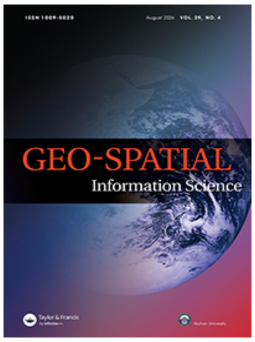

Geo-spatial Information Science

ISSN: 1009-5020 (Print) 1993-5153 (Online) Journal homepage: www.tandfonline.com/journals/tgsi20

Construction method for topological integration of multi-scale datasets supported by a primal-dual data structure

Amin Gholami , Pawel Boguslawski & Martijn Meijers

To cite this article: Amin Gholami , Pawel Boguslawski & Martijn Meijers (17 Sep 2026): Construction method for topological integration of multi-scale datasets supported by a primal-dual data structure, Geo-spatial Information Science, DOI: 10.1080/10095020.2026.2731849

To link to this article: https://doi.org/10.1080/10095020.2026.2731849

Published online: 17 Sep 2026.

GEO-SPATIAL INFORMATION SCIENCE
https://doi.org/10.1080/10095020.2026.2731849

Taylor & Francis
by informa

OPEN ACCESS Check for updates

Construction method for topological integration of multi-scale datasets supported by a primal-dual data structure

Amin Gholami [a], Pawel Boguslawski [a] and Martijn Meijers [b]

[a]Institute of Geodesy and Geoinformatics, University of Environmental and Life Sciences, Wroclaw, Poland; [b]Section Digital Technologies, Faculty of Architecture and the Built Environment, Delft University of Technology, Delft, The Netherlands

ABSTRACT

Spatial datasets covering the same area often exist as separate layers, which differ in scale and source. The spatial relations between matching features located at different layers can be retrieved through a computationally demanding process based on geometry and other properties. When these layers are stored without structural links, they may suffer from redundancy and inconsistent updates, making it difficult to identify spatial relationships across representations. The CityGML concept or national cadastral data, when compared with buildings from OpenStreetMap, often show geometrical and semantic inconsistencies. This study introduces a method for the topological integration of spatial datasets, enabling the construction of a full 3D vario-scale representation from 2D individual multi-scale layers. This approach first generates intermediate shared partitions using a geometric overlay. This partition allows us to detect matching feature components and create topological relationships between them across layers. The integrated model is implemented using a primal–dual data structure, where the primal space stores the geometric integrated representation and the dual space maintains the hierarchical-topological relational structure between the corresponding feature instances. Further optimization introduces deep integration and reduction of geometrical redundancy by removing horizontal faces between feature components and merging vertical coplanar faces and colinear edges. The case studies included a mock and also real datasets: administrative boundaries and independent multisource building footprints available in a national database and open repositories. The resulting structure forms a general foundation for future spatiotemporal data modeling and integration of 3D models into a 4D representation.

ARTICLE HISTORY
Received 11 February 2026
Accepted 6 September 2026

KEYWORDS
Vario-scale representation; multi-scale representation; primal–dual data structure; Dual Half-Edge (DHE); topological integration

1. Introduction

Geospatial datasets covering the same area and spatial objects often exist as independent layers that may include features at different scales. In addition, their geometrical and semantic properties may differ because of different methods of processing the same data (Cheng et al. 2026; Yang et al. 2024) or different data sources as inputs (Oude Elberink and Vosselman 2011). Analyses that combine multiple spatial datasets often rely on computationally expensive geometric operations to identify correspondences between features. A common example is the generalization of a road represented as a face at a large scale into a polyline at a smaller scale. When information must be transferred between these representations, additional computations are required to establish feature correspondence, increasing computational cost and reducing efficiency.

Disconnected layers lead to duplicated geometry and inconsistent updates (Davis and Laender 1999) and obscure how features split, merge or change over time (Buttenfield 1993; Frank and Timpf 1994). Misalignment between scales also requires frequent update propagation, sometimes triggering chain reactions across layers (Ai and Li 2009; Zhang et al. 2018). Integration of datasets may also be beneficial to ensure consistency in cases where updates are introduced at a specific layer without proper propagation of changes to other layers. The levels of detail concept in the CityGML standard (Gröger and Plümer 2012; Kutzner, Chaturvedi, and Kolbe 2020) is an example of individual representations of the same spatial object stored separately. A building removed from one level may remain at another level. Addressing these challenges requires the integration of datasets into a unified structure capable of representing shared geometry and cross-layer relationships. This is essential for maintaining consistency (Steiniger and Weibel 2007) and reducing redundancy, which negatively affects the data management (Karim et al. 2022). The importance of such integration is further amplified by emerging

CONTACT Amin Gholami amin.gholami@upwr.edu.pl

multidimensional data models when the management of space is extended by scale, time, and sometimes version (Paul, Bradley, and Breunig 2013; van Oosterom and Stoter 2010), which requires higher dimensional data structures for modeling.

The proposed work integrates two spatial layers into a coherent 2D + 1D representation, where the additional dimension (+1D) encodes relationships across scale. This is achieved by constructing an intermediate Level of Detail (LoD) through map overlay and feature subdivision, producing a shared spatial partition that enables precise identification of corresponding feature parts and supports the establishment of explicit horizontal (within-layer) and non-horizontal (cross-layer) connections, together forming a continuous topological structure. This integrated framework is implemented using a primal–dual data model, following the principles of the Dual Half-Edge structure (Boguslawski and Gold 2016), which supports both geometric representation and hierarchical relationships. In this study, deep integration refers to the process of merging spatial primitives across multiple scales into unified topological entities, reducing data redundancy while preserving their relationships and semantic traceability.

This study makes three main contributions. First, it introduces a general method for 2D + 1D integration that is applicable to datasets with different scales and sources. Second, it introduces a procedure for generating intermediate LoDs. This step establishes feature correspondence and cross-layer connectivity without relying on a preexisting hierarchical model. Third, it demonstrates how a primal–dual data structure can represent the resulting integrated representation efficiently, supporting analyses such as attribute propagation, change detection, and topological validation, and complementing previous work on multi-dimensional 4D-based models (Ohori et al. 2015).

Conceptually, this study provides a pathway toward a 2D(spatial) + 1D(scale/time) (3D) representation as a step toward a fully integrated 3D(spatial) + 1D(scale/time) (4D) representation. Comparable directions were explored by Ohori et al. (2015), who modeled a 3D city model through multiple levels of detail, van Oosterom and Meijers (2014), who proposed a 4D space scale cube (SSC) connecting two 3D SSCs via trans-scale boundaries, and Boguslawski and Gholami (2024), who advanced toward constructing 4D cell-complex models using DHE-based extrusion processes.

The remainder of this paper is organized as follows. Section 2 reviews the related work on multi-scale representations, as well as vario-scale and topological data structures. Section 3 presents the methodology in detail. Section 4 reports the practical implementation and tests. Section 5 concludes the paper and outlines directions for future work on extending the framework to 3D + 1D and 4D representations.

2. Related work

This section reviews the foundations of vario-scale modeling, the integration of multi-source spatial data, and the topological data structures that support these complex frameworks.

2.1. Multi-scale and vario-scale foundations

Multi-scale representation uses discrete LoDs, each stored as a separate dataset or a generalized map. Traditional structures often duplicate geometry across scales and support only a limited set of discrete intervals, thereby limiting their suitability for continuous generalization (Meijers 2011; Meijers and van Oosterom 2011; van Oosterom 2005; van Oosterom and Meijers 2014). Smooth transitions between scales are essential for modern interactive maps, and achieving seamless and consistent scaling has long been a key challenge in cartography (Gao, Li, and Chen 2020; van Oosterom and Meijers 2014). Vari-scale approaches aim to maintain geometric and semantic consistency as the scale changes (Liu, Jiao, and Liu 2011).

Early work on multiscale topology introduced hierarchical generalized maps supporting 2D, 3D, and 4D representations (Breunig et al. 2007; Thomsen and Breunig 2007). The lifespan model extended this idea by anchoring data at key scales and interpolating intermediate representations to create smoother transitions (Ai and Li 2009). Davis and Laender (1999) argued that traditional GIS architectures, which typically store only a single representation of each geographic object, cannot adequately support scale-dependent requirements and inevitably lead to redundant layers when multiple levels of detail are needed.

Previous studies have highlighted the challenges of managing multiple representations across scales, including data redundancy, synchronization difficulties, and limited cross-scale querying (Buttenfield 1993; Frank and Timpf 1994; Karim et al. 2022). Several approaches have been proposed to address these issues, such as spatio-temporal pyramid models (Qiang and Van de Weghe 2019) and LoD integration using intermediate levels and linking structures (Ohori et al. 2015).

Such frameworks are essential for scaling local 3D models to nationwide coverage while ensuring that map objects are visible across different LoDs (Stoter et al. 2013). When transitioning from higher LoDs to lower LoDs, specifically within the CityGML framework, generalization strategies must be applied to reduce data volume while preserving semantic information (Baig and Abdul-Rahman 2013).

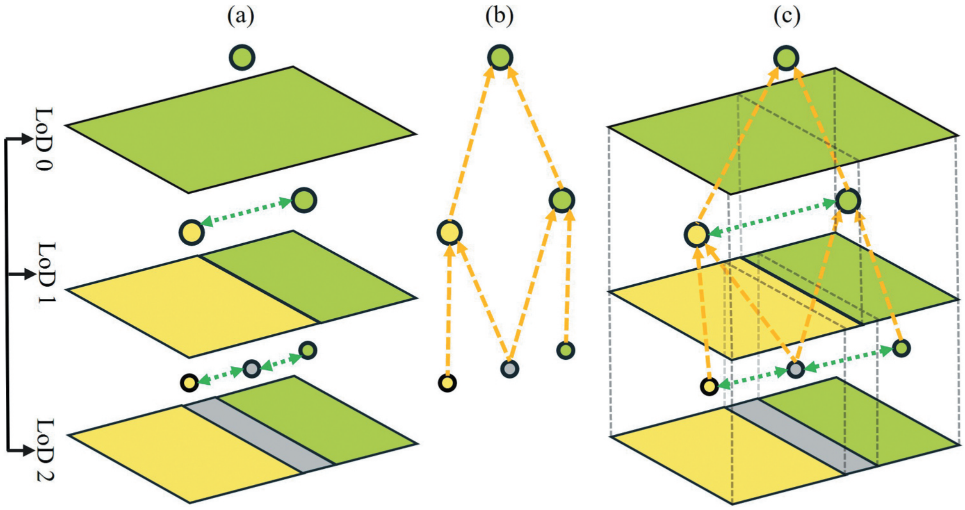

Figure 1. (a) Multi-scale, (b) tGAP, and (c) vario-scale representations. Colors distinguish features; circles mark nodes. Orange/green dashed lines indicate parent–child and topological links, respectively.

Figure 1 illustrates the conceptual transition from discrete multi-scale representations (Figure 1(a)) to a continuous vario-scale structure by topological Generalized Area Partitioning (tGAP) structure (Figure 1(b)). The input consists of three distinct levels of detail (LoDs): LoD2 (the most detailed), LoD1, and LoD0 (the coarse level). The features are represented by specific geometries, where roads, farmland, and forest are depicted as gray, yellow, and green faces, respectively.

As the scale decreases (moving from LoD2 toward LoD0), these features undergo generalization processes – such as merging adjacent faces of the same type or simplifying complex boundaries – to reduce data density while maintaining topological consistency (Meijers and van Oosterom 2011; van Oosterom 2005; van Oosterom and Meijers 2014). Specifically, Figure 1 demonstrates how the integration of the tGAP framework allows these discrete levels to be linked into a unified vario-scale representation.

