Displacement Methods based on Field Analysis

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Abstract As an important operator in polygon cluster generalization, the displacement has two applications. One is to resolve the proximity conflicts to guarantee the legibility constraint. Another is to act as the operator prior to other generalization operators, such as aggregation. This paper presents a field based method to deal with the displacement of polygon cluster in both aspects above. On the basis of the skeleton of Delaunay triangulation, a displacement field is built in which the propagation force is taken into account. Taking the building cluster as the example, the study offers the computation of displacement direction and offset distance for the building displacement, which is driven by the street widening. The vector operation is performed based on the grade and other field concepts.

Keywords map generalization, displacement, field analysis, Delaunay triangulation, building cluster

1. Introduction

Due to the impact of context, the generalization of an object cluster is much more complex than that of a single object. The constraints aiming at the whole cluster system and at every element object have to be considered simultaneously with respect to position retaining, structure pattern maintenance, statistic principles preservation, etc. Usually the constraints from different viewpoints contradict with each other and no solution could be found to satisfy all constraints. In recent years, the research of object cluster abstraction is active in the community of map generalization and the interest focusses on the displacement of objects within a cluster.

In the GIS domain, the spatial data model includes object oriented and field oriented models. From the latter viewpoint, the space is regarded as the field with associations between different spatial objects. We can image that the associations are the result from some forces, just like the gravity force in the gravity field or the electromagnetic force in the electromagnetic field. The change of force balance will lead to the position readjustment of the objects involved. The map abstraction can be thought of as the "force balance change " during the process of removal, exaggeration, and aggregation of part of the objects. So, the post-processing is required to maintain the balance. Ruas (1998) called this kind of post-operation reactive displacement. From the point of legibility view, displacement to handle too close objects is called active displacement. In both cases, the displacement acts as an independent operation. However, the displacement operation can also be used as sub-operation prior to other operations such as aggregation.

In the area of displacement study the field idea is popular. Ruas (1998) viewed the displacement as a set of localized distortions in a continuous field composed of a set of objects whose internal geometry is fixed. Hojholt (2000) presented a displacement approach using the finite element method in mechanic structure subject. The finite element method contains the idea in which the force acts on the field. Based on the neighbor analysis, Mackness (1994) developed a method to detect conflicts and to displace the objects with the offset decreasing from the conflict center. The key question of the implementation of field theory in the displacement consists of the field modeling and the force action modeling. Based on different understandings, we can build different field models and use different field concepts. This study presents a displacement field model based on the geometric construction similar to the Voronoi diagram. Through the analysis of the adjacency degree in neighbor relationships, the force action is propagated in the field with the magnitude decreasing, which is not settled in the finite element method.

The remainder of this paper is structured as follows. In section 2 the research motivation is given which is to process two kinds of displacement in building cluster generalization. Section 3 presents the method of displacement field construction. The displacement as the proximity resolving and the prior operation of aggregation is discussed in section 4 and section 5 respectively. Finally section 6 concludes with future works.

2. The Motivation for Building Cluster Generalization

Multi-scale representation and map generalization have to take into account spatial object properties in geometrical, semantic and topological aspects. The objects with the same geometric type, but different geographic meaning should be executed with different generalization strategies and algorithms. Recently the study of geo-oriented generalization is active, which aims at specific geographical categories. The research on urban building abstraction and multi-scale representation is an example. As the polygon object with human culture characteristics, the building has different properties in spatial distribution, shape structure and Gestalt nature compared with natural features such as soil parcel, vegetation, lake, etc. Building generalization involves the simplification of independent buildings, the aggregation and the displacement of building clusters. From the point of view of legibility, Regnauld and Edwardes (1999) discuss three operations for building simplification focusing on shape maintenance. Based on the divide-and-conquer idea, Guo and Ai (2000) give an algorithm to simplify a building polygon through separating a building into multiple hierarchical organizations of rectangle elements. For building cluster aggregation, Regnauld (1996) developed a method to detect building pattern groups by applying the minimum spanning tree (MST) model from graph theory.

