

The 3D Voronoi Diagram: A Tool for the Modelling of Geoscientific Datasets

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ABSTRACT

A lot of efforts and money are being spent, both by commercial companies and by the academic world, for the development of tools to model 3D geographical information. In most cases, the "3D GISs" built are systems where 3D objects can be visualised (in a 3D environment), and where in some cases primitive forms of spatial analysis are available (e.g. overlay between objects, buffer zones and viewsheds). One such example is a virtual city model where realistic 3D buildings are added on top of a topographic map; the emphasis here is on the representation of different objects such as buildings, pipelines and roads. While the tools developed are useful in many application domains, they are not in others, for example in geoscience (oceanography, meteorology, geophysics, etc.) where the objects of interest are usually not man-made, but rather the spatial distribution of three-dimensional continuous geographical phenomena such as the salinity of a body of water, the humidity of the air or the percentage of gold in the rock. Such datasets have traditionally been modelled with raster structures (regular and irregular grids), but, as we argue in this paper, doing so has many shortcomings.

In this paper, we present an alternative solution for the modelling of geoscientific datasets: the Voronoi diagram (VD) and its dual the Delaunay tetrahedralization (DT). We first discuss the VD/DT model and its properties, and we show that it has many advantages over already existing models/structures for analysing and extracting meaningful information from datasets. We also describe a series of concrete real-world applications where the VD/DT model can be used to solve problems.

1. INTRODUCTION

Much of the initiatives, in the industry and in academia, spent towards building a 3D geographical information system (GIS) focus on the representation of discrete objects such as buildings, pipelines and bridges. An example is a virtual city model, in which realistic 3D buildings, roads and other features of a city are added on top of a topographic map (see Figure 1 for an example).

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Figure 1: Example of a CityGML model. (Figure taken from www.citygml.org.)

Users can explore the virtual city by “navigating” in it (zoom, rotate and pan functions are present), and can usually query the database linked to the model to know more about the objects. There is also research being done for adding topological relationships to the objects (see for instance Egenhofer (1995), Ellul and Haklay (2006) and Zlatanova et al. (2002)) so that more elaborated forms of spatial analysis are possible (e.g. overlay between objects, buffer zones or viewsheds).

While the tools just described are useful for many 3D applications, they cannot be used in some other applications. One such example is geoscience (oceanography, meteorology, geophysics, etc.), where instead of having an empty space populated by discrete objects, we have phenomena that are continuous in 3D space (think of the temperature of the air, or the percentage of a certain chemical in a body of water). In technical terms, a continuous phenomenon is referred to as a *field*, and a field-view of the world is one where there are no ‘holes’ (every location in space has a certain value). The techniques developed for city models are therefore of no use for the representation of geoscientific datasets. It should be said that, as explained briefly in Section 2, there exist different solutions for dealing with such kind of data (mainly raster structures), but that the special properties of the datasets in geoscience hinder their use in practice.

As an alternative to using rasters for representing and modelling 3D geoscientific fields, we propose a new spatial model based on both the Voronoi diagram (VD) and its dual the Delaunay tetrahedralization (DT); the two structures, and their relationships, are defined in Section 3. In the following, we first introduce the concept of fields and their representation in computers, then we describe the VD/DT model and its properties, and we finally present a series of potential applications where this model can be useful. Note that this paper is a brief overview of the work carried out by the first author for his PhD thesis; readers interested in more details are referred to Ledoux (2006) and Ledoux and Gold (2007).

2. GEOSCIENTIFIC FIELDS

A field is a concept rather difficult to define because it is not tangible and not part of our intuitive knowledge. It is easy for us to see and describe entities such as houses or chairs, but, although we can imagine fields, they are somewhat an abstract concept. The definition usually used in a GIS context is borrowed and adapted from physics. Physicists in the 19th century developed the concept of a force field to model the magnetic or the gravitational force, where a force (a vector with an orientation and a length) has a value everywhere in space, and changes from location to location. In the context of GIS, the vector is replaced by a scalar value, and a field represents for instance the spatial distribution of the temperature of the air in a room.

