The Potential of the 3D Dual Half-Edge (DHE) Data Structure for Integrated 2D-Space and Scale Modelling: A Review

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Abstract Scaling factor is one of the most crucial aspect in 2D and 3D models especially in computer graphics, CAD, GIS, and games. Different user or/and application need different scale models during various stages of the use of data, including visualization and interaction. There are some arisen issues on 3D data model especially to meet GIS requirements while minimize the redundancy of the datasets. In GIS modelling, various data structures and data models have been proposed to support variety of applications and dimensionalities, but only a few in scale dimension. Some of them have succeeded in modelling scale such as in Space-Scale Cube (SSC) model. The recently implemented Dual Half-Edge (DHE) data structure within the PostgreSQL database is suitable for any valid 3D spatial model; not yet being explored for other dimensional such as scale environment. Using the same vario-scale approach, the DHE data model is also capable to implement a variable Level of Detail (LoD) representation such as SSC model. Some advantages of the DHE are described in this paper such as the dynamic property (valid updates based on Euler operations) and topology approach in

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comparison with other existing data structures. The last section of this paper describes capability of the DHE data structure to provide a better platform for GIS integrated space-scale data model.

Keywords Scale dimension · Data structures · Spatial models · Level of details

1 Introduction

Traditionally, spatial data is usually presented in two-dimensional (2D) environment such as a printed map. But, with current technology and new research knowledge, three-dimensional (3D) Geographic Information System (GIS) is becoming a norm in modelling software.

As far as 3D modelling is concerned, commercial and open source software offers functionalities to model and represent a model in 3D environment. For instance, the latest version of Blender, Google Sketchup, FreeCAD, Paraview, Transmagic and others provide a lot of modelling and visualization functions, especially for animations, games and building models with a focus on geometry and graphic visualization. As a matter of fact, these general purpose 3D modelling techniques are not really suitable for 3D GIS spatial modelling since it does not fulfil some basic data validation requirements especially on topology and thematic semantic information. For example, a 3D solid must be completely closed, otherwise it is not possible to compute its volume (and many other operations). Thus, many basic and complex functions or analysis available in 2D GIS can be hardly implemented in the 3D GIS environment.

There are three basic requirements that need to be preserved in order to provide basic and complex analysis in n-dimensional ($n > 1$) spatial model representation. They are geometry, topology and thematic semantic integration within the same model. A particular data model should be able to provide these three key components in order to widen analysis functions in the 3D spatial model.

However, these user or application-oriented 3D spatial data models are only suitable for and satisfy a specific demand as reported by Liangchen et al. (2008). In order to fulfill these demands, many available data models had been designed and implemented. A data model can be defined as a method or a logical or mathematical way of visualizing and representing data (informational needs) in an information system. From GIS point of view, a spatial data model is needed to simplify sketch the real world and a pursue disperse model in geographical way of representation within a computer (Liangchen et al. 2008).

There are some drawbacks highlighted by Liangchen et al. (2008) from this application–oriented data model: (1) very hard to meet different situation demand and multiple coupling area requirements; (2) lack of mathematical completeness, redundancy and lack of topological relationships.

In general, 3D data models can be divided into four main groups: 3D geometric models, 3D topological and graph models, 3D city models and 3D CAD models (Lee and Zlatanova 2008; Boguslawski 2011). The spatial models are used in accordance with specific application requirements, for example using Constructive Solid Geometry (CSG) or Boundary Representation (B-Rep) in 3D environment of CAD system (Boguslawski 2011).

A spatial data model can be represented by different data structures (Ledoux 2006). For instance, tetrahedral meshes in 3D or triangular meshes in 2D can be represented with radial-edges, half-edges or other data structures (Boguslawski 2011). A data structure can be defined as a specialized format or approach for organizing and storing digital geographic dataset in any data model. It refers to the problem on how to systematically encapsulate the geometry, topology and thematic semantic information of GIS dataset.

The capability of the selected data structure to support the required geometrical or/and topological analysis will increase the performance of the data model and quality of the data. For example, Tse and Gold (2004) combine the quad-edge data structure and the boundary representation to implement the extended TIN for supporting holes and caves.

The next section explains GIS data structures with special focuses on: topological connections and type of data model (kinetic and dynamic). Section 3 describes the scale dimension in GIS and discussion on scale implementation approaches. Section 4 introduces the Dual Half-Edge (DHE) as one of the potential data structures in scale dimension representation. A summary and future plan for the 3D-scale DHE data model implementation is provided in the last section of this paper.

