Modeling Ore Textures and Mineral Liberation Using 3D Voronoi Diagrams

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Abstract

In this paper, we present a combined stochastic geometry model for mineral liberation of two-phase ore under fragmentation in breakage process. The texture modeling of unbroken ore with solid grains and its interaction with artificial fracture pattern are considered on the base of simulations with the 3D Voronoi diagram (VD) and the Poisson polyhedra mosaic (PM). An algorithm was developed and implemented to generate the VD of ore texture overlaying with PM fracture pattern to predict expected liberation spectra as bivariate grade-size distribution of polyhedral fragments.

Three types of breakage mechanism are examined: i) random crushing planes; ii) preferential cracking within waste matrix or gangue phase that has very low hardness and iii) intergranular disintegration along grain contacts on boundaries of Voronoi polyhedra.

Introduction

The mineral liberation of ore phases in crushing and grinding operations is necessary for separation and extracting valuable ingredients throughout mining and ore dressing processes. Voronoi polyhedra constructed ore textures as crystal grains that fill the whole available space are making quantitative geometrical modeling possible, thus leading to unique volumetric solutions for predicting potential mineral liberation degrees.

Some earlier mentioning of 3D Voronoi diagram to model rock mineralogy was described in many works on integral geometry [] and its more recent applications [,].

Thus the computational technique should include the following main steps:

1. Modeling of heterogeneous multiphase ore using the 3D Voronoi diagram
2. Modeling of breakage or fracture pattern using the 3D Poisson polyhedra mosaic for micro cracking.
3. Simulation of interrelationship for the two stochastic geometrical process to calculate liberation spectra in expected particles for every mineral phase by pairs (selected target mineral and all others in gangue or waste matrix).
4. Estimation of mineral liberation spectra for different mechanisms of breakages in crushing process for homogeneous and heterogeneous two phase ores (in second case we assume that hardness of target phase much higher then in gangue matrix and cutting fracture planes can propagate only along boundaries of Voronoi grains)
5. Calculation of integral parameters of ore-dressing process on given extraction probabilities for predicted grade-size distributions

We denote:

A –Target mineral phase

B – Gangue mineral phase

λ - Intensity of Poisson point process for Voronoi tesselation

τ - Intensity of Poisson point process for Poisson fragmentation

v – volume of Poisson fragment or voxel particle

g – grade o of Poisson fragment as volume concentration of target phase

α (v, g) – texture quantification of unbroken ore as mineral liberation spectrum;

β (v) – fracture breakage pattern characterization with the size distribution of Poisson polyhedron particles;

γ (v,g) –Product of interaction between the texture and fracture as bivariate size-grade distribution of polyhedron particles.

MLS - Mineral Liberation Spectrum

α (v, g) β (v) $= \gamma$ (v,g)

Or in a matrix form:

$$\mathbf{L} \cdot \mathbf{f} = \boldsymbol{\gamma},$$

where L – the mineral liberation spectrum for target and gangue phases, f – size distribution of particles; γ - the bivariate size and grade distribution of particles.

1. Integral geometry approaches

There were developed several approaches by King [], Davy [], Barbery [] and many other followers with attempts to predict particle composition distributions. Basically their stochastic methods extend the integral geometry concepts of Santalo [], Matheron [] and Serra [] for one- and two dimensional cases. The needs to characterize ore samples in sections by image analysis bring them to relate textures with complex probabilistic equations. In this approach the covariance function in ore texture (with 0 and 1 phases) is defined by conventional equation:

$$C(h) = E\{[f(x) - V_V][f(x+h) - V_V]\},$$

where *f(x)* is the indicating function for lag *h* between two points, being equal to 1 when point x is in phase 1; the mathematical expectation *E* is taken for the complete material volume. The local covariance according to Barbery [] can be assessed by:

$$C(h) = V_V^2\{\exp[\theta K(h)] - 1\},$$

where *K(h)* is the global covariance of the primary grains, defined by Serra [] as:

$$K(h) = \int_{R^n} k(x) \cdot k(x+h) dx$$

taken for all x and all directions on the primary grains.

The ore texture modeling and subdividing sample space may be based completely on random Poisson process [] as was proposed by Barbery in []. Barbary's random Poisson polyhedral fracture pattern interacts with binary random Poisson polyhedral texture. In the case we have for covariance with Poisson intensity λ_3:

$$K(h) = \frac{6}{\pi^2 \lambda_3^2} \exp(-\pi \lambda h)$$

But as was pointed out in [] the above expressions rather complex and do not provide close form solutions.