The modelling of fields as found in geoscience is problematic because they have properties that are usually not present in fields in other disciplines. The special properties come from the fact that it is usually impossible to measure continuous phenomena everywhere, and that we have to resort to collecting samples at some finite locations and reconstructing fields from these samples. Each sample indicates the value of the attribute studied at a certain location x - y - z in space and at a certain time t . In engineering, medicine or computer-aided design for instance, the studied object is usually easily sampled because we have direct access to it; on the other hand, in geoscience, samples can be very hard and expensive to collect because of the difficulties encountered and the technologies involved. To collect samples in the ground we must dig holes or use other devices (e.g. ultrasound penetrating the ground); underwater samples are collected by instruments moved vertically under a boat, or by automated vehicles; and samples of the atmosphere must be collected by devices attached to balloons or aircrafts. Because of the way they are collected, three-dimensional geoscientific datasets often have a highly anisotropic distribution: as shown in Figure 2, the distribution can be abundant vertically but sparse horizontally. Another peculiar property of geoscientific datasets is that the samples they contain represent the spatial variability of a given attribute only at time t , and many geographical phenomena in geoscience change and evolve relatively quickly over time.

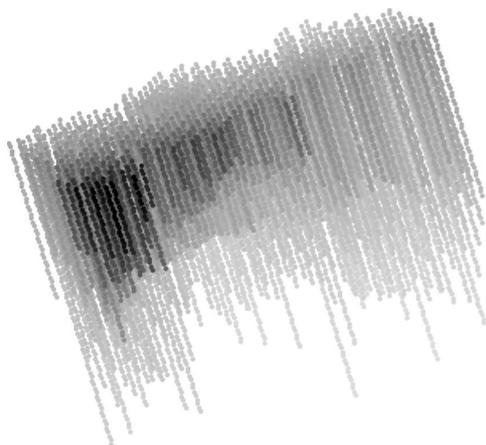


Figure 2: Perspective view of an oceanographic dataset, in which samples are distributed along water columns. Each sample has a location in 3D space (x - y - z coordinates) and one or more attributes attached to it.

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Since fields are continuous, they must be discretised—broken into finite parts—to be represented in a computer, and to be analysed and visualised. The space covered by a field can be partitioned, or tessellated, regularly or irregularly. In a regular tessellation, regions all have the same shape and size. The most common regular tessellation in GIS is by far the grid, where the elements are squares in 2D (usually called pixels as an analogy to digital images) and cubes in 3D (called *voxels*, a portmanteau of the words ‘volume’ and ‘pixel’). By contrast, the elements of an irregular tessellation can be of any shape and size; in three dimensions, each element is a polyhedron. Irregular tessellations usually follow the data points (the samples that were collected to study the field for instance), albeit this is not a requirement. Subdividing the space based on the samples has the main advantage of producing a tessellation that is adaptive to the sample distribution and to the complexity of the phenomenon studied.

Within the GIS community, regular tessellations have been for years more or less synonymous with continuous fields (Goodchild, 1992), and that in two and three dimensions. A few implementations of a voxel-based GIS exist, notably the open-source system GRASS in which diverse operations such as interpolation and visualisation are possible. Most of the earlier attempts at building 3D GISs were also using voxels, as explained in Raper (1989).

This wide popularity of regular tessellations is probably due to the fact that they permit us to integrate easily datasets from different disciplines, and also due to simplicity. Indeed, a grid is naturally stored in a computer as an array, and thus the spatial relationships between cells are implicit. The algorithms to analyse and manipulate (boolean operations such as intersection or containment) are also trivially implemented in a computer. On the other hand, raster representations have been severely criticised, both from a theoretical and a technical point of view. The issues involved when using pixels as the main element for geographical analysis are summarised in Fisher (1997). One problem is that a grid arbitrarily tessellates the space, without taking into consideration the objects embedded in it or the phenomena modelled. Kemp (1993) states that “[a raster representation] requires us to enforce a structure on reality rather than allowing reality to suggest a more appropriate structure for our analysis”. On the technical side, the problems most often cited are: (i) the meaning of a grid is unclear (are the values at the centre of each grid cell, or at the intersections of grid lines?), (ii) the size of a grid (if a fine resolution is wished, then the size of a grid can become huge, especially in three dimensions), (iii) grids scale badly and are not rotationally invariant, (iv) an entity is not represented explicitly, but by a group of grid cells with similar values, which means that its boundaries will be jagged.