2 GIS Data Structure

Nowadays, GIS is an important tool to help professionals in geography, science and engineering fields, especially for decision support system. Early GIS systems focused on the automating map making process and they provided simple analysis of two dimensional (2D) spatial data. Modern systems incorporate the capability of software, hardware and knowledge to perform GIS complex spatial analysis as well as the visualization in 3D spatial model. However, GIS is still relatively poor as far as the 3D is concerned; with most systems barely support 3D storage, analysis (functionalities) and visualization.

2.1 Existing 3D Data Structure

The CAD systems allow the model construction with a set of individual polyhedra, which are not topologically connected. Such connections are calculated from geometry each time they are required (Lee and Zlatanova 2008). Vertices, edges, faces and the relationships among them are defined within the solid boundary of B-Rep. Euler Operators are used to modify the geometry in CAD systems. There are various data structures used in B-Reps for solid objects (3D, some example in Fig. 1) such as:

1. Winged-edge of (Weiler 1988)
2. Half-edge of (Mäntylä 1988)
3. Quad-edge (Guibas and Stolfi 1985)
4. Doubly-Connected Edge List (DCEL)
5. Radial Edge (Weiler 1988)

2.2 *Kinetic and Dynamic Data Model*

According to Banks et al. (2009), there are two types of models; physical and mathematical models. As for GIS implementation, researchers such as Boguslawski (2011) prefer to use mathematical model since it can be easily analysed on a computer and effectively compared with a physical model. Physical models are still used in other disciplines: e.g. 'maquettes' in architecture and urban spatial planning or a scaled down version of a coastal area or river in lab basin for water management research. Further classifications of the mathematical (digital) models are: static or dynamic, discrete or continuous, deterministic or stochastic model (Banks et al. 2009; Boguslawski 2011).

For describing changes of the model, Gold (2005) divided data structures into two categories: dynamic and kinetic. A dynamic data structure refers to the capability to adapt any changes of the model after the batch construction phase. Gold (2005) defined a "dynamic" or updateable locally data structure if it offers the local insertion, deletion, movement and navigation in the model. This in contrast with some other models that are based on global criterion's during an update (which is practically not feasible for large data sets, with frequent changes).

On the other hand, the kinetic data model refers to static properties of the data structure such as lack of support for local modifications and non-dynamic user-end applications. It is also pre described as a static viewpoint or a set of objects and relationships, inability to make changes or interactive queries and perspective mood such as colour contras (Gold et al. 2004). For example, kinetic Voronoi diagram is

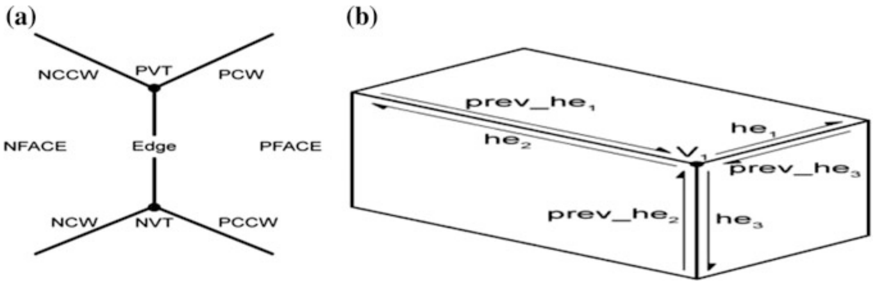

Fig. 1 Example of B-Rep data structure, **a** Winged-edge, **b** half-edge (Boguslawski 2011)

used in ship navigation (Gold et al. 2004) and Free-Lagrange for fluid flow simulation (Mostafavi and Gold 2004).

3 Scale Dimension

The research trend on scale dimension or multi-scale dataset in 3D models is observed not only in GIS field but also in fields such as 3D visualization of multi-scale geological models (Jones et al. 2009), Scale-Space Theory in computer vision (Lindeberg 1994), and cross scale dynamic and social ecology (Vervoort et al. 2012). Due to limitation of up scaling in geology, new multi-scale methods that incorporate fine-scale information into a coarse-scale equations has started to gain popularity (Aarnes et al. 2007).