In this framework, the most detailed geometry from LoD2 serves as the base layer. Each subsequent generalization step required to reach LoD1 and LoD0 is recorded in a tree structure that maintains the adjacency and inclusion relationships between the features (Dilo, van Oosterom, and Hofman 2009; van Oosterom 2005). This process transforms independent, redundant layers into a single, integrated dataset that supports smooth zooming and avoids the storage of multiple independent versions of the same geographic area (Bernard Marinus Meijers 2011; van Oosterom and Meijers 2014).

2.2. Integration and map-overlay techniques

Integrating space and scale, the SSC represents generalization by extruding 2D and 3D objects into higher-dimensional structures (Meijers and van Oosterom 2011). Production of this model provides consistency across scales, supports continuous zooming, and allows mixed-scale maps (Bernard Marinus Meijers 2011; van Oosterom and Meijers 2014).

Complementary to SSC, the tGAP data structure stores geometry at the most detailed level and generalizes it progressively for coarser scales (De Vries and van Oosterom 2007; van Oosterom 2005). The tGAP employs a hierarchical topological structure that supports efficient, progressive data transfer and web-based applications. Constrained tGAP has also been used to integrate medium-scale generalized representations with accurate large-scale geometry (Dilo, van Oosterom, and Hofman 2009).

SSC has been developed in two main variants: classic and smooth (van Oosterom and Meijers 2014). The classic SSC is an integrated multi-scale representation which represents discrete changes at threshold scales as horizontal slices, while the smooth SSC replaces these slices with sloped planes, allowing continuous transitions (van Oosterom and Meijers 2014). A feature denotes an individual spatial object defined by its geometry and attributes. In contrast, a class represents a group of similar features sharing the same type and attribute schema, such as roads, buildings, or forests. However, a feature's 2D + 1D representation can be expressed as a vertex (0D), edge (1D), face (2D), or cell (3D).

Steiniger and Weibel (2007) emphasize that effective map generalization requires explicit modeling of relations among map objects, distinguishing between horizontal and vertical relations. In Figure 1, these horizontal and vertical relations are shown with green and orange lines, respectively.

The fundamental logic for integrating heterogeneous spatial layers often relies on map overlay processes. In a conventional map overlay, the input consists of two or more topologically structured layers, and the output is a new layer where newly generated areas are attributed based on overlapping input features (Frank 1987; van Oosterom 1994). These overlays can be computed by intersecting point, line, and polygon features using the 9-Intersection Model (9IM) (Egenhofer and Herring 1990). In the present context, however, the overlay is adapted for the non-horizontal integration step.

Furthermore, high-dimensional topological invariants, such as the "connective degree," are used to maintain structural integrity during feature subdivision (Deng et al. 2007). This integration is increasingly supported by semantic knowledge graphs that bridge the gap between relational schemas and domain ontologies (Ding et al. 2025), as well as "Generalized Scale" logic for ranking feature importance in urban patterns (Niu and Cheng 2012), providing a foundation for multiway hierarchical structures that reduce geometric redundancy while maintaining the Gestalt nature (Ai, Wang, and Liu 2002) of urban building patterns.

2.3. Topological data structures and the dual half-edge

Effective vario-scale modeling requires data structures that can manage geometry, topology, and scale within a unified framework. Recent research has focused on implementing SSC and tGAP directly with the DHE (Dual Half-Edge) structure (Gholami et al. 2024; Karim et al. 2017). In the DHE structure, features are represented through topologically connected elements organized in primal and dual spaces based on 3D Poincaré duality (Boguslawski and Gold 2016).

Once the primal space is constructed from atomic elements, the dual space is generated automatically, enabling efficient navigation and evaluation of topological relationships (Boguslawski and Gold 2016). The connection between the two spaces is established through dual–primal pointers linking corresponding half-edges (Figure 2).

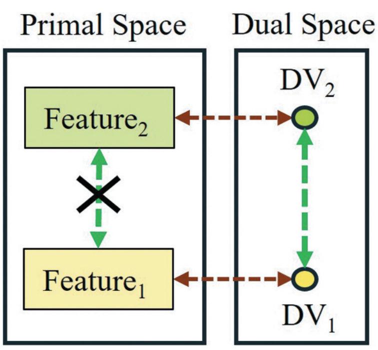

Figure 2. Navigation between two adjacent features via their corresponding dual vertices ($DV_{1 \text{ and } 2}$) in a cell complex; dashed brown and green arrows are *Dual* and *NextFace* pointers, respectively.

Adjacency between features sharing a vertex, edge, or face is represented in dual space rather than directly in the primal space through operators such as *JoinByVertex*, *JoinByEdge*, and *JoinByFace* (Boguslawski and Gold 2016). In this representation, dual vertices (DVs) correspond to cells, and loops of dual half-edges form dual faces, allowing traversal using the *NextFace* operator. The navigation between dual half-edges defines the dual connection represents adjacency in the DHE structure. Each dual vertex also serves as a semantic container, storing attributes such as identifiers and class information.

The DHE structure includes four primitives: vertex (0D), edge (1D), face (2D), and cell (3D) (see Figure 3(a–c)). An external cell is always present, enclosing all internal cells and ensuring topological consistency by providing valid adjacency for boundary elements. The dual vertex corresponding to the external cell is referred to as EDV in this paper. Two cells can be connected using the *JoinByFace* operator (Figure 3(d, e), while further merging of same-class cells (Figure 3(f,g)) is achieved through the *MergeByFace* operator, reducing redundancy and optimizing the structure.

Compared with G-Maps, DHE introduces duality and construction operators that G-Maps lack (Boguslawski and Gold 2016; Lienhardt 1989). Unlike compact half-edge or array-based structures that emphasize minimal memory (Alumbaugh and Jiao 2005; Salinas-Fernández, Fuentes-Sepúlveda, and Hitschfeld-Kahler 2024), DHE prioritizes completeness and adaptability. It also incorporates concepts from *Quad-Arc* and *Quad-Edge*, strengthening its ability to represent cell complexes and adjacency (Anton, Boguslawski, and Mioc 2014; Gold 2010). DHE enables reliable vario-scale representation by maintaining topological consistency and supporting incremental updates (Gholami et al. 2024, 2025). Beyond geometry, it manages semantic attributes through dual vertices. Its ability to reconstruct SSC and balance efficiency with adaptability makes it ideal for multiscale frameworks (Karim et al. 2017).

2.4. Research gap

Despite advancements in multiple representation management, significant challenges remain regarding data redundancy, synchronization, and limited cross-scale querying (Buttenfield 1993; Frank and Timpf 1994; Karim et al. 2022). Spatiotemporal pyramid models and LoD integration using linking structures have been proposed to address these issues (Ohori et al. 2015; Qiang and Van de Weghe 2019). However, existing approaches such as SSC and tGAP often rely on predefined hierarchical structures to maintain consistency. There is a lack of research focusing on a data-driven approach that can integrate independent 2D layers into a unified

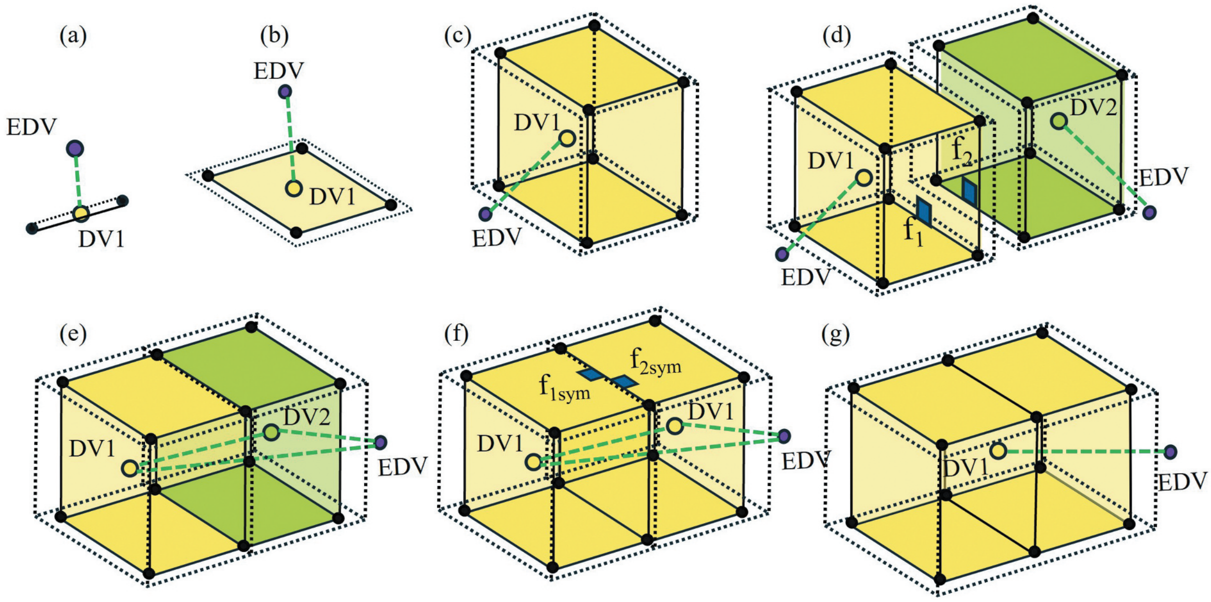

Figure 3. DHE construction operations: make (a) an edge; (b) a face; (c) a cell; (d) two separate cells; join two adjacent cells by a face (e) of a different class (f) of the same class; (g) merge two adjacent cells of the same class by a face; solid and dotted lines represent internal and external cells respectively, while a green line represents is dual connections between two DVs.

2D + 1D topological structure without requiring prior structural alignment or parent-child relationships. This paper addresses this gap by proposing a methodology that utilizes the DHE structure to automate the integration and attribute propagation between disparate, multi-source datasets also intended for different scales.

In addition, handling inconsistencies, redundancy, and misalignment between heterogeneous datasets remains a challenge. Existing solutions typically address these issues at the geometric level, without fully integrating topology and scale into a unified framework.

3. Methodology

This study employs multi-scale data to demonstrate the core concepts and workflow for transforming the multi-scale representation into a (deeply) integrated multi-scale representation using the Classic SSC and the DHE structure. While the focus is on scale-based generalization, the same framework can be extended to other ordered dimensions, such as time or thematic resolution or other hierarchical multiple layers, such as administrative boundary layers.

The paper categorizes topological integrations between features in a vario-scale representation into three types:

- **Intra-layer** Topological integrations (cross-feature or horizontal connections) link features within the same LoD (for example, two adjacent faces sharing a common boundary).
- **Inter-layer** Topological integrations (cross-scale or non-horizontal connections) link features across different LoDs (for instance, when several detailed faces at a larger scale are aggregated into a single feature at a smaller scale).
- **Compound** Topological integrations (intra- and inter-layer connections) occur when two features are connected both within the same LoD and across different LoDs (for example, two adjacent features at one level that merge into a single feature at an upper or lower LoD).

The primary objective of this methodology is to transform independent, multi-source spatial layers into a single, topologically consistent 2D + 1D vario-scale representation. Unlike traditional approaches that rely on preexisting hierarchical structures or parent-child relationships (van Oosterom 2005; van Oosterom and Meijers 2014). The proposed method generates structural links directly from the geometric and semantic properties of the input data. This is achieved by constructing a shared spatial partition at an intermediate level of detail, which serves as a topological bridge between the lower-bound and upper-bound layers.