3. THE VORONOI DIAGRAM AND THE DELAUNAY TRIANGULATION

The Voronoi diagram for a set of points is the partitioning of the space into regions such that all locations within any one region are closer to the generating point than to any other. In 2D,

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each cell around a data point is a convex polygon, having a defined number of neighbours; for example in Figure 3(a) the point p has 7 neighbouring Voronoi cells. In three dimensions, a Voronoi cell generalises to a convex polyhedron formed by convex faces, as shown in Figure 3(b).

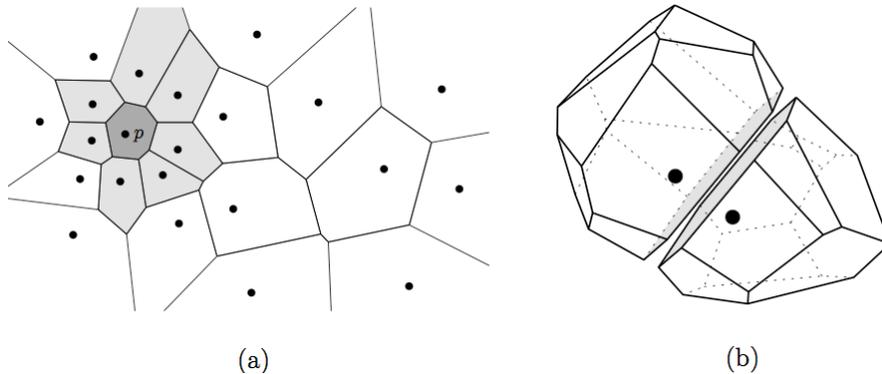


Figure 3: (a) The VD for a set of points in the plane. (b) Two Voronoi cells adjacent to each other in 3D (they share the grey face).

The VD has a geometric dual structure called the Delaunay triangulation, and that in any dimensions. That means that both structures represent the same thing, just from a different viewpoint. In 2D, this structure is defined by the partitioning of the plane into triangles—where the vertices of the triangles are the points generating each Voronoi cell—that satisfy the empty circumcircle test. That is a triangle is Delaunay if the unique circle on which lie its three vertices does not contain any other point in the set S ; Figure 4 illustrates a Delaunay triangulation. The generalisation to 3D of the Delaunay triangulation is the Delaunay tetrahedralization: each triangle becomes a tetrahedron that satisfies the empty circumsphere rule. The duality between the two structures implies that if one wants to work with the VD, he can create the DT and extract at any time the VD (the converse is also true).

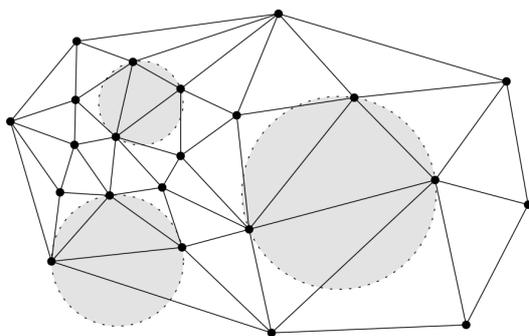


Figure 4: The DT of a set of points in the plane (same point set as (a)). Every triangle has an empty circumcircle, and for some (grey triangles) they are shown.

An important characteristic of the VD/DT is that local updates are possible, i.e. insertions, deletions or movements of points in the model can be made without recomputing the structure from scratch; see Ledoux (2006) for details. This is fundamental for the interactive exploration of a dataset, to model dynamic fields, and to maintain large datasets.

3.1 Advantages of Modelling 3D Geoscientific Datasets with the VD/DT

We argue in this paper that the tessellations obtained with VD and the DT have many advantages over other tessellations for representing fields. The main advantages are the following:

1. the VD offers a natural discretisation of space, which is based on the samples that were collected to study the field. The phenomena studied can thus be represented freely, and not enforced by a rigid structure like voxels. The shape and the size of the cells in the VD are adaptive to the spatial distribution of the samples, which is crucial when dealing with highly anisotropic distributions such as the ones found in the geosciences. Observe in Figure 3(a) that where the data distribution is dense the cells are smaller.
2. it gives a clear and consistent definition of the spatial relationships between unconnected points. The one-to-one mapping between the points and the Voronoi cells ensure that the original samples—the “meta-field” according to Kemp and Vckovski (1998)—are kept and not ‘lost’, as is the case when gridding.
3. the reconstruction of a field can be entirely based on the VD, as briefly explained in the next section. Such a reconstruction is efficient and entirely automatic, and that for discrete and continuous fields.
4. the knowledge of the VD implies the knowledge of its dual the DT, which helps greatly for manipulation operations, and also for some spatial analysis operations.
5. many GIS and spatial analysis operations in three dimensions, such as the extraction of isosurfaces, are possible and even optimised when the VD/DT of a set of points is constructed, as demonstrated in the next section.

4. APPLICATIONS IN GEOSCIENCE

This section describes a few spatial analysis operations based on the VD/DT, and also a few applications where the properties of the VD/DT are useful.

4.1 Spatial Interpolation

Given a set of samples to which an attribute is attached, spatial interpolation is the procedure used to estimate the value of the attribute at an unsampled location (thus filling the ‘holes’ in space). To achieve that, it creates a function that tries to fit the samples as well as possible. Interpolation methods are an essential operation in this paper because they permit us to reconstruct a field from the set of samples that were collected to study it. They are moreover required to model, visualise and better understand a dataset, and also to convert a dataset to another format (e.g. from scattered data to voxels).

An interesting interpolation method in the context of this paper is the natural neighbour interpolation method (Sibson, 1981), because it has been shown by different researchers to have many advantages over other methods when the distribution of samples is highly

anisotropic: it behaves consistently whatever the distribution of the samples is, and it is an automatic method that does not require user-defined parameters (Gold, 1989; Sambridge, Braun, and McQueen, 1995; Watson, 1992). This is a method entirely based on the VD for both selecting the samples involved in the interpolation process, and to assign them a weight (an importance). The neighbours used in an estimation are selected using the adjacency relationships of the VD, which results in the selection of neighbours that both surround and are close to x . That helps greatly when the distribution of samples is highly anisotropic. The weight of each neighbour is based on the volume (area in 2D) that the Voronoi cell of x ‘steals’ from the Voronoi cell of the neighbours in the absence of x ; Figure 5 shows a 2D example, and in 3D the idea is the same.

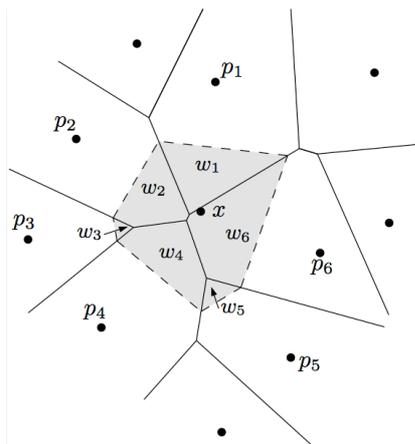


Figure 5: Natural neighbour coordinates of x in the plane. The insertion of x creates the new grey Voronoi cell.

The resulting function is exact (the samples are honoured), and also smooth and continuous everywhere except at the samples themselves. See Gold (1989) and Watson (1992) for further discussion about the properties of the method.

4.2 Visualisation

It is notoriously difficult to visualise 3D fields, even if a 3D computer environment offering translation, rotation and zoom is available. The major problem is that unlike 2D fields, where the attribute a can be treated as another dimension, we can not ‘lift’ every location by its value to create a surface in one more dimension and visualise/analyse it—we can not see in four dimensions! We therefore have to resort to other techniques to be able to visualise the spatial variation of trivariate fields and to extract meaningful information from them. We present here two such techniques.

Extraction of Isosurfaces

An isosurface is the 3D analogous concept of an isoline line in 2D: it represents the space where the attribute of a dataset has the same value. In 2D, isolines are usually extracted directly from a TIN or a regular grid. The idea is to compute the intersection between the

level value (e.g. 200m) and the terrain, represented for instance with a TIN. Each triangle is scanned and segment lines are extracted to form an approximation of an isoline. The same idea can be used in three dimensions to extract surfaces from a field, see the three examples in Figure 6.