In GIS, a multidimensional model may consist of one, two, three or more spatial dimensions that represent spatial objects (Gold 2005). It is important to distinguish the dimension of the embedding space and the dimension of the used primitives; e.g. a 0D point feature may be represented in 3D space. However, there are many approaches in defining higher dimensions in the model. The 2D geometry, that represents spatial dimensions, is accompanied by the third dimension to represent other non-spatial dimensions such as scale or time (Oosterom 2005). Gold (2005) suggest that 3D model may consist of a primitive (2D) map and unconnected data of points or objects. As far as the scale is concerned, the unconnected data can be understood as the reflection of viewpoint or observation of spatial objects (Zhou and Jones 2003).

Some researches attempted to add non-spatial dimension to the existing 3D spatial dimension (Worboys 1994; Raper 2000; Peuquet 2001; Ohori et al. 2013). Peuquet (2002) and Worboys (1994) focused on the time dimension, while Oosterom (2005; Li 1994) on the scale dimension. The implementation of the scale dimension faces many problems mostly due to limitation of the available data structures and models for three and higher dimension. Only a few models such as Multi-Scale Line tree (Jones and Abraham 1986), Arc-tree (Günther 1988), and Binary Line Generalization (BLG) (Oosterom 1990) integrate the geometric and scale aspects in one representation.

Since different applications and users need a specific fineness (number of details) of data representation, Sester (2007b) suggested that different representations or different Level of Details (LoDs) of the same reality have to be made available (with or without explicit relationships between corresponding features at the adjacent LoDs). Thus, there is a need to combine all level of detail into a single container called scale dimension.

Scaling dimension in GIS has gained some popularity in recent years due to the demand from users and applications. GIS research community is now moving forward to integrate a highly formal definition of geo-data (Oosterom and Stoter 2010) and thus focuses on designing the most efficient framework and implementation for scale and temporal dimension. Important functionality of these space-scale models is efficient zooming and progressive data transfer between server and client (in environment that avoids redundancy and inconsistency as much as possible). Currently, scale dimension can be categorized into two main approaches: multi-scale and vario-scale.

3.1 Multi-scale Approach

In urban modelling, a multi-scale concept is an important issue because different applications require different abstractions of building models (Sester 2007b) as well as city objects (represented as 3D), which can be visualized and analysed in multiple scales (El-Mekawy 2010). Typical frameworks for multi-scale integration are either storing individual LoD dataset in separate databases or using generalization techniques. Most of the national mapping agencies use the 2D digital data in different groups or databases such as 1:20,000 and 1:50,000 topographic maps, which can be used by a single application (e.g. web-mapping portal).

Multi-scale GIS spatial datasets may be maintained in two ways: separately maintained different databases with predefined scale-steps (Fig. 2) or/and maintaining only the most detailed data which can be automatically generalized into small scale data on the fly transition (Oosterom and Stoter 2012), such in Fig. 3.

The main drawback of the first maintenance method is data redundancy. Storage of the same dataset in different scale ratio (pre-defined datasets) and different databases will introduce data redundancy in terms of geometry and attributes. Thus, it will consume a lot of storage capacity as compared to use the second method; a single dataset (most detailed) storage and undergo any generalization process for coarse details. Furthermore, it also will cause a lot of upcoming problems such as difficulty in updating process (need more works to update certain area of the data for every pre-defined scale), introducing errors if it is not handled properly and problems with the geometry and topology consistency.

The main drawback of the second approach, on the fly generalization, is that this is a very hard and computation intensive. It may not be feasible to use this approach with sufficient cartographic quality in an interactive setting especially with many users of the service.

While in 3D spatial modelling, a particular generalization technique (involve aggregation, simplification and others) or a set of pre-defined LoDs (e.g. CityGML); will be selected to represent as the scale dimension. The main drawback of these techniques is the 3D objects are not topologically connected or connected with a very minimal connectivity.

A 3D spatial multi-scale concept, such as CityGML, introduces two kinds of possible transitions: generalization and progressive transmission as shown in Fig. 4. Multi-scale generalization can be described as a process of transmigration of a spatial object from a detailed level to the less detailed level within the same