3.1. Overview of the integration framework

The overall workflow is the logical progression from raw multi-scale input to a deeply integrated 2D + 1D model. The methodology is divided into three primary phases:

The first phase, preprocessing and extrusion, transforms input features into volumetric cells within the DHE framework. In the extrusion process, each 2D feature is extruded along the scale dimension to create 3D volumetric cells (Boguslawski and Gold 2016; Gholami et al. 2024). This extrusion effectively represents the "lifespan" of a feature across the space-scale cube (Ai and Li 2009; Meijers and van Oosterom 2011).

The second phase performs topological integration through map-overlay operations that generate an intermediate LoD and establish intra-layer and inter-layer connections. To establish connections between independent layers, an intermediate LoD is generated. This phase involves a map-overlay process where the geometries of the lower bound (LB) and upper bound (UB) layers are intersected using the 9-Intersection Model (9IM) (Egenhofer and Herring 1990; Frank 1987). This intersection creates a shared partition that aligns the geometries of both layers, enabling the identification of corresponding feature parts and the establishment of intra-layer (horizontal) and inter-layer (non-horizontal) topological links.

The third phase focuses on optimization and deep integration, reducing geometric redundancy through merging operations while preserving topological consistency. By applying DHE operators such as *MergeByFace*, redundant horizontal faces between features of the same class are removed (Boguslawski and Gold 2016; Gholami et al. 2024). This optimization transforms the initial integrated model into a "deeply integrated" representation, which minimizes storage requirements while maintaining a unified navigation framework for spatial analysis.

The first and second phases generate an integrated representation, while the third phase, if applied, produces a deeply integrated representation for further optimization. Figure 4 illustrates the first and second phases for constructing an integrated representation of 2D geographic data along an additional dimension, without requiring a preexisting hierarchical structure that defines how features are generalized across scales.

3.2. Generation of intermediate LoDs

To integrate two LoDs that lack parent–child relationships, two intermediate LoDs are created between the lower bound (LB) and upper bound (UB) layers. These intermediate layers share identical geometry but may differ semantically, forming a common partition that supports cross-layer correspondence. For each LoD interval, the method generates an integrated representation through first and second phases.

Notably, the test dataset used in this study corresponds to the multi-scale dataset illustrated in Figure 1, where the yellow, green and gray polygons represent farmland, forest, and road features, respectively. Therefore, the methodology in this paper focuses on how to construct a integrated multi-scale representation – that is an integrated representation – from an initially separate multi-scale dataset. The construction workflow for the integrated multi-scale representation of two LoDs (see Figure 5(a)) using the DHE structure and the 2D + 1D representation is illustrated in Figure 5.

An intermediate LoD will be added to the layers (see LoD1.5 in Figure 5(b)) to find the proper inter-layer topological integration between features across layers in three steps: Extrusion, establishing the intra-layer topological integration, and map-overlay. The feature extrusion generates a 3D representation, as a result, the extrusion of a polygon, line, and point features are 3D, 2D, and 1D cell primitives, respectively.

Using the DHE construction operators, each face representing an individual feature is extruded to a 3D cell by duplicating the face, shifting to the LoD1.5 level. Then connect the corresponding half-edges from the original and extruded faces by new vertical edges (see gray lines in Figure 5(b)). In the next step the intra-layer topological integrations between features (now represented as a 3D volume) are established in case they share a vertical face.

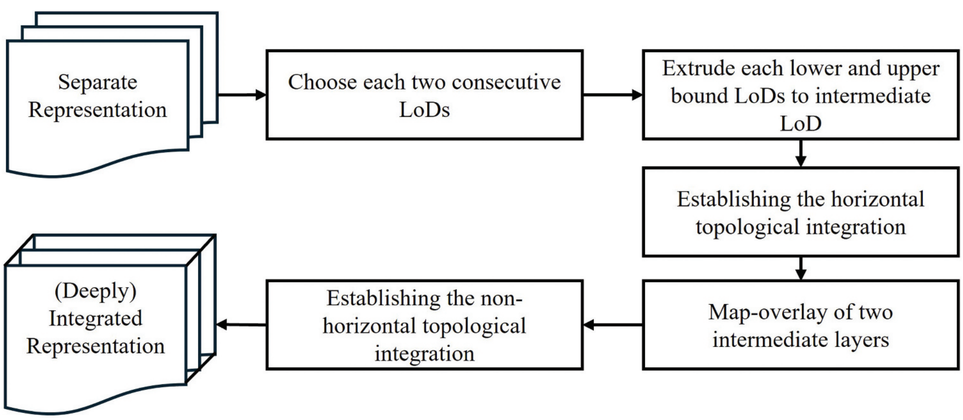

Figure 4. Proposed methodology for constructing the integrated representation of 2D geographic data along 1 dimension.

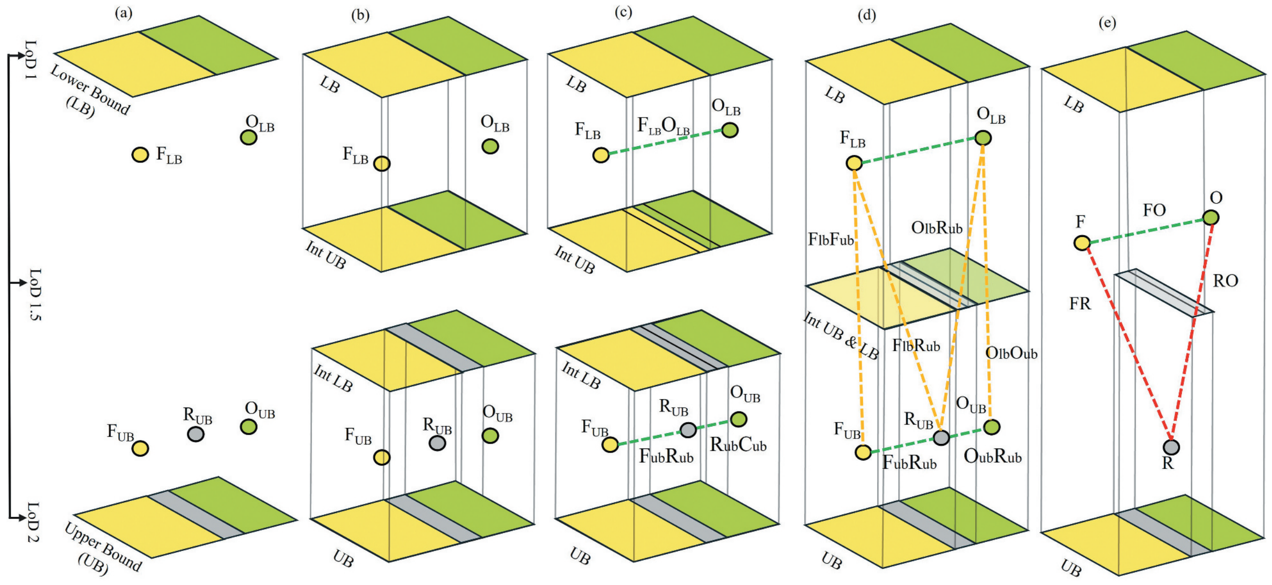

Figure 5. Workflow for constructing the vario-scale representation of two LoDs using the DHE structure and the 2D + 1D integrated representation: (a) construction of a horizontal face for each feature in both layers by DHE; (b) extrude each horizontal face to the intermediate LoD to form SSC; (c) establishing the intra-layer topological integration and map-overlay; (d) establishing the inter-layer topological integration; (e) deeply integrated representation by removing horizontal intermediate faces with the same class; green, orange and red dashed lines are intra-layer and inter-layer and compound topological integrations, respectively.

3.3. Establishment of topological integrations

For intra-layer topological integration, the *JoinByFace* operator is applied. As a result, dual connections between features are created (e.g. see green dashed lines between F_{LB} and O_{LB} in Figure 5(c)). In Figure 5 the [intermediate LB, UB] LoD interval, two adjacent relationships exist, which are defined by the shared vertical faces between the road cell and the farmland and forest cells. Similarly, in the [LB, intermediate UB] LoD interval, one adjacent relationship is established through a shared vertical face between the farmland and forest cells. After all intra-layer topological integrations are established separately for the [LB, intermediate UB] and [intermediate LB, UB] LoD intervals, a map-overlay process is applied to identify the corresponding topological integrations.

Before inter-layer connections are introduced between the intermediate LB and intermediate UB, the layouts of the intermediate LB and intermediate UB must be the same to find the shared vertices, edges, or faces through generalization transition. These two intermediate layers are overlaid – the intersection of features is calculated using the 9IM. New vertices and half-edges are introduced in such a way that features from both layers become subdivided. Then these two layers are geometrically aligned, although their attributes (e.g. feature identity, classification, or level of generalization) may remain distinct. This map-overlay and establishing inter-layer allocation process is illustrated in Figure 6.

In Figure 6, the dual-primal connections are visualized using color-coded non-horizontal lines that

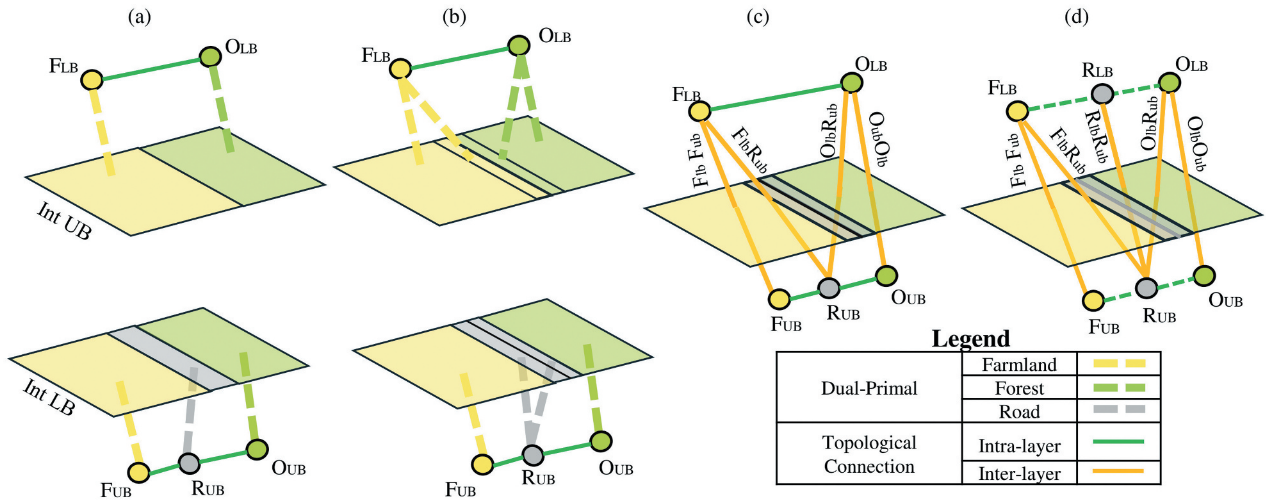

Figure 6. Map-overlay process. (a) Initial layers. (b) Layers after projection and subdivision. (c–d) Establishment of inter-layer topology for partitions without (c) and with (d) a collapsed road feature. Green horizontal lines indicate intra-layer links; multicolored and orange lines indicate dual-primal and inter-layer integrations.

correspond to each feature, illustrating the dual-primal connection between each face and its associated dual vertices. This dual-primal connection is illustrated only in Figure 6(a,b) to ensure the map-overlay step and inter-Layer allocation processes are well-understood.

In Figure 6, only the intermediate horizontal faces of features are depicted at each step of the process. In Figure 6(b), the feature subdivision step is performed. For example, the road face (highlighted in gray) in the intermediate LB is subdivided into two parts. Each resulting face corresponds to the projection of the farmland and forest features from the intermediate UB layer onto the original road face.