Building cluster generalization can be classified into three levels of decisions and operations. Grouping is the first level of decision-making, which is based on conflict detection, distribution pattern recognition and Gestalt nature cognition. The second level is the operator decision. And the third is the execution of geometric operations. This study focuses on the second and the third step and concentrates on the displacement operation.

The constraints of building generalization involve the maintenance of position accuracy, avoidance of short space distance, maintenance of the whole building area balance, preservation of Gestalt nature, and retaining of orthogonal shape. Due to the contradiction between different generalization constraints, it is difficult to find a solution satisfying all of them. One proper method is to remove spatial conflicts and to respect the other constraints above as much as possible during the process. The compromise strategy requires sacrificing each constraint partly, not respecting anyone completely.

From the point of view of legibility, when the distance between buildings is shorter than the cognition tolerance, we consider this as a spatial conflict. To resolve a conflict, the following candidate operators could be used: deletion, displacement and aggregation. Deleting some buildings leaves space for the remaining neighbor buildings and the conflicts between original buildings may be resolved. Displacement is valid just within relatively large (empty) space. When scale changes largely, in limited space one displacement may result in new conflicts and it is very hard to find an appropriate position for each building polygon. Direct aggregation makes the conflict between original buildings disappear, but increases the total building area. Furthermore the conflicts between new combined results still exist, unless all conflicting buildings are combined to one big block. To avoid the case that the space between conflicting buildings becomes the building area in the aggregation, we can execute both displacement and aggregation. It means that the displacement acts as the operation prior to aggregation in order to guarantee the balance of building area. Moving two or more buildings together and then aggregating them into one leads to the conflicts between them disappear. On the other hand, movement also gives in the opposite direction more room and the potential conflicts between new just generated building and context neighbors may also be resolved. This strategy guarantees the balance of the whole building area in some degree, but destroys the position accuracy of the moved buildings. The largest offset distance of displacement should be restricted within position accuracy tolerance. Generally, the prior movement cannot guarantee that two neighbor buildings will seamlessly share one common boundary. So there may still be some gap area between the buildings. The aggregation result still has the tendency to increase the area a little bit. Considering this fact, the next step, independent simplification can be controlled to prefer to reduce the building area.

The displacement has two applications in building cluster generalization. One is to resolve the proximity conflicts to guarantee the legibility constraint. Another is to act as the operator prior to other generalization operators, such as aggregation. The previous can be either an active or a reactive displacement in Ruas' (1998) classification. For the first type of displacement operation, the study object is the whole building cluster with associations to each other.

For the latter type of displacement operator, the study object is a set of the building objects within a local group, which will be combined. Both cases can be supported by field analysis. This paper will describe the construction of such a field model to process the two kinds of displacement respectively.

3. Constructing Displacement Field

We suppose that the force action exists in the whole building cluster, the displacement field. The force can be either repulsive or attractive. In the displacement field to resolve proximity conflicts, we suppose the repulsive fore drives the adjacent buildings to move along the propagation direction. In the displacement field to support post aggregation, we suppose that the attractive force drives close buildings together. The force association depends on the distance between one building and its neighbors. To model this kind of displacement field, we build a geometric construction similar to the Voronoi diagram on the basis of the Delaunay triangulation skeleton.

The Delaunay triangulation, which has (1) the empty circumcircle principle property and (2) the closest to equilateral property (Preparata and Shamos, 1985), plays an important role in spatial adjacency relationship analysis and results in series of achievements related to spatial neighbor assessment (Jones et al., 1995, Ware et al., 1997, Ai et al., 2000, Ai and Van Oosterom, 2002). A building cluster distribution contains much information associated with adjacency relationship within the context of the environment.

3.1 Constructing the Partitioning Model Similar to the Voronoi Diagram

For the building cluster within one street block, we construct the constrained Delaunay triangulation and just consider the triangles connecting different building objects. Those located within a building polygon or located in the

> concave area of a building polygon are removed. The reason for the latter removal is to avoid appearance of a dangling skeleton branch in the subsequent creation of the

> geometric partitioning. The remaining triangle sets are assigned into three types according to the number of neighbors. Those having one neighbor, two neighbors and

> illustrates the selected triangles between buildings, shaded with light green and marked

with Roman numerals. Type I triangle appears on the exit of building cluster, type II can

be found distributing between two buildings and type III triangle appears on the region



Figure 1. Selecting specific triangles and assigning type.

of three buildings meeting together.