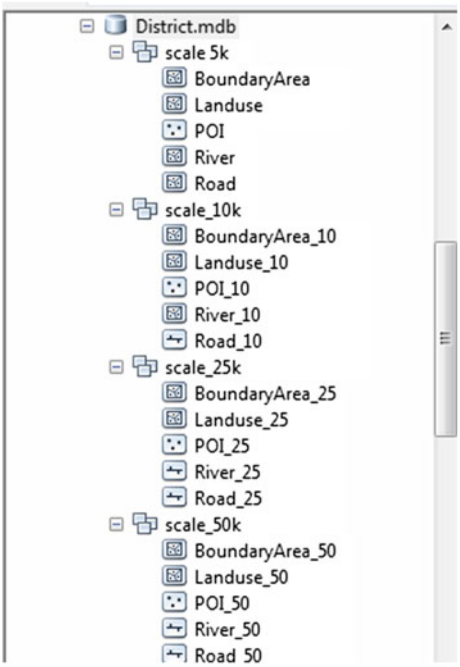

Fig. 2 2D redundancy problem in the multi-scale approach, which stores the same object in separate LOD datasets

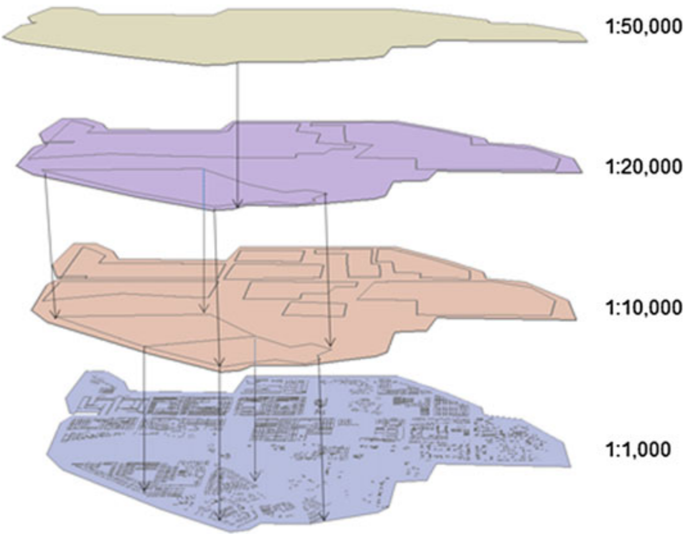

Fig. 3 Illustration of a typical multi-scale representation using generalization technique

Fig. 4 Multi-scale concept in CityGML (Sester 2007a)

application or model. While, a progressive transmission is the inverse process for generalization; the use of generalization chain (history; maximum and minimum elementary generalization operations) to revert the previous generalization levels. Many multi-scale generalization approaches has been implemented for 3D city models. Most of the researchers attempted different generalization methods in order to produce less-detailed LoDs by hiding less important details and reducing the data storage volume for efficient 3D analysis and visualization (Baig et al. 2011). It should be noted that these LoD representations are independent (no explicit relationships between features at different LoDs).

3.2 *Vario-Scale Approach*

In variable-scale approach, the basis is formed by a data structure in which each area of the map can be represented by a topological face (Meijers 2011). In principle, any topological data structure, such as Winged-Edge can be used. The true vario-scale structure, Space-Scale Cube (SSC), is the extension from the topological Generalized Area Partitioning (tGAP), which uses the hierarchical tree structure. The vario-LoD is defined as an additional dimension (third dimension) as for a series of maps at a range of scales; horizontal slice planes can be used. Similar to tGAP terminology, the third dimension is represented by the concept of "*importance*" (are diver for scale, LoD content) (Meijers 2011).

The importance of the objects is highly depending on the feature classification and the size of the object. For instance, a small city is more important than a large area of forest; a large residential area is more important than a small water body. The new importance value is calculated after the generalization (merging) from two predecessor objects as illustrated in Figs. 5 and 6.

The tGAP model helps to overcome the data redundancy problem since it was constructed using the most detailed dataset to the most coarse one. It also preserves the topology of the horizontal plane and provides the LoDs within the scale axis.