Conversely, the projection of the road face onto the farmland and forest polygons in the intermediate UB is also performed. This mutual projection results in two geometrically identical road segments represented in both intermediate layers. While they share the same spatial geometry, their semantic attributes remain distinct. For example, as illustrated in Figure 6(b), the road face in the two intermediate LoDs (intermediate LB and intermediate UB) are geometrically coincident after subdivision. However, each subdivided portion of the road face will connect in next section with a dual vertex in its corresponding intermediate layer (see Figure 6(c)).

Consider the case where the road feature remains present and is collapsed between the forest and farmland features at LoD1 as a line segment feature. Figure 6 illustrates how this configuration appears from the perspective of non-horizontal (cross-scale) topological integrations. In this situation, the union and overlay of the road features at LoD2 with the forest, road, and farmland features at LoD1 result in a subdivision of the road face into its overlapping regions. The outcome consists of two subdivided faces corresponding to the farmland and forest features at LoD1, and a single edge representing the road feature at LoD1.

Once feature subdivision through map overlay is completed, cross-scale (non-horizontal) topological integrations can be established. These connections link corresponding feature partitions across scales, ensuring proper allocation of features among consecutive LoDs. The connections are created using the *JoinByFace* operator when two adjacent horizontal faces are geometrically equal (see the shared horizontal faces in Figure 6(c,d)). When two adjacent edges are geometrically equal (see the shared edge in Figure 6(d)) we use the *JoinByEdge* operator.

It should be noted that after joining the intermediate LB and intermediate UB layers the corresponding cells become one cell complex, where matched features are linked by dual edges (see Figure 5(d)), it is called the *integrated representation*.

The resulting 2D + 1D model introduces additional entities, e.g. new vertices, edges, faces, and cells. The storage size is then bigger than in the case of the original multi-scale representation. The model can be optimized by reducing data redundancy, through the removal of shared horizontal faces of the same class. The *MergeByFace* operator is applied on the shared horizontal faces that have the same class (here, same dual vertex ID), which will remove these faces, and two cells will become one with a single dual vertex. For instance, the O_{LB} cell from the LB LoD and the O_{UB} cell from the UB LoD are merged into a single cell represented by the dual green vertex in Figure 5(e).

3.4. Deep integration and optimization

To further reduce data redundancy, after removing the redundant horizontal faces, each associated half-edge (e_i) within those faces can also be deleted, along with its symmetric half-edge (e_i.sym) and the corresponding start and end primal vertices (e_i.vertex and e_i.sym.vertex). This operation minimizes storage requirements by eliminating unused geometric elements (vertex, edge, or face) while preserving the overall topological integrity of the DHE structure. At this stage, after reducing data redundancy, the resulting integrated multi-scale representation can be regarded as the *deeply integrated representation*.

In the merging process the inter-layer and intra-layer links between two dual vertices of the same class will be merged into one link. For instance, O_{UB} and O_{LB} have both inter- and intra-layer topological integrations with another cell before merging (e.g. R_B in Figure 5(d)). After merging, their inter- and intra-layer topological integration merges into one compound topological integration (*RO* red line in Figure 5(e)). On the other hand, there will be disadvantages of removing the horizontal faces of the same class, e.g. by merging two cells, some important attribute will be lost during merging.

During the merging process, the inter-layer and intra-layer topological integrations between two dual vertices of the same class are combined into a single compound topological integration. For example, prior to merging, in Figure 5 the $O_{LB}R_{UB}$ and $O_{UB}R_{UB}$ (inter-layer and intra-layer topological integrations, respectively) are unified into one compound topological integration (the *RO* in Figure 5(e)).

3.5. Non-horizontal integration of two consecutive LoD intervals

The methodology described earlier focused on constructing the integrated multi-scale representation for two LoDs at a time, producing either a deeply or not-deeply integrated representation. In this subsection, we examine how multiple integrated multi-scale models can be integrated vertically through the levels of detail.

For the vertical integration process, the multi-scale representation shown in Figure 1 was selected, comprising three LoDs. These are organized into two LoD intervals: [0, 1] and [1, 2] (Figure 7). For each LoD interval, its UB layer and LB layer are treated as the corresponding multi-scale layers (see Figure 7(a)). Consequently, LoD1 appears twice – once as a UB LoD in the [0, 1] LoD interval and once as a LB LoD in the [1, 2] LoD interval – resulting in two dual vertices for each feature at LoD1 (Figure 7(a–c)). After constructing the integrated representation for each LoD interval individually (Figure 7(b)), these representations can be integrated vertically through their shared LoD – in this case, LoD1.

Vertical (non-horizontal) integration and is also used for creating non-horizontal topological linkages between two integrated multi-scale representations. These representations must cover consecutive LoD intervals and share one common LoD. Vertical integration is applied where they also share horizontal faces of the same class at either the LB or the UB LoD. For example, in Figure 6, two integrated representations for the [1, 1.5] and [1.5, 2] LoD intervals are connected and glued through their shared LoD at 1.5, producing a single integrated model.

To integrate the integrated multi-scale for the [0, 1] LoD interval and the integrated multi-scale for the [1, 2] LoD interval, the *JoinByFace* operator in DHE is applied to all faces in its common LoD1. This produces an integrated multi-scale representation for [0, 2] LoD interval. The result shown in Figure 7 is a unified integrated multi-scale representation for the [0, 2] LoD interval. This vertical integration process is carried out by establishing the inter-layer topological integration between individual representations.

In the DHE structure, dual vertices can store attribute and semantic information through pointer-based representations, which is particularly useful in this context. For example, each dual vertex can hold an attribute such as the "LoD interval," which can be updated dynamically whenever a new inter-layer topological integration is established.

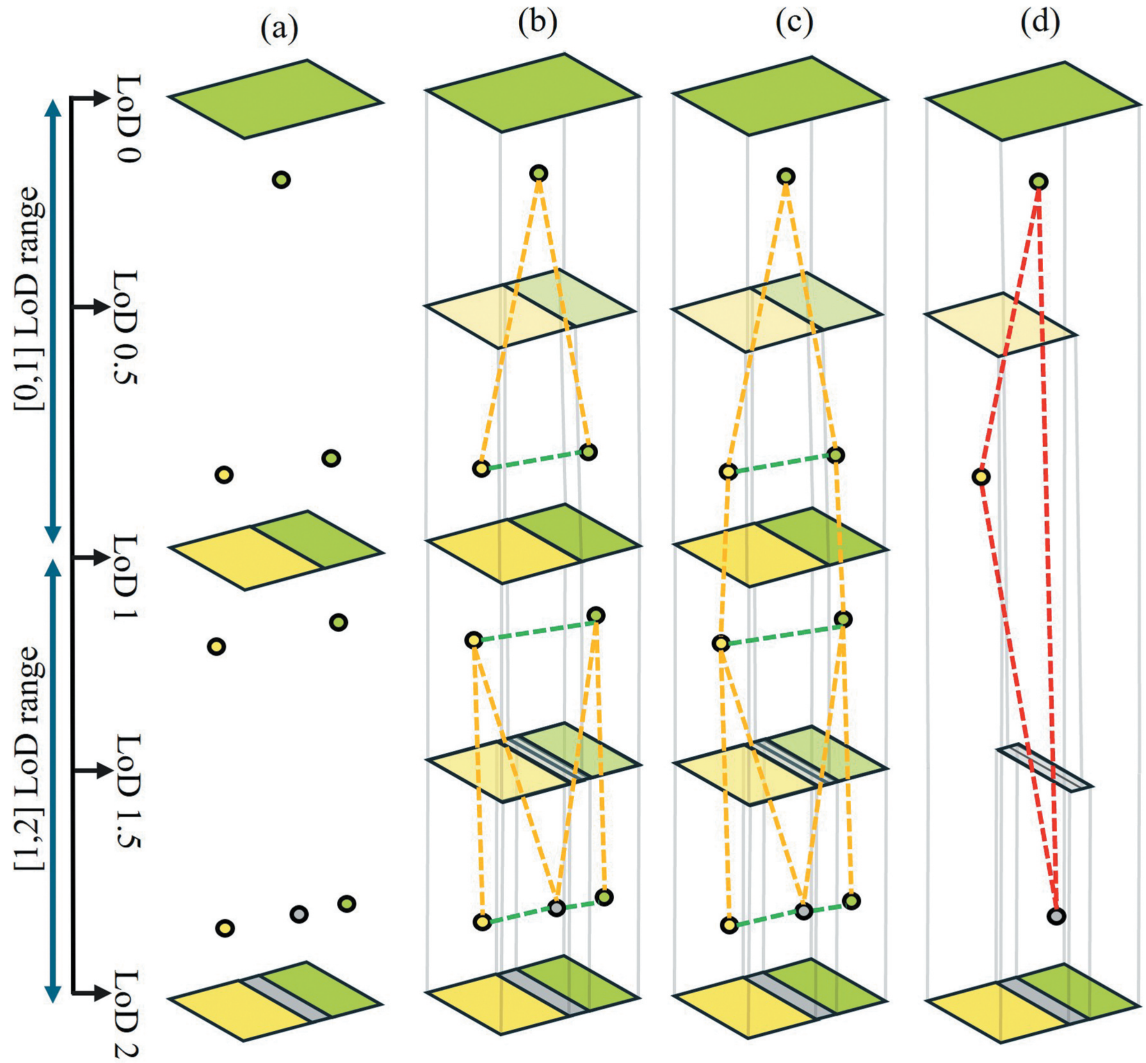

Figure 7. Vertical integration between vario-scale representations with consecutive LoD intervals; (a) multi-scale representation (b) individual vario-scale model for each interval, (c) integrated representation, (d) removing the shared vertex, edge and faces to acquire the deeply integrated representation.

It should be noted that, there are two adjacent surfaces on an intermediate LoD (see Figure 6) – the upward surface belongs to the lower LoD range, and the downward surface belongs to the upper LoD interval. For instance, there are upward surfaces at LoD0.5 that belong to the forest feature, and downward surface belong to the farmland feature (see Figure 7(d)).

3.6. Horizontal integration of line and point features into the model

The method also supports lines and points (Figures 8–9). The extrusion of a line produces a vertical face with a 2D volume, whereas in the DHE structure, it generates two joined internal faces and two joined external faces (Figure 8(a)) which act as a 2D volume primitive. Similarly, the extrusion of a point forms a vertical edge (a 1D volume primitive), but in the DHE structure, it creates two symmetric internal half-edges and two symmetric external half-edges (Figure 8).

In the case of adjacent relationships for an extruded line feature (Figure 8(a,b)), a line feature may be positioned between the boundaries of two adjacent features. For example, if a road face in LoD2 collapses into a road line situated between forest and farmland features in LoD1, the resulting vertical road face (represented by two symmetric faces in the DHE structure) lies between the forest and farmland cells (Figure 8(a)). After applying the *JoinByFace* operator to each side of this shared vertical face – linking it with the corresponding forest and farmland cells – an adjacent relationship is established between them (Figure 8(b)).

Similarly, for adjacent relationships involving an extruded point feature (Figure 8(c,d)), a point feature may also occur between two feature boundaries. For instance, if a shared vertex of the forest and farmland features is represented as an independent object (e.g. a shaft point), its extrusion forms a vertical shaft edge (represented by two symmetric half-edges in the DHE structure) located between the forest and farmland cells (Figure 8). After applying the *JoinByEdge* operator to each half-edge of this shared vertical edge – linking it to the corresponding half-edges of the forest and farmland features – the adjacent relationship between them is established (Figure 8).