We propose a technique based on modeling heterogeneous granular ore textures using 3D VD in order to calculate liberation spectra for mineral phases when the material undergoes random size reduction in crushing and grinding operations. The basic idea is to simulate multiphase ores taking in account results of modal analysis from geological sampling and then subdivide the space with a fracture pattern to calculate volumetric grade-size distributions of random Voronoi or Poisson polyhedra for product particles.

2. Modeling ore texture

We use the crystallization technique by a nucleation/growth method for construction ore texture as describe in [] and shown in Fig. 3.

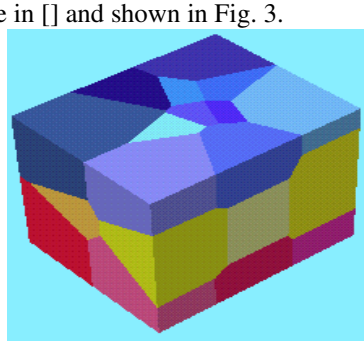


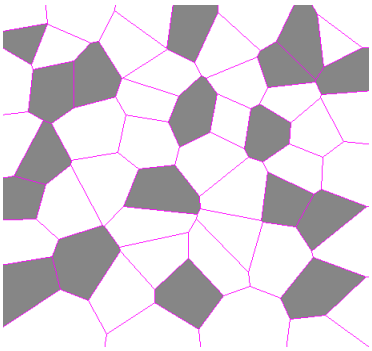
Fig. 3. The ore texture simulation with 3D VD

3. Creation fracture pattern

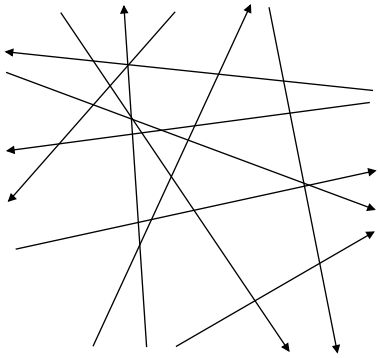
The manifold stochastic methods of modeling microstructures and fracture patterns recently were discussed in []. While common mathematical techniques for Poisson random fields and crack growth generation described comprehensively but studies were concerned mainly with strength properties and not liberation of phases in random microstructures.

To estimate liberation results after interaction between textures and fractures we investigated two different cases:

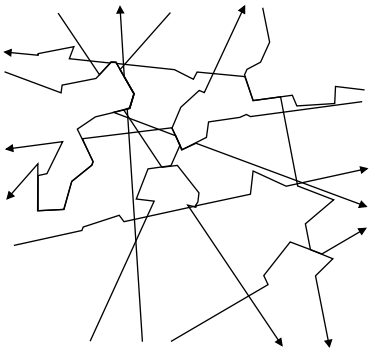
1. 3D Grid approach. Allocation or placing cubic particles of given voxel size (in defined range of sizes) in random positions inside the container box
2. Unstructured grid approach. Poisson space subdivision of texture in simulating fracture patterns in three cases: i) without any physical interactions of Poisson splitting planes and boundaries of Voronoi grains: ii) with postulation that Poisson splitting planes would not cross inside target mineral phase and iii) with postulation that breakages occurs only along facets or of boundaries of Voronoi cells.



Original two phase texture



i



ii

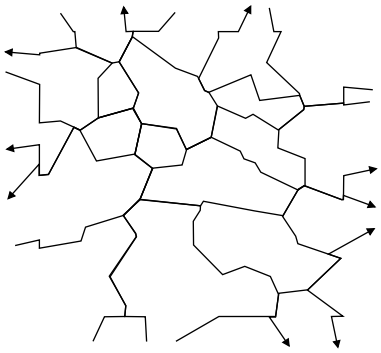


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Fig. 3. Illustration of Poisson fracture patterns: i- random crushing planes; ii- cracking within waste matrix or gangue phase that has very low hardness; iii- preferential disintegration along boundaries of Voronoi polyhedral grains

Interrelationships for the 3D VD of multi-phase ore texture model with Poisson fracture patterns make polyhedral particles, that are classified by size and grade to define liberation spectra L(v,g) and density distribution γ (v,g).

4. Algorithms and computer realization

Some earlier works on the stochastic simulation of mineral liberation in 1D case were examined by R.King [] and many others [, ,]. One of the computational approaches has the next steps []:

1. Generate along X axis a long enough line with multi-phase ore's intervals as 1D texture
2. Analyze the line by pairs of target mineral and others (considering coverings of pixels in series 1, 2, 4, 8, ..., max in the line)
3. Calculate the mineral liberation spectra (actually LengthOfSegment-ContentOfInclusionsInsideSegment distribution) for 12 classes of grades (concentrations from 0 to 100%) and 16 classes of sizes (see Fig. 3).
4. Multiply the mineral liberation matrix on given particle size distribution (as forthcoming ground product) to calculate the size-grade distribution
5. Define integral grade, yield and recovery of target mineral phase in extraction product.