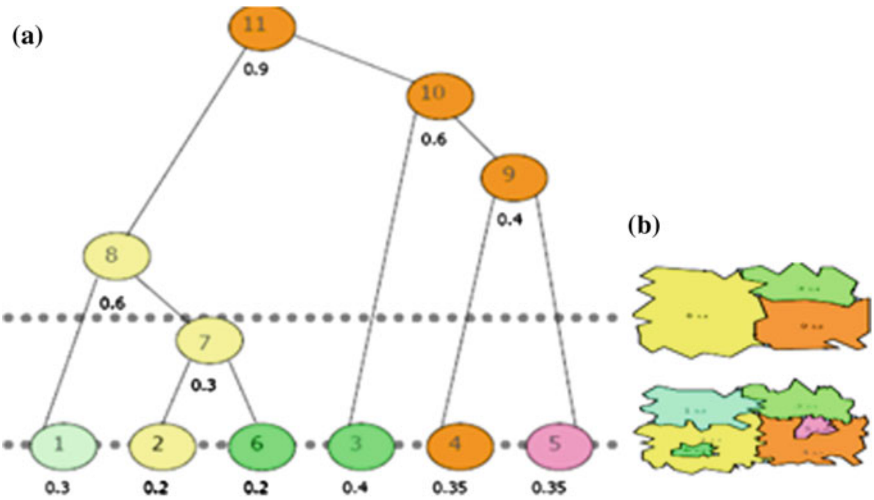

Fig. 5 Structure of variable-scale model, **a** tGAP hierarchical tree structure for the thematic semantic information and importance value, **b** respective slice planes (Meijers 2011)

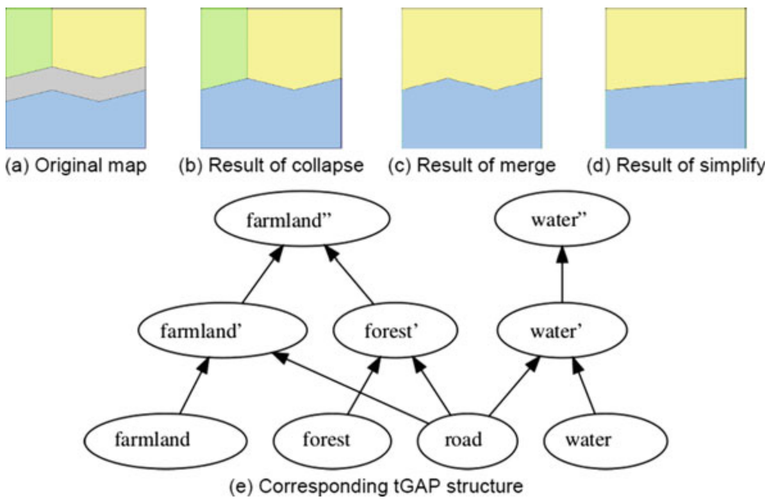

Fig. 6 The process of generalization and hierarchical tGAP structure (Meijers 2011)

The original tGAP structure (Oosterom 2005) capable to link the objects at different levels of detail.

The 3D, integrated 2D space and scale, tGAP model is represented in the Space-Scale Cube (SSC), which can be selected as a data model for the tGAP structure. While for the 2D dataset, it can be obtained by slicing the SSC model in horizontal plane. In the SSC model (see Fig. 7), the topology of the vario-scale

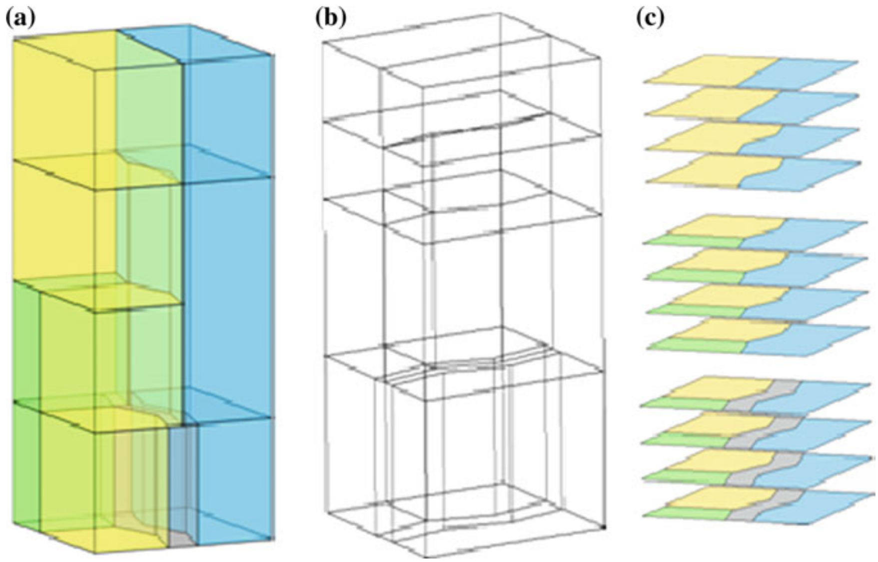

Fig. 7 Classic tGAP model, **a** classic SSC, **b** wireframe model, **c** slicing planes for several LoDs (Meijers 2011)

model in the vertical axis highly depends on the hierarchical tGAP structure in a database. The tGAP structure in vario-scale approach can be used in a web-based environment and/or in a desktop environment. For analyzing the content of a tGAP dataset ('data debugging'), 3D views can be generated; e.g. via ParaView software (Suba et al. 2013).