The process described for establishing adjacent relationships between a line or point feature and a single polygon feature is identical to that used for two polygon features. The following section describes the containment (within) relationships where a polygon, line, or point features (see Figure 9(b–d)) is enclosed by a polygon feature (see Figure 9(a)).

In case of a feature containing another feature, the containment/within relationship should be arranged. This relationship can be decomposed into some adjacent relationships by adding a bridge edge, face or cell in DHE structure (Boguslawski and Gold 2016) in order to connect inner and outer boundaries. In this context a bridge cell with a zero volume is applied to connect the outer boundary shell to inner boundary shell as shown in Figure 9(a), where purple bolded cells are bridge cells, and the inner boundary shell (the

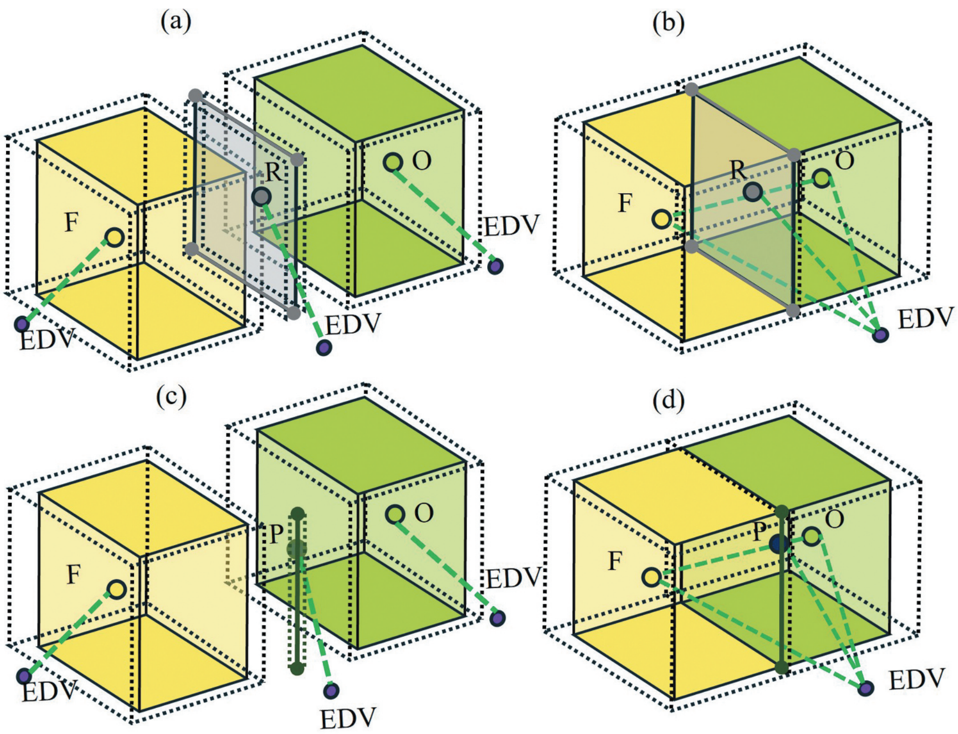

Figure 8. Adjacent relationships for extruded lines and points: a vertical face or edge lies between two cells before (a) and after (b) establishing adjacency.

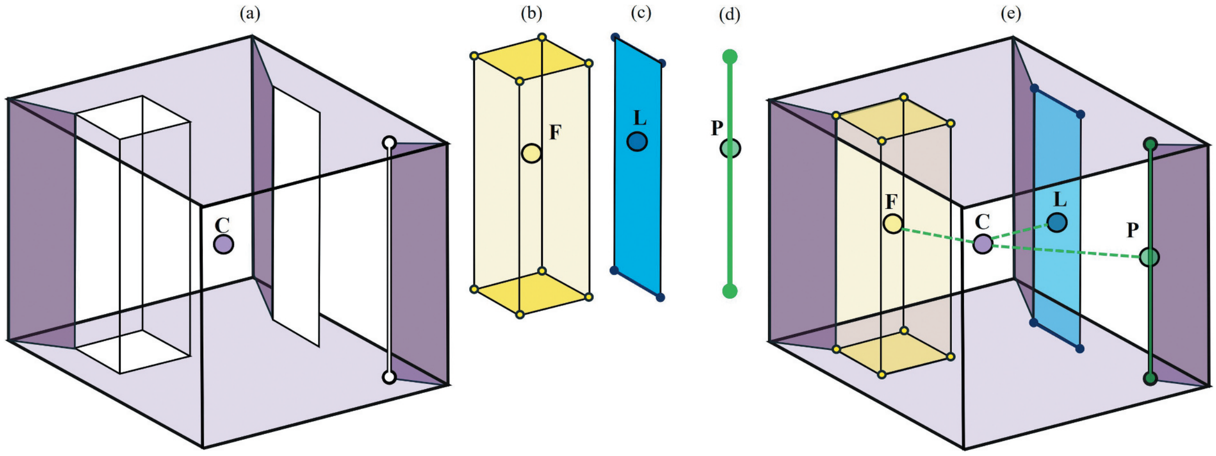

Figure 9. Containment relationships before (a–d) and after (e) integration. Bold purple elements indicate zero-volume bridge cells.

white bolded shells) of each feature is considered as hole inside the container cell. However, after filling these holes with corresponding features, their topological relationships should be applied by establishing the adjacent relationship between the shared vertical faces and edges of container features with other features (Figure 9(e)).

4. Results and discussion

This section evaluates the proposed 2D + 1D integration framework using three datasets: (1) a test multi-scale example, (2) administrative boundaries, and (3) multi-source data.

(a) **Test Multi-Scale Dataset**: A controlled dataset with three LoDs (0–2) was created to illustrate the method.
(b) **Administrative Boundaries of Poland**: The method was applied to hierarchical administrative boundaries of Poland – national (LoD0), voivodeship (LoD1), county (LoD2), and municipality (LoD3). The experiment focuses on LoD1 and LoD2 (see Figure 10(a)) for the Lower Silesian Voivodeship, with a simplified

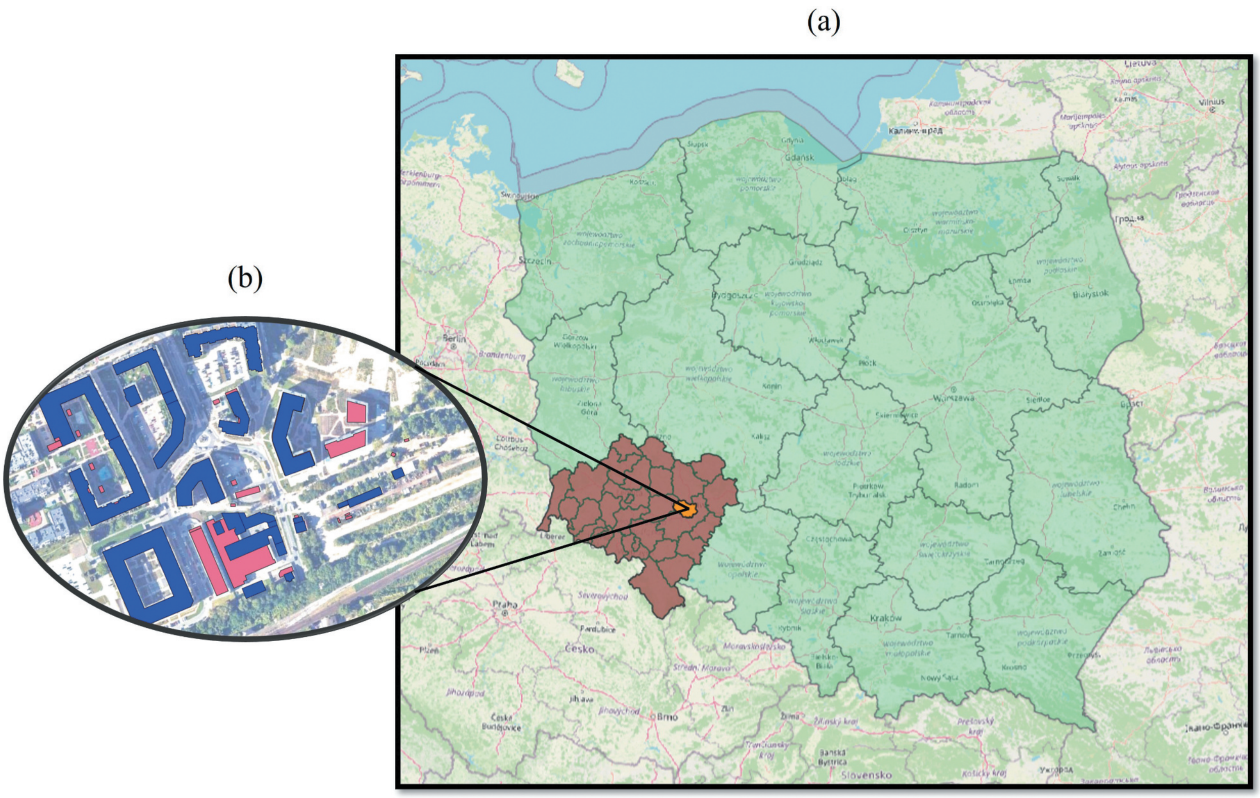

Figure 10. Case studies: (a) administrative boundaries of Poland – green and brown faces show LoD1 and LoD2; the orange face marks Wrocław. (b) Building footprints for the Kleczków district – BDOT[1] shown in pink-red and OSM in purple-blue.

LoD0 included to demonstrate integration in cases where outer boundaries are not perfectly aligned.

(c) **Multi-Source Building Footprints**: This case study integrates building footprints from two independent sources – BDOT10k and OpenStreetMap – for the Kleczków district in Wrocław (Figure 10(b)). Both datasets represent LoD0 but differ in geometry, completeness, and positional accuracy.

4.1. Analysis of deep integration

The progressive transformation of three separate spatial layers into a unified integrated multi-scale representation is demonstrated on Figure 11. The sequence shows how horizontal integrations are first established within each layer (see Figure 11(b)), how non-horizontal integrations are then formed across layers using the shared intermediate partition (see Figure 11(c)), and how redundant geometry primitives (faces, half-edges and vertices) are incrementally removed through vertical merging and optimization (see Figure 11(d)).

To highlight the differences between each stage in Figure 11 from a data-storage perspective, Table 1 reports the number of DHE primitives produced at each step of the workflow. As introduced earlier, horizontal and non-horizontal (vertical) integrations are established through join operations applied to shared primitives between two features.

