. Then spatial relationship abstraction g is needed. As shown in figure 8 C, when resolution reduced, detailed distance relationships need to be assigned to the higher level representation selecting a typical representative from low relationships. Here too short distance within object edge has to be assigned to a zero distance, $g(e_i r' e_j) \to e_i r e_j$. The street generalization of classifying streets into grades according to street width belongs to this case. Changing each polygon of lake cluster from the disjoint to exact touch also belongs to spatial relationship mapping. What drives the map generalization is the relationship constraint rather than entity constraint. For this kind of mapping, some of them can be obtained by the operation displacement, see figure 8 C, but others such as street classification generalization and lake cluster generalization are not able to be described as displacement. From the classification of 20 generalization operators which is presented by Mackness(1994) or 12 operators as classified by Shea & McMaster(1989), it is difficult to find a proper operator to explain this kind of generalization. The reason is that the operator classification only considers entity oriented operation, neglecting the relationship side.

In spatial relationship mapping, SRR plays the main role. For figure 7, which represents a building cluster within a street block, when the street is widened, the boundary b moves and destroys the relationship between street edge and neighbor buildings. How far away the moving b impacts and to which strong degree it has its impacts on different regions depends on the SRR consideration, *adjacency degree*. Based on the iso-distance-relationship model, the displacement problem between street edge and buildings could be resolved through adjacency degree loop analysis and the concept field in physics science could be borrowed. The iso-distance-relationship contour is similar to the iso-dynamic of magnetic field.

4.3 Both spatial entity and spatial relationship against constraints

This is the mixture of the two former cases. The relationship mapping has to take into account constraints from two sources: 1. the destroyed original relationship possibly resulting from entity mapping; 2. the undistinguished relationships in existing relationship representation. Generally spatial entity mapping f executes first. Subsequently the relationship mapping $g(f(e_i) r' f(e_j)) \to f(e_i) r f(e_j)$ on the one hand performs the relationship abstraction, whereas on the other hand the damaged relationship is corrected. Sometimes, relationship abstraction implicitly has satisfied the constraint "remaining relationship unchanged" under low resolution recognition. This process contains two comparisons, one the parallel state between neighbors, another the historical state between after and before mapping.

5. CONCLUSION

Based on algebra mapping theory and according to two categories of information contained in a geographic database, this paper presents a new map generalization model in which spatial relationship generalization gets much emphasis compared with traditional ones. This model provides generalization with a standard framework on how to classify generalization operators, which is an important question in this field. Recently the research of spatial relationship computation and reasoning based on a certain model of spatial relationship representation, such as 9 intersection model is active in the GIS community. As a special relationship operator, spatial relationship abstraction in map generalization has to consider an important concept, spatial relationship resolution. This paper based on Egenhofer's conceptual neighbor of the 9 intersection model, Frank's cardinal direction model and our iso-distance-relationship model respectively discusses the resolution construction for topological, orientation and distance relationship.

The future works involve of:

1> Further formalization the generalization conceptual model and separate spatial entity mapping, spatial relationship mapping deeply according to different constraints to construct a detailed formalized generalization operator classification system.

2> Development of the integrated representation of three sorts of spatial relationship aiming at a relationship resolution change in map generalization.

3> Building methods to detect spatial representation conflicts based on spatial relationship evaluation and apply relationship mapping approach to resolve conflicts in generalization.

6. ACKNOWLEDGMENTS

The authors would like to thank the anonymous referees whose comments materially improved the paper. Special thanks go to Elfriede M. Fendel for editing the paper.

7. REFERENCES

[1] Bjorke,J. T. Framework for Entropy-based Map Evaluation, Cartography and Geographic Information systems, 23(2):78-95,1996.

[2] Brassel, K. E. and Weibel, R. A Review and Framework of Automated Map Generalization, Int. Journal of Geographical Information Systems, 2(3): 229-244, 1988.

[3] Clementini, E., Di Felice, P., and Oosterom, P. van, A Small Set of Formal Topological Relationships for End-User Interaction. in: D. Abel and B. C. Ooi (Ed.), Advances in Spatial Databases - Third International Symposium, SSD'93. Lecture Notes in Computer Science LNCS 692, pp. 277-295, Springer-Verlag, Singapore, 1993.