4 The Potential of DHE Data Structure in Scale Modelling

For the higher dimension modelling, there are several data structures available, which are able to represent models in four or more dimensions such as polytopal meshes (Sohanpanah 1989) and decompositions of polytopes (Bulbul et al. 2009). However, despite the fact that they are able to maintain various topological relationships, none of them provides a navigable network for the efficient navigation within the model.

Ohori et al. (2013) had identified two candidates of data structures that are able to model higher dimensions; Generalized Map (G-Maps) and the DHE. The DHE is a spatial 3D GIS data structure related to the radial-edge, facet-edge and half-edge data structures (Boguslawski 2011) and is based on the Augmented Quad Edge (AQE) date structure (Ledoux 2006; Ledoux and Gold 2007). In this paper we will further explore the DHE for realizing the vario-scale SSC.

4.1 Geometry, Topology and Thematic Semantic Integration

The DHE uses two structures: the dual and the primal graphs. The dual structure of the DHE model is associated with the topology, while the primal represents the geometry of any object in 3D model. The presentation of the entity (vertex, edge, face, and volume) in the primal space with its dual space conforms to the rules of the 3D Poincaré duality. Thus, a cell in the primal is presented as a single dual vertex; a face as a dual edge, an edge as a face and a vertex will be presented as a cell. Figure 8 illustrates the concept of 3D Poincaré duality used in DHE data structure.

The DHE is also capable of handling the thematic attributes-semantic integration of information. For instance, a room is presented by a dual vertex, while the attributes describing primal geometry (e.g. a room name) can be assigned to the dual vertex of the primal cell. The thematic attribute semantic information is embedded in the DHE data structure (see Fig. 8) to support GIS spatial data requirements.

Navigation in DHE data structure is possible because all of elements of the model are connected using pointers. A half-edge (HE) is the concept used to rep-resent edges in the model, where all edges are split into two directed halves. The DHE has five navigation operators: around a symmetrical edge, shared vertex, face and edge and adjacent edges as shown in Fig. 9 respectively. The navigation is

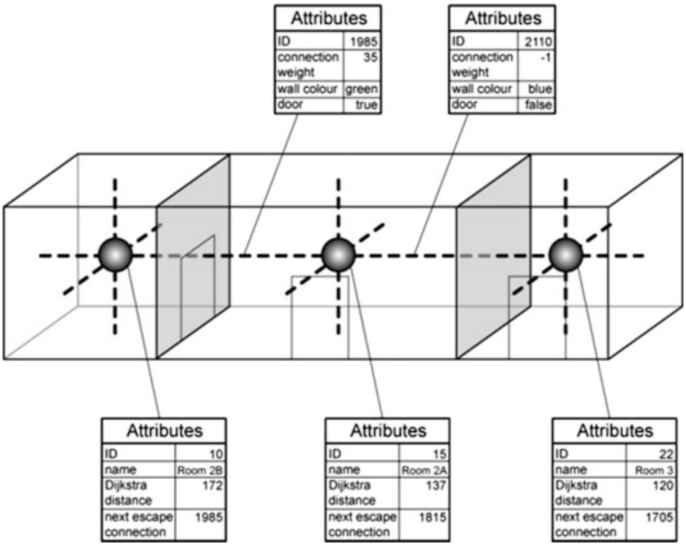

Fig. 8 Integration of Poincaré duality (*vertex* and *dotted edge*) with the primal space to support both navigation and the geometry of a 3D cell complex model (Boguslawski 2011)