As expected, once two cells are joined in the DHE structure, their external cells are merged into a single one, resulting in a cell complex comprising of two internal cells and one external cell. During this process, some external half-edges and faces are removed, while the internal primitives remain intact. Furthermore, after redundant shared faces, half-edges, and vertices are eliminated in the optimization stage, the number of internal and external primitives decreases substantially. This progression is clearly reflected in the values shown in Table 1. Consequently, when stored in a database, the integrated representation requires significantly more

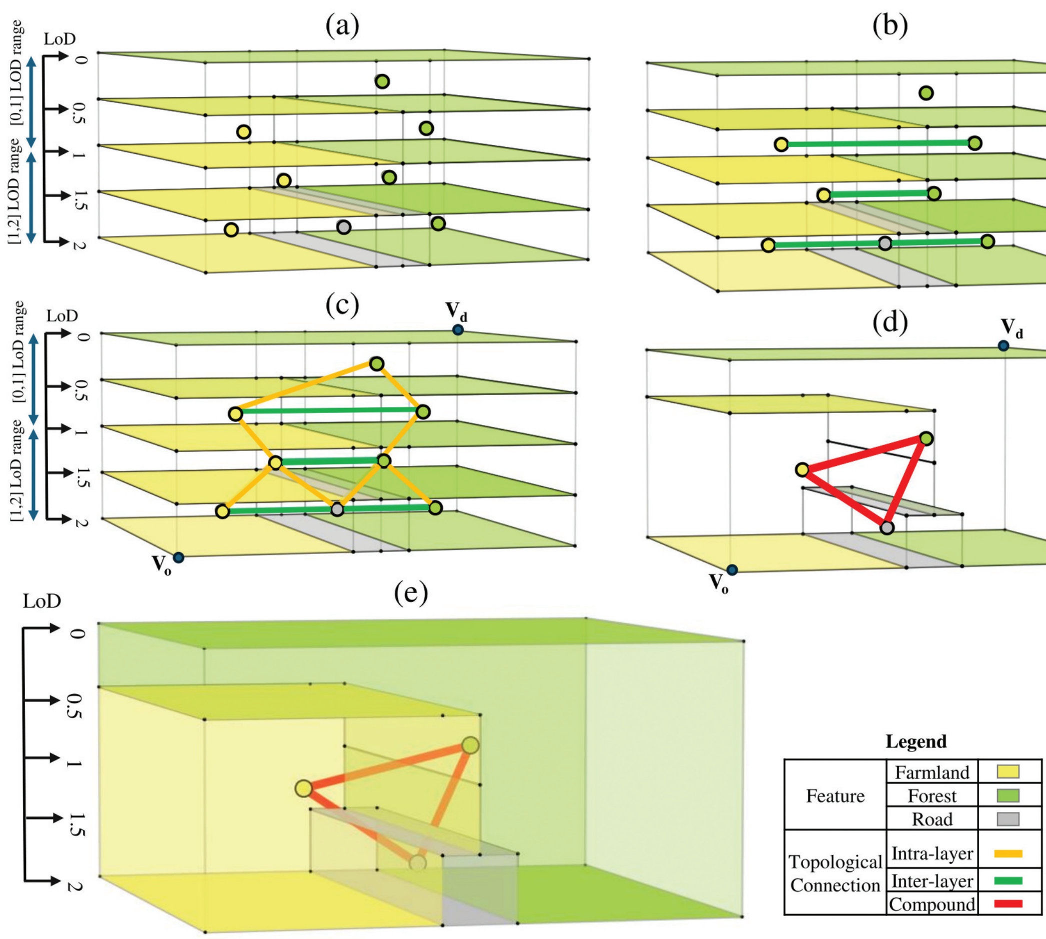

Figure 11. Implementation of the test dataset; (a) Initial 2D + 1D representation before establishing horizontal and non-horizontal topological integrations; (b) after establishing horizontal (within-layer) topological integration; (c) after establishing non-horizontal (cross-layer) topological integrations to generate integrated representation; (d) deeply integrated representation through the removal of redundant shared faces, half-edges and vertices; (e) volumetric view of the resulting 2D + 1D representation corresponding to (d). In subfigures (a)–(d), vertical half-edges and faces are depicted as light-gray lines and uncolored faces.

Table 1. Number of DHE primitives in the test dataset at each stage of the integration workflow shown in Figure 11. The table reports the number of internal and external half-edges, faces, and cells after each stage shown in Figure 11(a–e)). The progressive reduction in primitives reflects the consolidation of shared geometry and the removal of redundant topological elements.

Number of	Stage Type	Initial 3D representation (Figure 11(a))	Horizontal integration (Figure 11(b))	Vertical integration (Figure 11)	Deep integration (Figure 11 (d,e))
Half-Edge	Internal	576	576	576	272
	External	576	512	352	152
	Sum	1152	1088	928	424
Face	Internal	66	66	66	27
	External	66	58	42	13
	Sum	132	124	108	40
Cell	Internal	8	8	8	3
	External	8	4	1	1
	Sum	16	12	9	4

storage space and involves more complex cell navigation will be involved.

Furthermore, the integrated representation contains a greater number of topological integrations, including intra-layer, inter-layer, and compound types. As a result, cell navigation in the integrated structure often requires longer and more intricate traversal paths compared to the deeply integrated version. For instance, to evaluate navigation complexity, two vertices (V_0 and V_d) were selected as the origin and destination (see Figure 11(c,d)), and the shortest path between them was computed for both representations. In the non-deeply integrated structure, the traversal required 27 DHE navigational operations, whereas in the deeply integrated representation, only 9 operations were needed. This demonstrates a significant reduction in traversal complexity achieved through deep integration.

However, the position of the intermediate LoD can vary depending on the relative importance assigned to the less detailed (LB LoD) and more detailed (UB LoD) levels in the integrated multi-scale representation. In this paper, both levels are considered equally important, so the intermediate LoD is set to the average of the LB and UB LoD values. Because each LoD in the [0, 2] interval is extruded to intermediate LoDs during processing, the feature cells for a given LoD are extruded twice; once to a half-step lower LoD and once to a half-step upper LoD. For example, for LoD 1, the feature cells are extruded into the [1, 1.5] interval and again into the [0.5,1] interval.

Once the inter-layer topological integration is established, it modifies the recorded lifespans of some features. For instance, the farmland feature extends its lifespan from LoD1 to LoD0.5, and the road feature extends from LoD2 to LoD1.5 (see Figure 11(a–c)).

To preserve semantic integrity during deep integration, an attribute propagation mechanism is introduced. Each merged cell maintains references to its originating primitives, allowing attributes to be inherited and aggregated rather than discarded. In cases of conflicting attributes, aggregation rules (e.g. majority voting or union) or multi-valued storage are applied. This ensures that all original semantic information remains traceable within the integrated structure.

4.2. Real-world datasets and models

The input data and integrated representation are shown on Figure 12. However, only LoD1 and LoD2 of the Lower Silesian Voivodeship in southwestern Poland are used, as including all LoDs and provinces would make the figure overly complex. Additionally, a simplified version of LoD1 is introduced as LoD0 to account for cases where the outer boundaries of consecutive LoD layers are not perfectly aligned (Figure 12(a)).

The map overlay of the two administrative boundary layers (LoD1 and LoD2) produces a common intermediate LoD, allowing the construction of the corresponding integrated representation despite the significant gap in levels of detail between them. Similarly, the map overlay of LoD0 and LoD1 produces another common LoD, which helps identify the shared horizontal faces between them, even when their outer boundaries are not spatially aligned. Once the [0,1] and [1,2] LoD intervals are constructed, they can be vertically integrated via their common LoD1, even though they originate from different separate representations (see Figure 12(b,c).

The performance of the integrated model was evaluated by quantifying the Dual Half-Edge primitives and calculating storage requirements based on the pointer-based DHE structure (Boguslawski and Gold 2016). While the initial 3D representation required 10 pointers per DHE element, the establishment of topological linkages via the *JoinByFace* operator reduced this count to eight (Boguslawski and Gold 2016).

To quantify the storage requirements of the integrated 2D + 1D model, the dataset shown in Figure 12 was analyzed, consisting of 32 internal cells and one external cell. The model contains a total of 117,000 half-edges, including 82,160 internal and 34,840 external half-edges.

Assuming a pointer size of 4 bytes, the memory required for internal half-edges is approximately 2,629,120 bytes, while the external cell requires

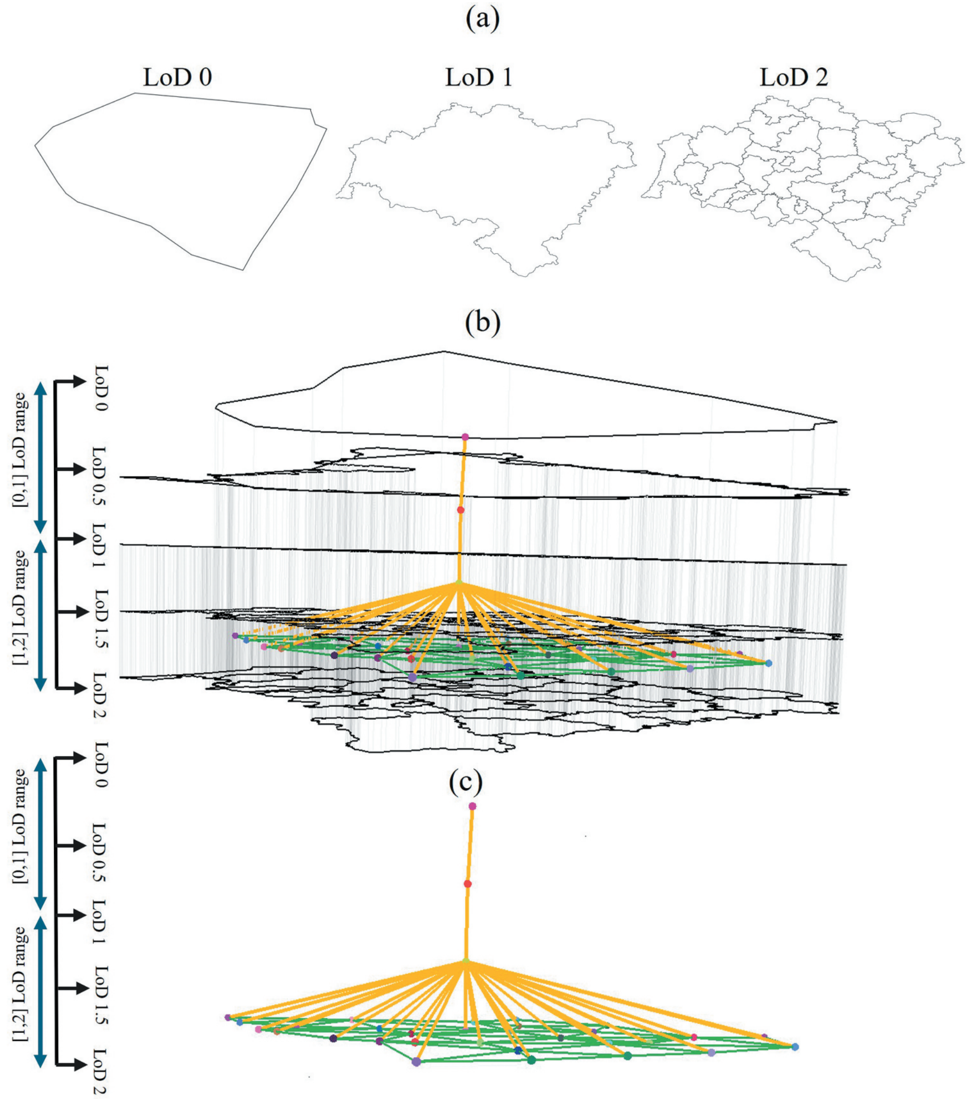

Figure 12. Construct integrated representation for second dataset, different LoD intervals of the administrative boundary layers of Poland: (a) separate representation, (b) integrated representation; and (c) intra- and inter-layer topological integration in corresponding dual space.

1,114,880 bytes. Overall, the total storage requirement of this integrated multi-scale representation is approximately 3,744,000 bytes (≈3.74 MB).

The third dataset in Figure 10(b) exhibits substantial differences: Some buildings differ significantly in geometry or semantic attributes, while others disappear entirely – due to errors and inconsistencies in the data sources. Detecting such situations is valuable for organizations responsible for data quality and change monitoring. The two datasets also show boundary misalignments, caused by differences in positional accuracy between sources; as a result, the overlay operation does not produce perfectly aligned partitions.