In 3D case the mineral liberation simulation could be implemented in following steps:

Algorithm ml_1: Cubic particles in voxel model of texture (without cutting planes)

Input: A VD in R3 as multiphase texture of mineral grains. v- volume of particle.
We can generate 1000 random points with proportions (contents) of phase m (mineral): Ca=10, Cb=30 and Cc=60 percents in the box of size Nx*Ny*Nz (1024*1024*1024) units.
This would be a proto particle and then we can subdivide it by cutting planes.
Output: MLS (Mineral Liberation Spectrum) and SGD (Size-Grade Distribution) of particles for every mineral phase.

1.Generate random germs for grains of minerals in container box
2.Construct the 3D VD for multi-phase ore texture
3. Convert VD 3D to voxel model

3.1. For all voxels with *ijk* inside a Voronoi grain assign index of particle *l* and index of phase *m*.
4. Insert a random cubic particle from size distribution f(v) in range [1..BoxSize] inside the box, align it on the grid and define grade.
5. Calculate MLS
6. Define integral grade, yield and recovery of target mineral phase in extracted product.

Thus for 3D case with regards to Poisson process of planes we have the following algorithm:

Algorithm ml_2: Poisson polyhedra created by cutting planes.
Input: VD in R3 as multiphase texture of mineral grains. v- volume of particle.
The average size of particles in volume units v_{av}
We create TParticles: array of TParticle, where
TParticle = class
…
ID_Particle;
Grains: array of TGrain // Voronoi cells in proto particle and then simply polyhedra
procedure Volume;
procedure Grade;
procedure SizeAndGradeFractionBelongings
…
end.

Output: MLS (Mineral Liberation Spectrum) and SGD (Size-Grade Distribution) of particles for every mineral phase

1. Create a proto particle as the cubic box were all Voronoi grains have particle Identifier *l* = 1.
(from now we don't need Voronoi diagram structure because it will be destroyed by cuttings)
2.Generate a random breakage plane using 3 random points
3. Find all particles that cut the current plane
3.1. Case 1: Define all grains of phase *m* in this particle that random plane cuts as a scissor
3.1.1.Divide edges and add facets to two new particles
3.2. Case 2: Define all grains of phase *m* in this particle that random plane cuts with preferencial mild phase
3.2.1. Find all facets if phase *m* in particle when dividing on two new ones
3.3. Case 3: Define all grains of phase *m* in this particle that random plane cuts with intergranular boundaries
3.2.1. Find the facet path through all *M* phases inside the particle when dividing it on two new ones
4. Increase the Index of every of two particles by one and the whole number of particles
5. Define the average size of particles and if it less then Vav goto step 2.
6. Calculate mineral liberation spectrum
7. Calculate the volumetric size-grade distribution of polyhedral particles

After calculations we are getting the next spectrum of mineral liberation for homogenous two-phase ore (Fig. 3)

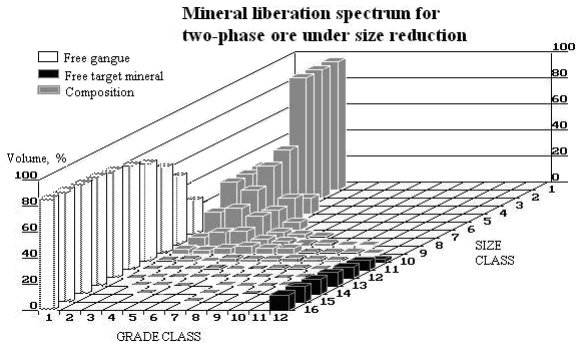

Fig. 3. The liberation spectrum of two-phase ore under size reduction process

The spectrum of mineral liberation for two phase system (Fig. 3) was calculated using random voxel particles in the size range 1-2048 inside the cubic sample container. It is implemented according to algorithms_ml1.
Implementation of algorithms_ml2 for interaction of ore texture with Poisson fracture pattern offer mechanism to simulate main physical properties of ores, such as hardness and strength of intergrowth in mineral grains.

Conclusion

In our approach of solving mineral liberation problem we don't use integral geometry expressions derived from analyzing multiple stochastic characteristics of objects involved as was proposed in [, , ,], but instead we have developed a direct computational algorithm for processing every Poisson polyhedron's volume and grade in final product of fragmentation.
The model of mineral liberation could be compared with stereological measurements for ore sample sections from linear and areal methods of scanning electron microscopy.
The developed approach provides a reliable 3D

VD/PM model to predict particle compositions in ore ground products before mechanical size decrease operations and could be applied for advanced geostatistical mapping and reserve estimation of mineral deposits.

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