# Updating Features in a TEN-based DBMS approach for 3D Topographic Data Modelling 

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## INTRODUCTION

In earlier research a TEN (Tetrahedronised Irregular Network)-based DBMS approach for modelling 3D Topography was developed. This topological 3D data model relies on Poincaré algebra. Topographic features are modelled within the TEN as set of constraints. This paper focuses on the way in which these constraints can be inserted (incrementally). It is shown that 16 theoretical cases can be identified, in which 9 unique cases will be distinguished. A more comprehensive discussion can be found in the accompanying Technical Report (Penninga and Van Oosterom 2006).

## BACKGROUND: MODELLING APPROACH

The volumetric approach as introduced in (Penninga 2005, Penninga et al. 2006) has the basic assumption that in the case of topography we consider the real world as a volume partition (analogously to a planar partition): a set of non-overlapping volumes that form a closed modelled space. The most important consequence is that objects like earth and air are explicitly part of the real world and thus have to be modelled. The UML class diagram of our topographic data model is given in Figure 1. Note that the Tetrahedron, Triangle and Edge class are all directly specified by an ordered list of Nodes. This is a direct result of the implementation of the boundary operator of Poincaré's simplicial homology (Poincaré 1895, Giblin 1977 and Hatcher 2002) and in contrast with the earlier models based on the direct relationships between tetrahedron and triangle (and triangle and edge, and edge and node), which are now all derived. Another important aspect is that our approach deals with multiple features in a single TEN, modelled by constraints. Using constraints of multiple features in a TEN structure rarely happens in the adjacent field of computational geometry (more specific: finite element meshing research (e.g. Shewchuk 1997,2004 ) as they often focus on a single 3D object.

## UPDATING FEATURES BY THE INSERTION OF CONSTRAINTS

The insertion process of the volume features needs the following aspects (example in Figure 2):

- Its outer boundary needs to be triangulated and all resulting edges (and faces) should be treated as constraints


Fig. 1: UML class diagram of the TEN structure (discussed in Penninga and Van Oosterom 2006)



Fig.2: Triangulation of boundary and internal tetrahedronisation

- The interior needs to be tetrahedronised. This tetrahedronisation can be performed either directly in the TEN or separately, after which all resulting edges can be inserted into the TEN. Input in both cases is the set of constraint edges of the outer boundary.
- Regardless which of the two previous options is used, local tetrahedronisation might be necessary in order to optimize the structure by creating better-shaped tetrahedrons.
- Updating the relevant feature table(s).

Within this procedure the basic operator can be identified: the insertion of constraint edges (representing the feature's boundary) into the TEN. As this edge consists of two nodes, inserting an edge is equivalent to the insertion of two nodes and connecting them. In inserting a new node in an existing TEN structure four different cases can be distinguished, as a the node can lie within a tetrahedron, on a triangle, an edge or a node. In case of inserting the start- and end node of the edge $4 * 4=16$ cases can be identified of which are 6 symmetric (see Table 1).

Tab. 1: Options of inserting a constrained edge into a tetrahedron

| Node lies on... | Node | Edge | Triangle | Tetrahedron |
| :--- | :---: | :---: | :---: | :---: |
| Node | 0 | 1 | 2 | 3 |
| Edge | $(1)$ | 4 | 5 | 6 |
| Triangle | $(2)$ | $(5)$ | 7 | 8 |
| Tetrahedron | $(3)$ | $(6)$ | $(8)$ | 9 |

If the constraint edge crosses several tetrahedrons, it is split into a combination of these cases. For instance an edge crossing three tetrahedrons results in cases 8-7-8 (Figure 3: node inside tetrahedron to additional node on boundary triangle, from this node to another additional
node on boundary triangle (thus crossing the third tetrahedron) and from this additional node to the endnode inside the third tetrahedron. As a result, case 0 is not a real case, as in this case the edge is already present (one of the edges of the tetrahedron).


Fig. 3: Insertion of a edge trough 3 tetrahedrons (top: before, bottom: after)

## BASIC OPERATORS

In (Penninga and Van Oosterom 2006) the nine unique cases are all considered. In this abstract we confine ourselves to one example (case6): One node lies on an edge, the other node lies in the middle of one of the $n$ involved tetrahedrons (n tetrahedrons involved): First the node on the edge is inserted. Added are +1 node, $+(n+1)$ edges, $+2 n$ triangles, $+n$ tetrahedrons. In Figure 4 the case $n=4$ is illustrated. As one can see, each original tetrahedron in split into two new tetrahedrons. Since one knows in which original tetrahedron the second node would be inserted, two subclasses can be distinguished, related to these two new tetrahedrons:

- The second node lies in the middle of one of the two tetrahedrons, changing the totals to: +2 nodes, $+(n+5)$ edges, $+(2 n+6)$ triangles, $+(n+3)$ tetrahedrons.
- The second node lies in the triangle between the two tetrahedrons, changing the totals to: +2 nodes, $+(n+6)$ edges, $+(2 n+7)$ triangles, $+(\mathrm{n}+4)$ tetrahedrons. Similar to case 2(b)
In both cases the constraint edge is one of the newly formed edges. Further, it is impossible for the second node to lie on an edge (after split) as the second node lies in the interior of an original tetrahedron and not on its boundary.


Fig. 4: Insertion of a node in case $n=4$

## CONCLUDING REMARKS

In this paper a further step is taken in the development of a 3D topographic TEN-based data structure: the inclusion of constraints due to the features. The TEN structure is very well accessible through the use of the boundary operator of Poincaré's simplicial homology.

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