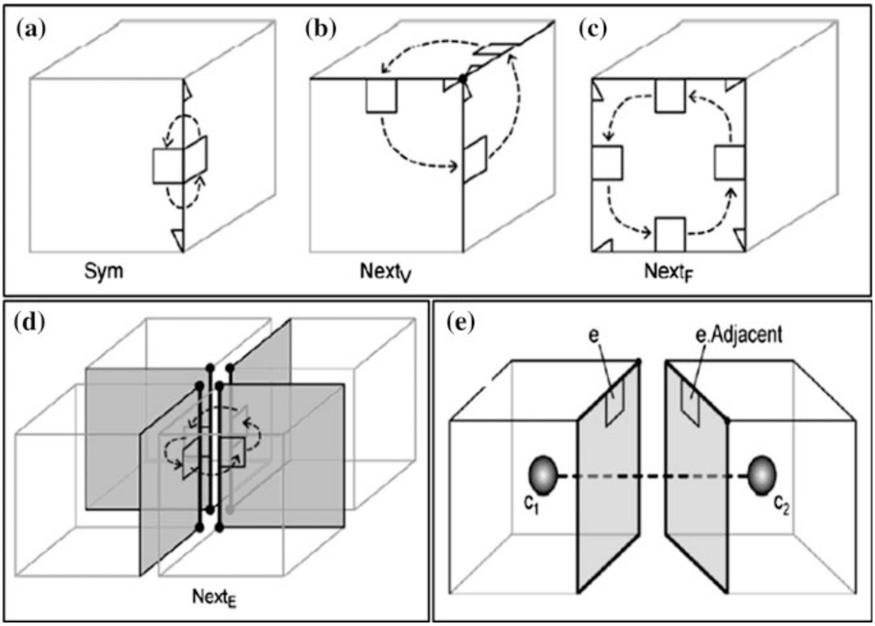

Fig. 9 Navigation of the dual half-edge (Boguslawski 2011)

performed from half-edge to half-edge. There are four basic operators available for navigation: Sym, NextV, NextF and Dual and compound operators: NextE and Adjacent based on the basic ones (see Fig. 9).

4.2 Concept of the DHE Vario-Scale Data Model

The implementation of the true vario-scale of tGAP structure is based on the SSC model (can be used with clients in desktop-based, web-based or mobile environments), using DHE with scale implementation may be categorized as a 3D model. The DHE data structure is capable to support the geometry, topology and attribute with additional scale dimension as replacing the Z (height) axis by adding some conversion processes (Fig. 11a, b) from original 2D scale dataset. The relationships between the representations at the previous and the next level as directly available in the variable scale DHE will be exploited to minimize the redundancy in scaling datasets. The concept of DHE vario-scale model is illustrated in Fig. 10.

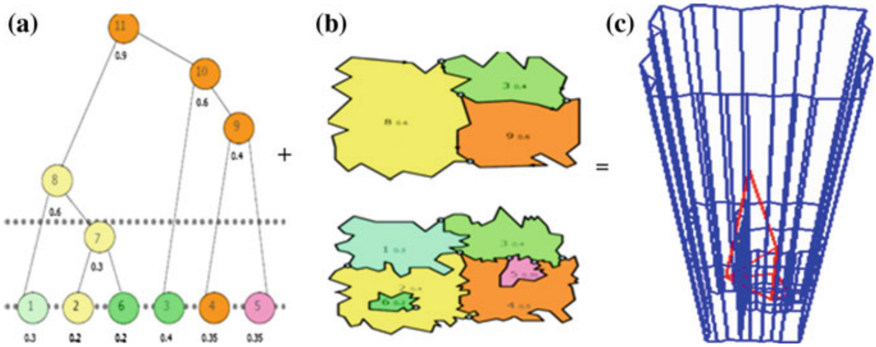

Fig. 10 Vario-scale data model, **a** hierarchical tree, **b** horizontal navigation, **c** idea of the DHE vario-scale data model. **a** and **b** are taken from (Meijers 2011)

To have an access query across LoDs, it is necessary to link object in one LoD to its respective object in the previous or next LoD (Paul et al. 2013). The DHE dual edge, which associated with the primal face (a loop of XY coordinates), can be used for hierarchical navigation and query purposes as well as storing the attribute and thematic semantic information (see Fig. 11c). While, the horizontal faces representing different LoDs can be illustrated in Fig. 11d.

Based on the PostgreSQL, foundation of the 3D DHE extension has been added to this DBMS. This is motivated by the fact that geographic data sets are often quite voluminous and do not fit in main memory for efficient use. Especially large scale base maps are large and these are also the types of maps that are used as basis for a range of scales (LoDs): either resulting in a multi-scale or vario-scale representation. Also, these geographic datasets are used my many persons and also maintained by multiple persons. These aspects, data size and multiple users, require a DBMS approach.

However the implementation of 3D DHE database using PostgresSQL and navigation operators will not being discussed in this paper. Goudarzi et al. (2015) discussed the implementation of 3D DHE database for a big dataset.