The creation of the skeleton segments for three types of triangles is described in figure 2, where P₁, P₂, P₃ are the midpoints of corresponding triangle edges, and O is the triangle weight center. Link skeleton segments by means of next paths:

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Type I:
           A \rightarrow P_1 \text{ or } P_1 \rightarrow A
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Type II: P_1 \rightarrow P_2 \text{ or } P_2 \rightarrow P_1
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Type III: $O \rightarrow P_i$ or $P_i \rightarrow O$, i=1,2,3

Through the polygon topological organization based on linking the skeleton segments, we obtain the special geometric construction as illustrated in Figure 4.

This partitioning model has the following properties:

- i> *Each partitioning polygon contains one building;*
- *Each node relates to three skeleton edges;* ii>
- *iii>* Each edge has a left and a right building, separating the space between the buildings equally;
- iv> If the number of type I, type II, type III triangles isn_1, n_2, n_3 respectively, then the number of edges is



Type I Type II Type III Figure 2. Skeleton connection ways for three



Figure 3. Based on the triangles between building polygons, the skeleton connection gets a special geometric construction similar to the Voronoi diagram.

$(n_1+3n_3)/2.$

Property i, ii, iii are valid except for the border area of the polygonal cluster. Adding an outside closed boundary can guarantee that the border objects are also within one partitioning polygon. Triangles locating in the concave part have been removed in the previous selection process. This implies that some of the outside concave area is also regarded as belonging to object polygon, for such as object polygon *B* in figure 3. It is similar for the method of raster operation to get the polygonal cluster Voronoi diagram. This is the reason why we do not directly use skeleton based on all triangles outside the object polygon to get this kind of geometric construction.

This geometric construction looks like the Voronoi diagram (VD). But according to the strict definition of the VD (Preparata and Shamos 1985), it is not a Voronoi diagram. Originally the Voronoi diagram is point cluster oriented and has geometric properties that (1) partitioning cell polygons are convex and (2) the connection of neighbor center points results in its dual, the Delaunay triangulation. For line and polygonal cluster, it is more difficult to give a strict definition of the Voronoi diagram in computational geometry and it usually depends on the construction method. One of the known methods is based on raster data expansion (Li and Chen 1999) to construct the Voronoi diagram of line cluster and polygonal cluster. For the purpose of application rather than the strict theory of computational geometry, we can think it as a Voronoi diagram if it equally partitions the space between cluster elements.

The partitioning polygons can be thought of as the growth region of corresponding object polygons, covering the whole area with neither gaps nor overlapped regions. We can think of this as the result of each object competing outwards for growth range and this competition has to consider context impacts. The neighborhood relationship between original object polygons is now mapped as the topological touch relationship between partitioning polygons in this partitioning model. Based on the relation of partitioning polygons, see figure 3, we can find every object polygon's neighbor candidates. Some are relatively far away from each other but the triangle connection makes them possible neighbors, and if the distance between them is less than the tolerance then they are real neighbors.

3.2 Constructing Displacement Field

In map generalization, the displacement of geographic objects has to take into account the propagation influence in the context. One object receives the driven force moving itself and also pushes the force to its neighbors with some magnitude reduction. This process is similar to the phenomena of magnetic or electronic field in physics. To model the displacement field, we need to decide the force source and the propagation behavior. For every building object, we need to find the fore propagation direction and its associated neighbors receiving the propagation force.