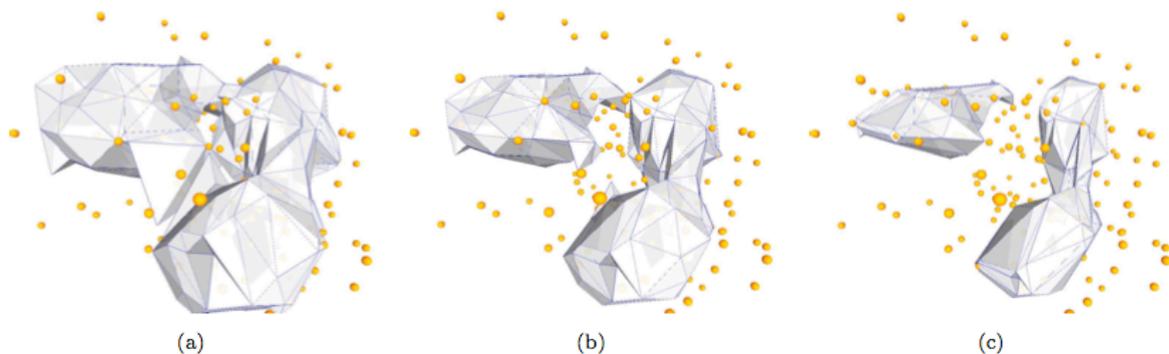


Figure 6: An example of an oceanographic dataset where each point has the temperature of the water, and three isosurface extracted (for a value of respectively 7C, 8C and 9C) from this dataset.

The principal and most known algorithm for extracting an isosurface is the *Marching Cubes* (Lorenson and Cline, 1987), which was designed to work with a voxel input only. This algorithm can nevertheless be easily rewritten to work with a set of adjacent tetrahedra instead of cubes: each tetrahedron of a DT is visited and the intersections between the implicit surface and each edge of the tetrahedron are computed by linear interpolation. This method is simple, robust, and yields good results in practice.

Slicing a Dataset

Although the slicing method is rather simple, it can still help users to have a greater understanding of a dataset, and therefore reduce the analysis time. Slicing involves ‘cutting’ a dataset according to a certain plane, and then visualising the spatial variation of a given attribute on that plane, for instance with the help of colours. Figure 7 shows an example where a field (represented by its samples), is sliced according to a certain plane, and interpolation is performed at regular intervals on the plane with natural neighbour interpolation (but any other method could also be used). In the case here, red means that the value of the attribute is higher.

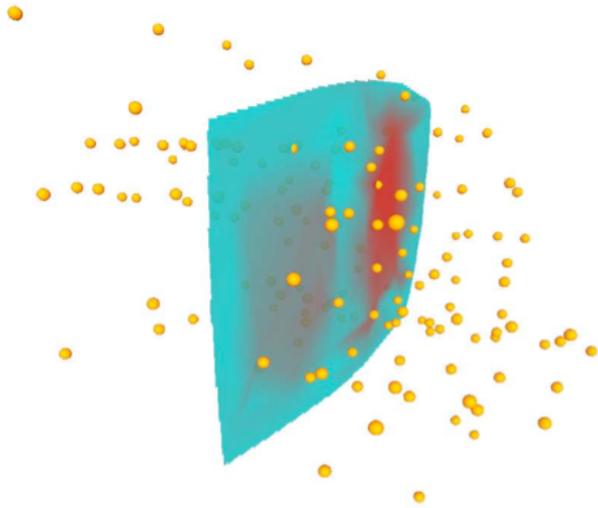


Figure 7: A dataset, representing a continuous field, sliced according to a plane.

Visualisation tools can also be combined and with the help of new techniques developed in recent years in computer graphics, it is possible for instance to draw many isosurfaces and view them all by using ‘transparency’ techniques, assigning different colours to each, ‘peeling off’ surfaces and navigating inside and outside to see the shape. For instance, Figure 8 shows one case where a cutting plane and an isosurface for a high value of the attribute are displayed. With the help of such tools and visualisation tools to ‘navigate’ in the scene, a user can understand better the spatial variation of the attribute being studied.