Despite these errors and inconsistencies, the proposed methodology can construct a valid integrated representation (Figure 13(b)). The DHE structure successfully builds the corresponding horizontal and cross-layer topological integrations, enabling the detection of invalid or missing relationships through future query analyses. For example, if a dual vertex lacks a cross-layer (non-horizontal) connection (as shown as orange lines in Figure 13(b)), this may indicate a missing building. Likewise, differences in positional accuracy can be identified in the intermediate LoD, where misaligned boundaries generate very small or narrow faces during the subdivision step. By assigning area attributes to the resulting faces in the DHE structure, such anomalous faces can be detected automatically by comparing their size to a specified tolerance.

The current framework primarily focuses on the detection of geometric anomalies (e.g. small or narrow faces) rather than their explicit geometric correction. Incorporating topology-preserving repair operations, such as area-based merging or simplification, remains a direction for future work. Nevertheless, the proposed approach mitigates this issue by assigning these small or narrow faces at the intermediate LoD to corresponding features in the LB or UB LoDs. In cases

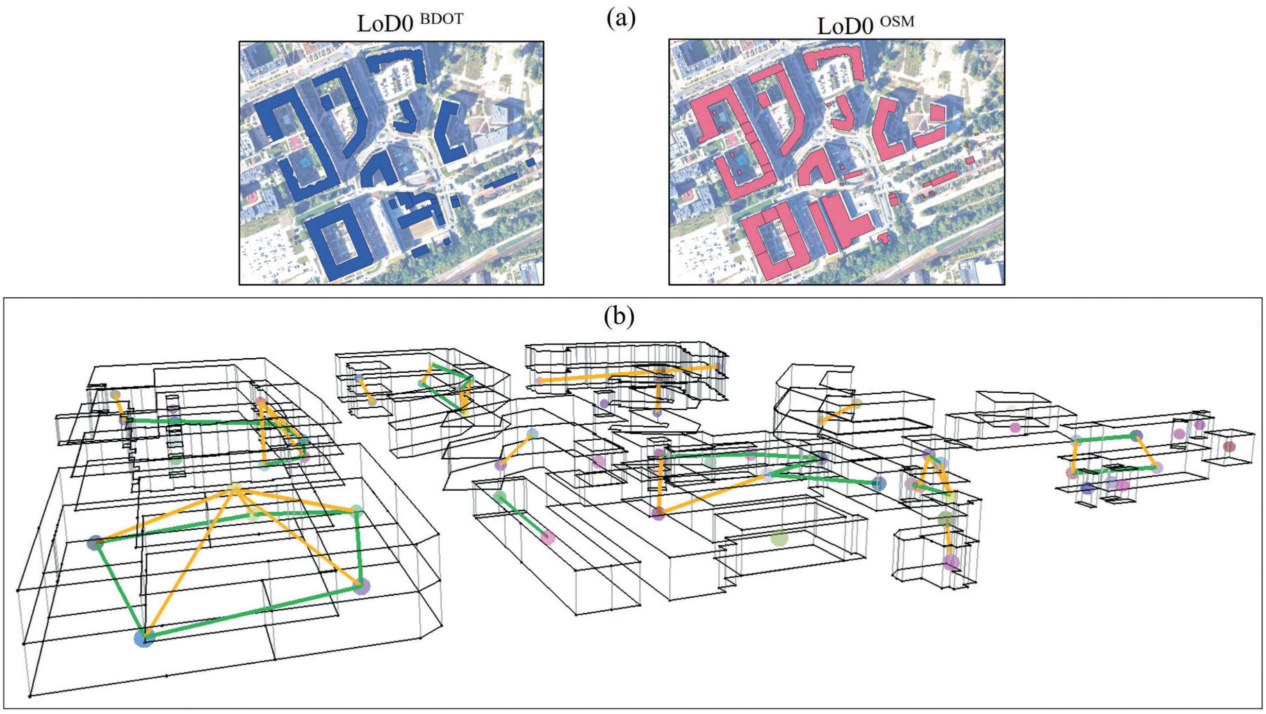

Figure 13. Construction of the methodology for integration representation of two independent multi-source datasets; (a) separate representation: $LoD0^{BDOT}$ and $LoD0^{OSM}$ building footprint layers, representing two independent sources; (b) the resulting integrated representation.

where no overlap with existing features is found, these faces are assigned to the external cell complex.

In the lower-left part of Figure 13(a), five features from the $LoD0^{OSM}$ layer are vertically integrated and linked to a single feature in the $LoD0^{BDOT}$ layer. This indicates that an aggregation relationship exists between the two independent sources. Notably, even though no direct semantic correspondence is available between these datasets, the proposed integration representation is still able to identify and construct the correct topological linkage between features. This demonstrates that the method can establish meaningful connections across heterogeneous sources, in addition to supporting integrated representation.

These forms of invalid topological integrations can serve as triggers for a user or an automated system, to support decision-making about data correction, validation, or further investigation.

Intermediate LoDs provide a practical way to link scales with different granularity by creating shared partitions that enable valid cross-scale topological integrations. This preserves adjacency and lineage without requiring a predefined hierarchy and supports heterogeneous geometries through 9IM-based overlay. Although intermediate LoDs increase complexity and may introduce redundancy when they coincide with existing layers, they remain an effective tool for achieving topologically consistent multi-scale integration across diverse datasets.

In summary, this study investigates the potential of a primal–dual data structure for extruding real-world features (0D, 1D, and 2D cell primitives) into a 3D representation, where attribute and semantic information is assigned to each feature through its corresponding dual vertex. The topological relationships between features are defined within dual space, through connections established between their dual vertices; these together describe cross-feature, cross-scale, and compound topological integrations. Consequently, by extruding these 3D cells along an additional 1D dimension, a 4D representation becomes achievable, laying the groundwork for future extensions toward fully integrated spatio-temporal and multi-dimensional vario-scale modeling.

5. Conclusions

This paper presents a methodology for constructing a 2D + 1D integrated multi-scale representation through the topological integration representation of spatial datasets, using a primal–dual data structure to unify geometry and relationships across layers. The method introduces the concept of generating intermediate LoDs from two input layers and using this shared partition to establish explicit horizontal (within-layer) and non-horizontal (cross-layer) topological integration. By eliminating the need for a preexisting hierarchical model such as tGAP, the approach offers a flexible, data-driven solution for integrating multi-scale spatial datasets.

The methodology unifies multiple discrete layers of spatial data into a single, topologically consistent framework, establishing valid cross-feature (horizontal) and cross-scale (non-horizontal) integration through the creation of intermediate LoDs. The resulting deeply integrated representation reduces geometric redundancy by merging shared faces and removing unused topological elements, yielding a more compact and efficient model.

Furthermore, this study investigates the potential of the primal–dual data structure for extruding real-world features into a 3D representation, where attribute and semantic information are assigned to each feature through its corresponding dual vertex. While the proposed method is demonstrated using the Dual Half-Edge (DHE) structure, it is not limited to it. The framework can be implemented with other topological or non-topological data structures, as its core novelty lies in generating intermediate layers to establish cross-scale correspondences without a preexisting hierarchical model. The DHE structure, however, offers unique advantages: its primal space defines the geometric integrated multi-scale representation, while its dual space captures the tree structure, integrating both aspects into a unified representation.

Results across all datasets show that the method preserves topology, supports semantic propagation, and enables cross-scale integration as well as inconsistency detection. While the integrated representation increases storage requirements due to the generation of intermediate cells, the deeply integrated representation decreases the number of entities and primitives, thereby reducing the overall storage requirements.

Several directions for future research emerge from this work. The proposed framework can be extended to support additional generalization operations, including simplification, aggregation, displacement, and typification. Further developments may include the design of new DHE-based construction operators for both classic and deeply integrated models and the incorporation of smooth SSC representations through inclined faces. Another promising direction is the application of the framework to temporal data in order to unify spatial and temporal scales. Future studies should also investigate methods for validating generalized layers and ensuring topological consistency across scales. Finally, the methodology could be extended toward fully integrated 3D + 1D (4D) representations by extruding 3D volumes along an additional dimension.

Note

1. Topographic Objects Database (BDOT10k).

Acknowledgments

The authors thank Prof. Peter van Oosterom for his valuable insights, advice, and constructive discussions regarding this research. The article is part of a PhD dissertation titled "New Multidimensional Data Structure for Vario-scale Spatial Model Representation", prepared during the Doctoral School at the Wrocław University of Environmental and Life Sciences. The APC/BPC is financed by Wrocław University of Environmental and Life Sciences.

Author contributions

CRediT: **Amin Gholami:** Conceptualization, Formal analysis, Investigation, Methodology, Software, Validation, Visualization, Writing – original draft; **Pawel Boguslawski:** Conceptualization, Funding acquisition, Methodology, Project administration, Supervision, Writing – review & editing; **Martijn Meijers:** Methodology, Resources, Supervision, Writing – review & editing.

Disclosure statement

No potential conflict of interest was reported by the author(s).

During the preparation of this work the authors used ChatGPT (OpenAI) in order to improve the clarity, organization, and language of the manuscript. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

Funding

This work was supported by the National Science Centre (NCN), Poland [Grant number 2021/41/B/ST10/03178].

Notes on contributors

Amin Gholami is a doctoral researcher at the Institute of Geodesy and Geoinformatics, Wroclaw University of Environmental and Life Sciences (UPWr). His research focuses on developing a topological data structure to support multidimension representation.

Pawel Boguslawski is an associate professor at the Institute of Geodesy and Geoinformatics, Wroclaw University of Environmental and Life Sciences (UPWr). He is an expert in topological data structures and the developer of the DHE model.

Martijn Meijers is an assistant professor at the Faculty of Architecture and the Built Environment, Delft University of Technology (TU Delft). He specializes in automated map generalization and multi-scale spatial modeling.

ORCID

Amin Gholami http://orcid.org/0000-0002-2792-4962
Pawel Boguslawski http://orcid.org/0000-0002-1914-9214
Martijn Meijers http://orcid.org/0000-0003-0923-5423

Data availability statement

The research findings were supported by three distinct datasets:

(1) **Synthesized Mock Data**: A dataset created by the authors, inspired by the vario-scale modeling principles and scenarios proposed by van Oosterom and Meijers (2014) (https://doi.org/10.1080/13658816.2013.809724).
(2) **National Authoritative Data**: Administrative boundaries and BDOT10k datasets obtained from the Polish Head Office of Geodesy and Cartography (GUGiK) via the national geospatial portal (https://mapy.geoportal.gov.pl/).
(3) **Crowdsourced Data**: OpenStreetMap (OSM) datasets retrieved via the Overpass Turbo API (https://overpass-turbo.eu/).

The processed topological models and the specific configurations used for model construction in this study are available from the corresponding author, Amin Gholami, upon reasonable request.

References

Ai, T., and J. Li. 2009. "The Lifespan Model of GIS Data Representation Over Scale Space." *Proceedings of the 17th International Conference on Geoinformatics*, 1–6. Fairfax, VA, USA: IEEE. https://doi.org/10.1109/GEOINFORMATICS.2009.5293369.

Ai, T., H. Wang, and Y. Liu. 2002. "Multi-Scale Representation of Building Feature in Urban GIS." *Geo-Spatial Information Science* 5 (2): 37–44. https://doi.org/10.1007/BF02833884.

Alumbaugh, T. J., and X. Jiao. 2005. "Compact Array-Based Mesh Data Structures." In *Proceedings of the 14th International Meshing Roundtable*, edited by B. W. Hanks, 485–503. Berlin/Heidelberg: Springer. https://doi.org/10.1007/3-540-29090-7_29.

