Piecewise Linear Complex represenation through Conforming Delaunay Tetrahedronization

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The boundary of three-dimensional objects is usually represented by Piecewise Linear Complexes (PLCs). A PLC is a set of vertices, segments and facets. The definition of PLCs requires that they must be closed under taking intersections; two segments can intersect only at a shared point, and two facets are either completely disjointed or intersecting only at shared segments or vertices. Another restriction of the facets of PLCs is that the point set which defines a facet must be co-planar. These PLCs are input for a tetrahedral mesh generator like (Tetgen, 2006). TetGen generates a Conforming Delaunay Tetrahedronization (CDT) from these PLCs by adding Steiner points where necessary to obey the Delaunay empty circum sphere criterion.

Once the CDT is generated, the PLC is part of this CDT. If a boundary marker is assigned to each facet of the PLC, all boundary faces on that facet in the final mesh will have the same boundary marker.

This condition can be used to represent a PLC just by a list of all nodes of the CDT together with a list of the boundary faces in this CDT. The PLC could be regenerated by a default Delaunay Tetrahedronization (DT) mesh generator, given the list of the nodes of the CDT and the list of the boundary faces of the CDT to re-assemble the faces of the PLC.

The list of boundary faces of the CDT can be encoded in a very efficient way if and only if the node numbering of the DT is identical to the node numbering of the CDT, and the tetrahedral mesh of the DT is identical to the tetrahedral mesh of the CDT. Each face of the CDT can then be flagged by true (1) of false (0) whether it is a boundary face or not. If these faces are sorted according a certain scheme, only the bitmap setting (a list of 0's and 1's) of these flagged faces has to be stored.

This process is shown in figures 1 to 4. Figure 1 shows a PLC representation of the Schönhardt Polyhedron. The CDT of the Schönhardt Polyhedron is shown in figure 2. Note: the Schönhardt Polyhedron could not be tedrahedronized constrained, thus adding Steiner points is here necessarily anyhow (Schönhardt, 1928). The PLC can now be reconstructed by a Delaunay Tetrahedronization of just the nodes of the PLC in conjunction with the added Steiner points, and the bitmap of the flagged faces (see figure 3). The DT of this input is show in figure 4, where the boundary faces are set according to the given bitmap of the flagged faces. These flagged faces represent the original PLC of the Schönhardt Polyhedron.

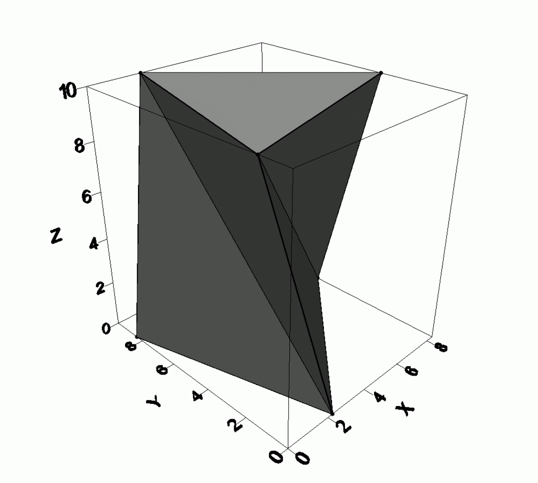

Figure 1:
PLC of Schönhardt Polyhedron

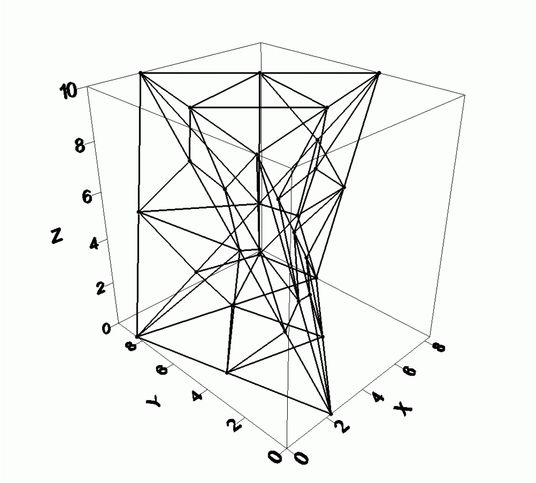

Figure 2:
CDT of Schönhardt Polyhedron

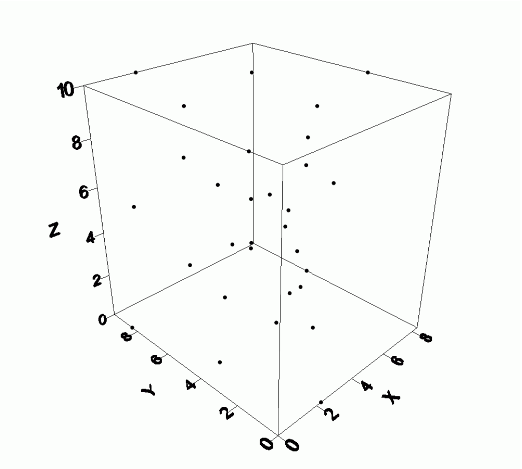

Figure 3:
Nodes & Steiner Points of CDT

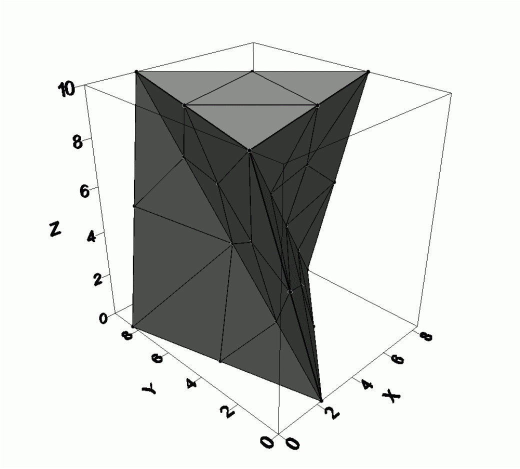

Figure 4:
Reconstructed PLC by DT

This way of representing PLCs should be considered as a 3D implementation of the method of encoding and decoding of Planar Maps through Conforming Delaunay Triangulations (Verbree, 2006). The main gain of this kind of PLC-representation through a CDT is the guaranty that the PLC is watertight, as the CDT could not be constructed anyhow. A second achievement is the efficient storage of PLC's by the set of nodes and the bitmap of boundary faces only. This method is therefore another approach to compress 3-Dimensional Urban Models, as described in

(Eppinger, 2005). A drawback is, however, the cost of the added Steiner Points and some processing capacity as the PLC has to be reconstructed by applying a default Delaunay Tetrahedronization mesh generator.

REFERENCES

Eppinger, F., and V. Coors (2005). Compressing 3-Dimensional Urban Models, DMGIS'05 – 4th ISPRS Workshop on Dynamic and Multi-Dimensional GIS, University of Glamorgan.

E. Schönhardt. (1928) "Über die Zerlegung von Dreieckspolyedern in Tetraeder", Math. Annalen 98 309-312

Tetgen (2006), http://tetgen.berlios.de/

Verbree, E. (2006). Encoding and decoding of Planar Maps through Conforming Delaunay Triangulations, ISPRS Workshop on Multiple Representation and Interoperability of Spatial Data, Hannover.