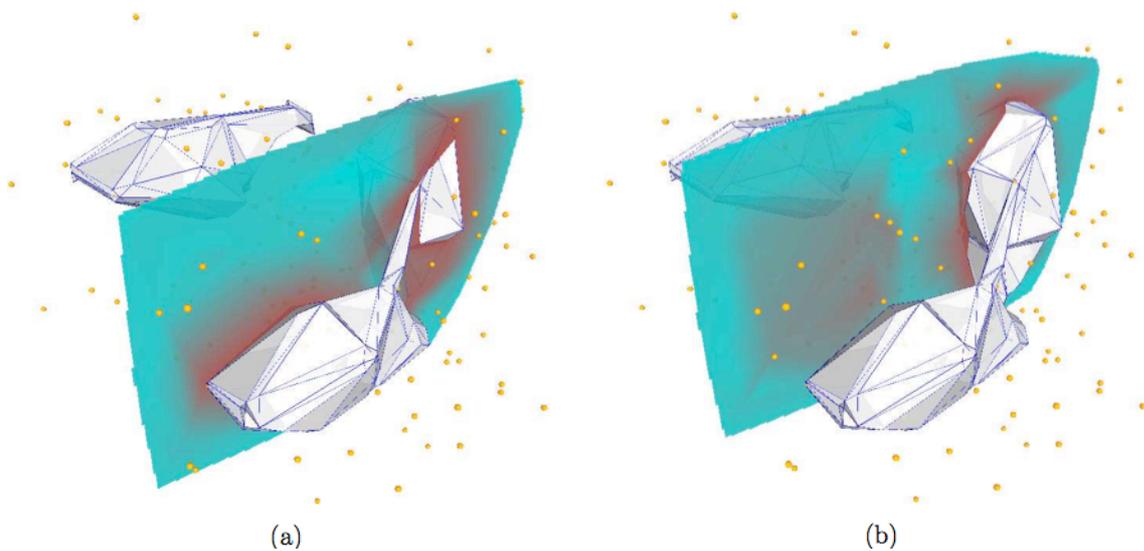


Figure 8: Two slices taken at different locations in the same field (red indicates a high value). For both cases, an isosurface for a high value is also shown.

4.3 Oceanography

The most common type of volumetric data in the marine environment are CTD data (conductivity, temperature and depth of the water), which were traditionally gathered through water columns. Volumetric information about the seawater can also be collected with remotely operated vehicles and autonomous underwater vehicles (Nichols et al., 2003). In both cases the samples collected have highly anisotropic distributions, similar to that in Figure 2.

Examples of applications in oceanography have already been covered in a previous paper (Ledoux and Gold, 2006), and we present here only a summary. Briefly, three main applications were presented. The first one is the concept of real-time applications, i.e. when data are collected at sea, quickly processed and then added directly to the GIS. This concept permits us to obtain directly at sea a first check of the quality of a survey, and to correct mistakes or collect new data when some are missing (Nichols et al., 2003). The second one is biogeography, which is the science that deals with the spatial distribution of species. It helps us understand where animals and plants live, and tries to understand why they live there; its link with GIS is obvious. The third one is the study of upwellings, which refers to the movement of subsurface water to the surface. Su and Sheng (1999) studied this process by visualising water properties over a certain period of time. Different isosurfaces, for each property, were created and then animated; the location and pattern of these isosurfaces helped scientists to determine upwelling characteristics. For all these examples, the dynamic and kinetic properties of the VD as presented in this article could be very useful. The gridding of data (as it is usually performed) could be circumvented, and the data visualised and analysed directly from the VD/DT.

4.4 Meteorology

Datasets collected to study the properties of the atmosphere are closely related to those in oceanography. Although collecting data in the atmosphere is somewhat simpler than underwater, obtaining a complete representation of the atmosphere is still difficult. The vast majority of datasets in meteorology have a highly anisotropic distribution similar to that in oceanography (Betancourt, 2004).

A spatial model based on the VD/DT to represent and analyse atmospheric data would basically bring the same advantages as in oceanography: the gridding of the samples could be omitted and the data could be analysed directly. The dynamic/kinetic properties of the spatial model are particularly useful here as the atmosphere tends to change quickly over a short period of time—the analysis and weather forecasting must usually be performed quickly. Concrete applications such as the analysis and prediction of the trajectory of air parcels (Barjenbruch et al., 2002), and the simulation of winds (Ciolli et al., 2004) are good concrete examples.