[4] Clementini, E., Sharma, J. and Egenhofer,M. Modeling Topological Spatial Relationships: Strategies for Query Processing, Computers and Graphics, 18 (6): 8 15-822, 1994.

[5] Egenhofer,M.J. and Franzosa, R. D. Point Set Topological Spatial Relationships, International Journal of of Geographical Information Systems, 5(2):161-174, April-June, 1991.

[6] Egenhofer, M. J. and Mark, D. Modeling Conceptual Neighborhoods of Topological Line-Region Relationships, International Journal of Geographical Information Systems, 9 (5): 555-565, 1995.

[7] Egenhofer, M. J. and Franzosa, R.D. On the Equivalence of Topological Relationships, International Journal of Geographical Information Systems, 9 (2): 133-152, 1995.

[8] Frank, A.U. Qualitative Spatial Reasoning about Distances and Directions in Geographic Space, Journal of Visual Languages and Computing, 3: 343-371, 1992.

[9] Frank, A.U. Qualitative Spatial Reasoning: Cardinal Directions as an Example, International Journal of Geographical Information Systems, 10 (3): 269-290,1996.

[10]Goodchild, M. F., Egenhofer, M., Kemp, M., Mark, D., Sheppard, E. Introduction to the Varenius Project, International Journal of Geographical Information Science, 13(8): 731-745, 1999.

[11]Hernandez, D. and Clementini, E. Qualitative Distance, Proceedings of COSIT'95, Semmering, Austria: 45-57, 1995.

[12] Lonergan, M. E., Jones, C. B. and Ware, J. M. Optimal Map Generalization: Saving Time with Appropriate Measures of Imperfection, CD-Rom Proceedings 19th Int. Cartographic Conference, Section 8, Ottawa,1 999.

[13] Lamy, S., Ruas, A., Demazeau, Y., Jackson, M., Mackaness, W., and Weibel, R. The Application of Agents in Automated Map Generalization, CD-Rom Proceedings 19th lnt. Cartographic Conference, Ottawa, 1999.

[14] Mackness, W. An Algorithm for Conflict Identification and Feature Displacement in Automated Map generalization, Cartography and GIS: 21(4), 1994.

[15] McMaster,R. and Shea,S. Cartographic Generalization in Digital Environment: When and How to Generalize, Proceedings AUTOCARTO 9, 1989.

[16] Muller, J. C., Lagrange, J.-P. and Weibel, R. (eds.), GIS and Generalization: Methodology and Practice. London: Taylor & Francis, 1995.

[17] Oosterom, P. van, The GAP-tree, An Approach to On-the-Fly Map Generalization of An Area Partitioning, in Muller,J-C., Lagrange, J-P., Weibel, R (Ed). GIS and Generalization: Methodology and Practice. Taylor & Francis, London,120-132, 1995.

[18] Peng, W. Automatic Generalization in GIS, Ph D thesis, ITC Publication Series (The Netherlands), 1995.

[19] Ruas,A. A Method for Building Displacement in Automated Map Generalization, International Journal of Geographic Information Science, 12(8):789-803,1998.

[20]Ruas, A., Mackness, W. Strategies for Urban Map Generalization, Proceedings of the 18th International Cartographic Conference, Stockholm, vol.3: 1387-1394,1997.

[21]Ware, J. M. and Jones, C. B. Conflict Reduction in Map Generalization Using Iterative Improvement, Geoinfomatica, 2(4): 383-407,1998.

[22]Weber,W. Map Generalization---An Information Science Approach, In: Opheim, H. ed. Contributions to Map Generalization Proceedings, Oslo, Norway: Norweigian Computing Center, pp. 31-52, 1980.

[23] Weibel, R. and Dutton, G.H. Constraint-Based Automated Map Generalization. Proceedings of the 8 th Intema-tional Symposium on Spatial Data Handling, Vancouver, BC, pp. 214-244, 1998.

[24] Worboys, M.F., Metrics and Topologies for Geographic Space, in Advances in Geographic Information Systems Research II: Proceedings of the International Symposium on Spatial Data Handling, Delft, Kraak, M.J. and Molenaar, M. (eds.), Taylor &Francis, pp. 365-376, 1996.

[25] Wu, H. H. Structured Approach to Implementing Automatic Cartographic Generalization, Proceedings of the 18th ICC, Stockholm, Sweden, Vol.1: 349-356, 1997.