Anton, F., P. Boguslawski, and D. Mioc. 2014. "The Dual Half-Arc Data Structure: Toward a Universal B-Rep." In *Lecture Notes in Geoinformation and Cartography*, edited by Alias Abdul Rahman, Pawel Boguslawski, François Anton, Mohamad Nor Said, and Kamaludin Mohd Omar, 103–117. Cham: Springer. https://doi.org/10.1007/978-3-319-03644-1_7.

Baig, S. U., and A. Abdul-Rahman. 2013. "Generalization of Buildings within the Framework of CityGML." *Geo-Spatial Information Science* 16 (4): 247–255. https://doi.org/10.1080/10095020.2013.866617.

Boguslawski, P., and A. Gholami. 2024. "Building a 4D Cell Complex Topology Through the Extrusion Process." *Intnal Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciencesernatio* 48:93–98. https://doi.org/10.5194/isprs-archives-XLVIII-4-2024-93-2024.

Boguslawski, P., and C. Gold. 2016. "The Dual Half-Edge: A Topological Primal/Dual Data Structure for Cell Complexes." *ISPRS International Journal of Geo-Information* 5 (2): 19. https://doi.org/10.3390/ijgi5020019.

Breunig, M., A. Thomsen, B. Broscheit, E. Butwilowski, and U. Sander. 2007. "Representation and Analysis of Topology in Multi-Representation Databases." *Intnal Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciencesernatio* 167. https://doi.org/10.5445/IR/1000021239.

Buttenfield, B. P. 1993. *Research Initiative 3: Multiple Representations*. NCGIA Technical Report.

Cheng, L., H. Hu, L. Su, Z. Dai, L. Zhang, G. Yang, B. Xu, and Q. Zhu. 2026. "Simplifying the Bipartite Matching Topology Recovery for Vectorized Building Footprint Extraction." *International Journal of Applied Earth Observation and Geoinformation* 129:105071. https://doi.org/10.1016/j.jag.2025.105071.

Davis, C. A., and A. H. F. Laender. 1999. "Multiple Representations in GIS Through Generalization and Spatial Analysis." *Proceedings of the 7th ACM International Symposium on Advances in Geographic Information Systems*, 60–65. https://doi.org/10.1145/320134.320151.

Deng, M., Z. Li, G. Li, and X. Zhang. 2007. "The Four Intersection-And-Difference Model for Line-Line Topological Relations." *Geo-Spatial Information Science* 10 (4): 293–298. https://doi.org/10.1007/s11806-007-0095-z.

De Vries, M. D., and P. van Oosterom. 2007. "Model Generalization and Methods for Effective Query Processing in Web Architectures." In *Spatial Data on the Web: Modeling and Management*, edited by Alberto Belussi, Barbara Catania, Eliseo Clementini, and Elena Ferrari, 85–106. Berlin/Heidelberg: Springer. https://doi.org/10.1007/978-3-540-69878-4_5.

Dilo, A., P. van Oosterom, and A. Hofman. 2009. "Constrained tGAP for Generalization Between Scales: Dutch Topographic Data." *Computers, Environment and Urban Systems* 33 (6): 388–402. https://doi.org/10.1016/j.compenvurbsys.2009.07.006.

Ding, L., G. Xiao, A. Pano, M. Fumagalli, D. Chen, Y. Feng, D. Calvanese, H. Fan, and L. Meng. 2025. "Integrating 3D City Data Through Knowledge Graphs." *Geo-Spatial Information Science* 28 (2): 780–799. https://doi.org/10.1080/10095020.2024.2337360.

Egenhofer, M. J., and J. Herring. 1990. *Categorizing Binary Topological Relations Between Regions, Lines and Points*. NCGIA Technical Report 90-9.

Frank, A. U. 1987. "Overlay Processing in Spatial Information Systems." *Proceedings of the 8th International Symposium on Computer-Assisted Cartography*, 16–31.

Frank, A. U., and S. Timpf. 1994. "Multiple Representations for Cartographic Objects in a Multi-Scale Tree." *Computers & Graphics* 18 (6): 823–829. https://doi.org/10.1016/0097-8493(94)90008-6.

Gao, A., J. Li, and K. Chen. 2020. "A Morphing Approach for Continuous Generalization of Linear Features." *PLOS ONE* 15 (12): e0243328. https://doi.org/10.1371/journal.pone.0243328.

Gholami, A., P. Boguslawski, M. Meijers, and P. van Oosterom. 2024. "Evaluation of the Dual Half-Edge Data Structure for Vario-Scale Modeling." *Intnal Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciencesernatio* 48:25–32. https://doi.org/10.5194/isprs-archives-XLVIII-4-W11-2024-25-2024.

Gholami, A., P. Boguslawski, M. Meijers, and P. van Oosterom. 2025. "2D to 1D and 2D to 0D Geometry Generalization Based on the Straight Skeleton by the Dual-Half Edge Structure." *International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences* 48:513–519. https://doi.org/10.5194/isprs-archives-XLVIII-G-2025-513-2025.

Gold, C. 2010. "The Dual is the Context: Spatial Structures for GIS." In *International Symposium on Voronoi Diagrams in Science and Engineering* , edited by Mir Abolfazl Mostafavi, 3–10. Quebec, Canada: IEEE. https://doi.org/10.1109/ISVD.2010.9.

Gröger, G., and L. Plümer. 2012. "CityGML – Interoperable Semantic 3D City Models." *ISPRS Journal of Photogrammetry & Remote Sensing* 71:12–33. https://doi.org/10.1016/j.isprsjprs.2012.04.004.

Karim, H., A. Abdul Rahman, P. Boguslawski, M. Meijers, and P. van Oosterom. 2017. "Potential of the 3D DHE Structure for 2D-Space and Scale Modelling: A Review." In *Lecture Notes in Geoinformation and Cartography*, edited by Alias Abdul Rahman, 477–493. Cham: Springer. https://doi.org/10.1007/978-3-319-25691-7_27.

Karim, H., A. A. Rahman, S. Azri, and Z. Halim. 2022. "Multiscale Data Management for CityGml-Based 3D Buildings." *Geomatics and Environmental Engineering* 16 (1). https://doi.org/10.7494/geom.2022.16.1.71.

Kutzner, T., K. Chaturvedi, and T. H. Kolbe. 2020. "CityGML 3.0: New Functions Open Up New Applications." *PFG – Journal of Photogrammetry, Remote Sensing and Geoinformation Science* 88 (1): 43–61. https://doi.org/10.1007/s41064-020-00095-z.

Lienhardt, P. 1989. "Subdivisions of N-Dimensional Spaces and Generalized Maps." *Proceedings of the 5th Annual Symposium on Computational Geometry*, edited by Kurt Mehlhorn, 228–236. Saarbrücken, Germany: ACM Press. https://doi.org/10.1145/73833.73859.

Liu, Y., L. Jiao, and Y. Liu. 2011. "Effects of Scale and Land-Use Metrics on Generalization Indices." *International Journal of Applied Earth Observation and Geoinformation* 13 (3): 346–356. https://doi.org/10.1016/j.jag.2011.01.002.

Meijers, B. M. 2011. *Variable-Scale Geo-Information*. Delft, The Netherlands: NCG Delft.

Meijers, B. M., and P. J. M. van Oosterom. 2011. "The Space–Scale Cube: An Integrated Model for 2D Areas and Scale." *Intnal Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciencesernatio* XXXVIII-4/C21:95–102. https://doi.org/10.5194/isprsarchives-XXXVIII-4-C21-95-2011.

Niu, F., and C. Cheng. 2012. "Novel Approaches to the Multi-Scalar Display of Vector Data." *Geo-Spatial Information Science* 15 (4): 251–261. https://doi.org/10.1080/10095020.2012.714102.

Ohori, K. A., H. Ledoux, F. Biljecki, and J. Stoter. 2015. "Modeling 3D City LoDs as a True 4D Model." *ISPRS International Journal of Geo-Information* 4 (3): 1055–1075. https://doi.org/10.3390/ijgi4031055.

Oude Elberink, S., and G. Vosselman. 2011. "Quality Analysis on 3D Building Models Reconstructed from Airborne Laser Scanning Data." *ISPRS Journal of Photogrammetry & Remote Sensing* 66 (2): 157–165. https://doi.org/10.1016/j.isprsjprs.2010.09.009.

Paul, N., P. E. Bradley, and M. Breunig. 2013. "Integrating Space, Time, Version and Scale Using Alexandrov Topologies." *arXiv Preprint arXiv: 1303 2595*. https://doi.org/10.48550/arXiv.1303.2595.

Qiang, Y., and N. Van de Weghe. 2019. "Re-Arranging Space, Time and Scales in GIS: Alternative Models for Multi-Scale Spatio-Temporal Modeling and Analyses." *ISPRS International Journal of Geo-Information* 8 (2): 72. https://doi.org/10.3390/ijgi8020072.

Salinas-Fernández, S., J. Fuentes-Sepúlveda, and N. Hitschfeld-Kahler. 2024. "Polygonal Meshes in Compact Space." In *Lecture Notes in Computational Science and Engineering*, 3–24. Cham: Springer. https://doi.org/10.1007/978-3-031-40594-5_1.

Steiniger, S., and R. Weibel. 2007. "Relations Among Map Objects in Generalization." *Cartography and Geographic Information Science* 34 (3): 175–197. https://doi.org/10.1559/152304007781697866.

Stoter, J., L. van den Brink, J. Beetz, H. Ledoux, M. Reuvers, P. Janssen, F. Penninga, G. Vosselman, and S. Oude Elberink. 2013. "Three-Dimensional Modeling with National Coverage: Case of the Netherlands." *Geo-Spatial Information Science* 16 (4): 267–276. https://doi.org/10.1080/10095020.2013.866619.

Thomsen, A., and M. Breunig. 2007. "Topological Abstraction in Multi-Representation Databases." In *Lecture Notes in Geoinformation and Cartography*, edited by Alexander Popovich, Peter Schrenk, and Konstantin Korolenko, 234–251. Berlin/Heidelberg: Springer. https://doi.org/10.1007/978-3-540-37629-3_18.

van Oosterom, P. 1994. "An R-Tree Based Map-Overlay Algorithm." *Proceedings of EGIS*, 318–327. Vol. 94, Paris.

van Oosterom, P. 2005. "Variable-Scale Topological Data Structures: GAP-Face Tree and GAP-Edge Forest." *Cartography and Geographic Information Science* 32 (4): 331–346. https://doi.org/10.1559/152304005775194782.

van Oosterom, P., and M. Meijers. 2014. "Vario-Scale Data Structures for Smooth Zoom and Progressive Transfer." *International Journal of Geographical Information Science* 28 (3): 455–478. https://doi.org/10.1080/13658816.2013.809724.

van Oosterom, P., J. Stoter. 2010. "5D Data Modelling: Full Integration of 2D/3D Space, Time and Scale Dimensions." In *Geographic Information Science: GIScience 2010*, edited by S. I. Fabrikant et al., 310–324. Berlin/Heidelberg: Springer. https://doi.org/10.1007/978-3-642-15300-6_22.

Yang, B., M. Zhang, Z. Zhang, Y. Zhao, and J. Gong. 2024. "UniVecMapper: A Universal Model for Thematic and Multi-Class Vector Graph Extraction." *International Journal of Applied Earth Observation and Geoinformation* 128:103915. https://doi.org/10.1016/j.jag.2024.103915.

Zhang, X., W. Yin, M. Yang, T. Ai, and J. Stoter. 2018. "Updating Authoritative Spatial Data Using Timely Sources." *International Journal of Applied Earth Observation and Geoinformation* 72:42–56. https://doi.org/10.1016/j.jag.2018.05.022.