5. Discussion

A spatial model based on the VD and the DT, with the spatial operations mentioned in this paper, is a generic tool that can be used in a wide variety of domains and applications. Indeed, any applications where 3D fields—represented by their samples—are involved can benefit from the properties of the VD and the DT.

Several important aspects of the VD/DT have been left out in this paper. One of them is the implementation of such a model, which is arguably very important since 3D geometric computing is known to be plagued with special cases that make the ‘jump’ between theory and practice rather difficult (Hoffmann, 1989). Readers interested in implementation details for the VD/DT are referred to Ledoux (2006).

REFERENCES

- Barjenbruch DB, Thaler E, and Szoke EJ (2002). Operational applications of three dimensional air parcel trajectories using AWIPS D3D. In Interactive Symposium on AWIPS. Orlando, Florida, USA.
- Betancourt T (2004). Some concepts in atmospheric data. In The ArcGIS Atmospheric Data Modeling Workshop. Seattle, USA.
- Ciulli M, de Franceschi M, Rea R, Vitti A, Zardi D, and Zatelli P (2004). Development and application of 2D and 3D GRASS modules for simulation of thermally driven slope winds. *Transactions in GIS*, 8(2):191–209.
- Egenhofer MJ (1995). Topological relations in 3D. Technical report, University of Maine, Orono, USA.
- Ellul C and Haklay M (2006). Requirements for topology in 3D GIS. *Transactions in GIS*, 10 (2):157–175.
- Fisher PF (1997). The pixel: A snare and a delusion. *International Journal of Remote Sensing*, 18(3):679–685.
- Gold CM (1989). Surface interpolation, spatial adjacency and GIS. In Raper J, editor, *Three Dimensional Applications in Geographic Information Systems*, pages 21–35. Taylor & Francis.
- Goodchild MF (1992). Geographical information science. *International Journal of Geographical Information Systems*, 6(1):31–45.
- Hoffmann CM (1989). The problems of accuracy and robustness in geometric computation. *Computer—IEEE Computer Society Press*, 22:31–42.
- Kemp KK (1993). Environmental modeling with GIS: A strategy for dealing with spatial continuity. Technical Report 93-3, National Center for Geographic Information and Analysis, University of California, Santa Barbara, USA.
- Kemp KK and Vckovski A (1998). Towards an ontology of fields. In *Proceedings 3rd International Conference on GeoComputation*. Bristol, UK.

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- Ledoux H (2006). Modelling three-dimensional fields in geoscience with the Voronoi diagram and its dual. Ph.D. thesis, School of Computing, University of Glamorgan, Pontypridd, Wales, UK.
- Ledoux H and Gold CM (2006). La modélisation de données océanographiques à l'aide du diagramme de Voronoï tridimensionnel (in French). *Revue internationale de géomatique*, 16 (1):51–70.
- Ledoux H and Gold CM (2007). Modelling three-dimensional geoscientific fields with the Voronoi diagram and its dual. *International Journal of Geographical Information Science*. In Press.
- Lorenson WE and Cline HE (1987). Marching cubes: A high resolution 3D surface construction algorithm. *Computer Graphics*, 4:163–168.
- Nichols CR, Porter DL, and Williams RG (2003). Recent advances and issues in oceanography. Greenwood Press, Westport, USA.
- Raper J, editor (1989). Three dimensional applications in Geographic Information Systems. Taylor & Francis, London.
- Sambridge M, Braun J, and McQueen H (1995). Geophysical parameterization and interpolation of irregular data using natural neighbours. *Geophysical Journal International*, 122:837–857.
- Sibson R (1981). A brief description of natural neighbour interpolation. In Barnett V, editor, *Interpreting Multivariate Data*, pages 21–36. Wiley, New York, USA.
- Su Y and Sheng Y (1999). Visualizing upwelling at Monterey Bay in an integrated environment of GIS and scientific visualization. *Marine Geodesy*, 22:93–103.
- Watson DF (1992). *Contouring: A guide to the analysis and display of spatial data*. Pergamon Press, Oxford, UK.
- Zlatanova S, Abdul Rahman A, and Pilouk M (2002). 3D GIS: Current status and perspectives. In *Proceedings Joint Conference on Geo-spatial Theory, Processing and Applications*. Ottawa, Canada.

BIOGRAPHY

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