In the field analysis, we can define isolines to represent the model. The objects locating in the same loop between two adjacent isolines receive the identical force action in magnitude. Along the normal direction, when we move across one isoline, the forces will reduce/increase in grade. We use this idea to build an iso-distance-relationship model to represent the displacement field

In the partitioning model above we call the building polygon OP, which is surrounded by one partitioning polygon, called PP. With respect to a specific reference OP, the other OPs have a certain distance relationship to it depending on the context rather than just the metric distance. From the reference OP to the current OP, the path needs to go across a number of PPs. We define the minimum number of across PPs as the concept *adjacency degree*. If two PPs share a common boundary, the corresponding OPs are called immediate neighbor and have the adjacency degree 1. The topological relationship between PPs can be mapped to represent the distance relationship between OPs. This representation is based on the assumption that the space is isotropic.

We define the reference OPs as the force source in the displacement field. The force source could be one OP or a set of OPs. For the case of street widening, the buildings within one street block will be displaced in different ways and the force source can be thought to come from the street boundary. The immediate adjacent OPs with the street boundary can be defined as the reference OPs. For the situation that one building is too close to its neighbors, the center building can be assigned to the force source to push away its neighbors.

Based on the partitioning model above, we present the following algorithm to compute the adjacency degree of every OP in the building cluster. Assign the reference Ops (e.g. with respect to the street boundary, say *b*) to set *A*:

- 1> Let OPs in set A have adjacency degree 0, and initiate other OPs adjacency degree to -1;
- 2> Initiate A belonging to active object set, Initiate variable degree_count 0;
- 3> Repeat next steps until active object set NULL;
 - 3.1> Find all adjacent objects of active object set based on PP boundary extending search;
 - 3.2> Ignore those adjacent objects with adjacency degree greater than -1;
 - 3.3> degree_count adds 1 and assign the value to each valid adjacent object;
 - 3.4> Empty active object set and let valid adjacent objects belong to active object set;

Next we remove the PP boundary arcs with faces on the two side OPs having the same *adjacency degree*, represented as a yellow line in figure 4. Connecting the remaining PP boundary arcs to form the closed contour line, which separates OPs with *adjacency degree n* from those with *adjacency degree n+1*. The objects within the loop between two neighbor contour lines have the same *adjacency degree* with respect to boundary *b*. So, we call this kind of contour line the iso-distance-relationship contour, just like the altitude contour of terrain representation. The lower the

adjacency degree is between two OPs, the closer the relationship is to each other. Obviously this contour is different from the iso-distance contour, which is represented as progressive circle buffers with the same center and increasing radius. The iso-distance-relationship model considers the context environment and spatial distribution. An OP far away in metric distance possibly has a very low *adjacency degree* and close neighbor relationship with the reference boundary *b*. In this case the boundary acts as the force source to drive the displacement, and the force propagation passes across the contours. Figure 4 describes another example, which is referenced with a determinate OP, building A in the cluster center. Figure 5 illustrates the whole process of the displacement filed construction with respect to the street boundary.



Figure 4. The iso-distance-relationship contour with respect to the center object *A*.

4. Displacement as the Proximity Conflict Resolving

The street widening results in the spatial conflicts between the street boundary and the involved buildings. Under the operation of the street block boundary compressing, the buildings receive different forces to move their position. Based on the displacement field of iso-distance-relationship representation, we describe the method to compute the displacement direction and offset distance.

The force is propagated from the outer street boundary to the inner buildings. The force action on one OP is decided by the boundary properties of the corresponding PP. Except the center buildings, each OP faces some OPs with a low adjacency degree and on the other side faces OPs with a high adjacency degree. We call the first the active boundary and the latter the reactive boundary. For the same PP boundary, it may be reactive with respect to one OP, but active to its neighbor. The force is propagated from the active boundaries and the action result is to push the neighbor OPs through the reactive boundaries. In this process the yellow boundaries in the figure do not participate in the force propagation. Local conflicts between OPs within the same loop may be generated in this situation.

Through vector add operation, we compute the movement direction of each building driven by the propagation force from the active boundaries. Usually the boundary edge is a curve. We construct an approximated straight line for every edge using the least squares method and let the normal direction be the vector direction. In figure 5 c, the green arrow symbols represent the added vector direction of all vectors resulted from active boundaries.



Figure 5. The field construction and the displacement result (with 'force' from the boundary).