5 Discussion and Summary

In this paper a discussion on the spatial dimension in GIS modelling with some basic requirements and the use of data models/structures is presented. The existing 3D data models and structures in the context of capability of local modification are also discussed. The paper elaborates in detail on implementation approaches for representation of the scale-dimension as the third dimension: multi-scale and vario-scale models. The DHE is suitable for the vario-scale approach because of the

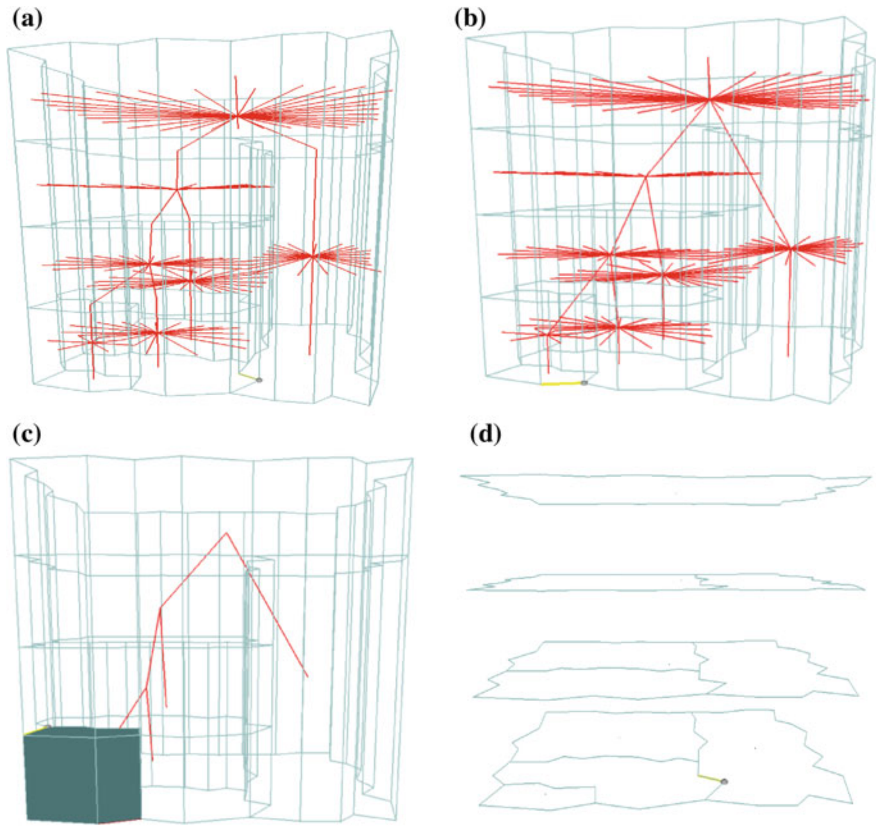

Fig. 11 Visualization of DHE-scale model dataset, **a** Original 3D scale DHE model (extrusion from single 2D dataset); the primal geometry in *light blue* and the dual in *red colour*, **b** intermediate process (merging by face), **c** Ideal DHE vario-scale model includes hierarchical navigation (the dual), attribute or semantic storing information in the primal or dual node, **d** 2D scale maps from slicing technique of SSC model

minimizing the redundancy of the dataset and preserving topological connections in each LoD.

The advantages of the data structure are its ability to perform local modifications and navigation using well defined operators (e.g.: Euler operators). It can be categorized as a consistent data model for scale modelling since it simultaneously updates both geometry and topology of the model via local modification.

Further implementation of the 3D (integrated 2D space and scale) DHE data model may produce better topological scale analysis such as zoom in and out for particular LoD using hierarchical query. Thus, a further study on this matter will be carried out to extend the capability and idea of the DHE data structure for scale dimension.

In future research we also intend to explore the 4D DHE for representing the integrated 3D space and scale model in order to realize true vario-scale 3D models. This in contrast to current 3D models, such as CityGML, which offer a discrete number of LoD with 'unconnected' and potentially inconsistent data.

The proposed research attempts to provide a new database-based service for web-based or desktop-based clients, exploiting the 3D (integrated 2D space and scale) data model which is suited for users and applications which require topological element in each level. It provides hierarchical and horizontal navigation, attribute storage and local modifications of the model with simultaneous and automatic update of the topology with the change of the geometry.

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