- (a) Original building cluster and street boundary.
- (b) Based on Delaunay triangulation and skeletons, construct the partitioning model similar to the Voronoi diagram.
- (c) The construction of the displacement field in which the force propagation direction and magnitude are computed and visualized in the graphic.
- (d) The displacement result driven by boundary compress. The nearer to the boundary, the longer offset the building has moved. The core buildings have no movement.

For the computation of offset distance, we define a decay function related to the adjacency degree x, say f(x) = c - kx. It implies the higher adjacency degree leads to the shorter offset distance. This function is the simplest one, the lineal type. The function form can also be another one according to the decay speed along the change of adjacency degree. This function corresponds to the grade concept in the field. The OPs within the same contour loop have the same grade and move the same offset. The core objects, which are the farthest away from the street boundary, could be controlled without offset keeping the original position. Figure 5 d is the displacement result based on the computation of displacement direction and offset in figure 5 c. Figure 6 gives a real application example based on the above displacement method.

One question is that too densely distributed buildings may overlap after displacement. The improvement is to consider the local force produced from the too close objects to each other. It means the displacement force is driven not only by the street boundary compression but also by very close buildings. When too close, the buildings generate the local repulsive force and the new vector is added to the vector operation. But when objects distribute very densely, the final combined vector approximates to zero, and the overlap does not yet avoid. In this case, only displacement generalization could not resolve the question and also aggregation or deletion is required. The next section will discuss the integrated operation of displacement and aggregation.



Figure 6 Real application in the displacement of street buildings during the street widening.

5. Displacement as the Operator prior to Aggregation

According to the discussion in section 2, the integration of displacement and aggregation is able to solve conflicts and simulantiously keep the area balance of all buildings. We still use the cluster partitioning model, but concentrate on a local region with conflicts caused by too close objects. In this case, we just consider the immediate neighbor buildings. The displacement force behaves as an attractive one.

5.1 Where are there Conflicts?

The distance between two neighbor buildings is often used to detect the conflicts. But how to compute the distance between objects when considering their geometric shape? What it means for A to be near B depends not only on their absolute positions (and the metric distance between them), but also on their relative sizes and shapes, the position of other objects, and the frame of reference (Hernandez and Clementini, 1995). We offer the following method to compute the weighted distance between two buildings. A PP boundary goes across a set of triangles, which divide the skeleton into segments. For each short segment, compute this local distance between OPs according to the triangle type and then sum the local distance weighted with the ratio of local segment length to the whole skeleton length. For three types of triangle, the local skeleton width representation, W_1W_2 is expressed in Figure 7. The computation function is:

$$\overline{W} = \sum_{i=0}^{k} \frac{||P_i P_{i+1}||}{||W_i W_k||}$$

where *l* is the whole skeleton length, *k* is the number of involved triangles. \hat{w} is also called skeleton width. This weighted distance computation based on skeleton takes into account the building



Type IType IIType IIIFigure 7. W_1W_2 skeleton width representation for
the three types of triangles.

shape structure, spatial distribution and the influence of other buildings. In the recognition of the building group, as the judgment parameter, the weighted distance is better than minimum distance.

The weighted skeleton width is used as the condition in the conflict detection. In the partitioning model, those skeletons with weighted width shorter than the predefined cognition tolerance are identified as conflict skeletons, and those building objects related to one or more conflict skeletons are defined as conflicting building objects. Figure 8 gives an example of detecting conflict skeletons, which are represented as wide red lines. According to PP connectivity, the conflict objects can be assigned into groups.

5.2 How to Displace?

The detection of conflicting buildings answered the question of what will be displaced. The next question is how far and in what direction the conflicting buildings have to be moved.

If the conflicting building has only one conflict skeleton, then the normal direction of the approximated straight line of the conflict skeleton serves as the displacement direction. Otherwise, using vector add operation computes the integrated moving direction. We suppose each conflicting building is attracted by its neighbor conflicting building and the attraction force is equal. When one building is attracted by neighbors from two opposite directions, or when it is surrounded by conflicting buildings, it will keep unchanged.



Figure 8. Experiment illustrations of conflict skeletons, visualized as wide line, and building displacement direction, visualized as arrow line and dark dot.

In an actual application, when the added vector length is shorter than a threshold, we can think no direction attraction is strong enough over other directions and also regard the object as fixed. In figure 8 the dark arrow symbol represents the displacement direction for the conflict objects and the dark dot represents the building fixed.

For the length of the displacement, firstly we assume that the position accuracy is not less than half of the conflict distance. It means the conflicting buildings moving face to face and meeting together in some position are not against the position accuracy. Parallel with the displacement direction, draw an extended line from each vertex of conflict OP and compute the distance between the start vertex and the intersection point of the extended line and PP boundary. Select the shortest distance as displacement offset length. This process guarantees each building moving within its own PP range, not overlapping with another building's PP. It means that the displacement will not result in a new conflict.

The purpose of displacement in building cluster generalization is to statistically maintain area balance. But usually after displacement, it has not yet been achieved that two buildings exactly share a common seamless boundary, and probably there still exists a gap area. An improvement is to execute rotation, but rotation angle and rotation scope are complex to decide and yet cannot resolve the problem completely. An alternative might be to do a second iteration of computation: recompute the PPs and displacement vectors.

5.3 Progressive Generalization Workflow

How to integrate displacement and aggregation in a complete generalization process depends on overall workflow control. Considering the fact that conflicts within a building cluster are related to each other, we cannot simply aggregate all the conflicting buildings, which are 'connected' to each other. Aggregation of a number of conflicting objects (group) and displacement may resolve the conflict between different groups. Especially when scale changes considerably, the relatively large conflict distance may lead to the situation that all buildings locating within one street block are in conflict. Obviously it is not the best solution to combine all buildings into a big one. The whole control workflow of building cluster generalization should be a progressive procedure to remove conflicts step by step.

If the distribution of skeleton widths (weighted distances) covers a broad range and the width values are able to be obviously distinguished, we introduce the MST method idea (Regnauld 1997) to control the generalization procedure. The workflow is described briefly as follows.

Set initial minimum distance tolerance value d_{tol} and repeat the following steps until step i> finds no conflict skeletons (conflicting buildings):

- i> Construct triangulation and based on the partitioning model find the conflict skeletons and the conflicting buildings.
- ii> Sort the conflict skeletons on weighted width from short to long.
- iii> Scan the sorted conflict skeletons to check the related left and right conflict OPs. Remove other conflict skeletons related to these two OPs from the list.

- iv> Aggregate all pairs of buildings between the current conflict skeleton with a distance tolerance less than d_{tol} .
- v> Increase d_{tol} for the next iteration.

The above workflow guarantees each conflict removal to happen exactly between two buildings. Figure 9 illustrates the progressive procedure of building cluster generalization. If the building distribution is more or less random and the conflicts are few, the workflow above can get proper generalized results. But questions exist regarding the following two aspects: 1. The early aggregated building will displace many times in the following processes and the position accuracy may be damaged. 2. The distribution pattern cannot be maintained completely. Further research is needed in this area.



Figure 9. The progressive aggregation of a densely distributed building cluster based on the partitioning model.

6. Conclusion

Based on the Delaunay triangulation skeleton, this study presents the partitioning model, which is similar to the Voronoi diagram. The property of equally separating space makes it a powerful tool to analyze the distribution within polygon cluster. A displacement field is constructed in which the displacement propagation (including decay) can be realized. Aiming at the resolving of proximity conflicts and also the displacement operator prior to aggregation, this study presented two displacement methods respectively. But for overall generalization, the two methods have to be combined. The selection of generalization operators and inter-execution of them belong to the complex decision-making at the macro level.

Independent building simplification gets some achievements. Building cluster generalization belongs to research still facing many problems. The representation and automatic recognition of spatial distribution pattern are the first questions to be resolved in future research. When the distance between buildings is generally the same, the judgment of building cluster mainly depends on non-distance fact. The Gestalt nature in building size, orientation, shape, and distribution structure will be an important consideration fact. How to involve the Gestalt principles in the pattern recognition of building cluster is our next research in the future.

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